SLAC-PUB-974
(TH) and (EXP)
October 1971

MULTIPARTICLE PROCESSES AT HIGH ENERGY*

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ABSTRACT

Unlike the study of two-body scattering processes, analysis of multiparticle phenomena will always involve a subjective element: the selection from a vast amount of primary data of some small subsample comprehensible to the human mind. Therefore diverse methods of viewing the primary data are of crucial importance. In pursuit of this goal we suggest light-cone variables and rapidity as useful variables, and use of 3-dimensional models for analyses of multibody events. We also discuss a possible analogue to statistical mechanics and the theory of liquids invented by Feynman and formally developed by Mueller which may be applicable to multiparticle phenomenology.

In this talk I shall consider the question of multibody phenomena from a very global point of view. Consequently the material is "soft"; I will not write down formulae with which data can be compared. In fact, after listening to the presentations of concrete ideas and hard facts here by people who have done a great deal of work, I feel a sense of chagrin in offering so little in this talk. Much of the inclusive-reaction data presented here will serve as an important cornerstone on which future work will build.

Nevertheless, before getting too deeply into the business of searching for ways to understand multiparticle phenomena, I feel it is still important to keep clearly in mind the nature of the problem, which is quite different from that of two-body phenomena. For two-body processes, the primary data $d\sigma/dt\,(s,t)$ can, in principle at least, be comprehended, i.e., placed on the pages of Physical Review. This is not true of primary data for multiparticle processes, which are a vast set of functions of many variables[1]

$$\rho_n(p_1 \cdots p_n) = E_1 \cdots E_n \,\frac{d\sigma(n,s)}{d^3p_1 \cdots d^3p_n} \tag{1}$$

Even were these ρ_n measured, we just could not display all the information in a way the human mind can comprehend. Of course, there may come the day when Physical Review is no longer published as sheets of paper and is instead published on magnetic tape and read only by computers. Although that may lead to a rise in its circulation, I do not look forward to that day.

In this conference we have seen many slides of multiparticle data, and what are they? They are mappings of the primary data, Eq. (1), into a

*Work supported by the U. S. Atomic Energy Commission.

(American Physical Society Divisional Meeting, Rochester, New York, August 30 - Sept. 2, 1971.)

space of one dimension (or occasionally two):

$$S(x) = \sum_n \int \frac{d^3p_1}{E_1} \cdots \frac{d^3p_n}{E_n} \rho_n(p_1 \cdots p_n)\, S_n(p_1 \cdots p_n, x) \equiv \mathcal{S}\rho \qquad (2)$$

where $\mathcal{S}$ is the projection operator which produces the slide S(x).

It is clear the first problem is not to fit the resultant function S(x) to a theory, but to be sure that we have chosen the projection operator $\mathcal{S}$ in the wisest possible way.

Theorists with insight and wisdom do propose functions S which reveal important and fundamental features of multiparticle processes. These include the inclusive distribution-functions, which are certain to play a central role in our understanding of these phenomena. But basically the main burden of choice still falls on the experimentalist, for he has the primary contact with the facts. The experimentalist is the one who has the ears. He is the one who can listen to what the data is saying, provided he understands the language and there isn't too much background noise. The theorist has only a mouth, and sometimes a mind. What then can the theorist do? First, he should invent some hearing-aids for the experimentalists. One of the most successful has been the Dalitz-plot. Another example is the Van Hove longitudinal phase-space plot. Another (invented by an experimentalist, Al Erwin) is the building of 3-dimensional models of multibody events; I will suggest a variant of that later on. Another hearing-aid is the judicious choice of variables. Rapidity is a good example, again one I shall return to. The theorist may also try to invent analogies with other multibody phenomena which are more familiar, albeit equally uncomputable, to provide inspiration and insights otherwise hard to obtain. An example of this is the so-called Feynman gas,[2] which should be called Feynman liquid. I shall also return to that. Finally there is no reason the theorist shouldn't continue in his traditional task of building models of the dynamics. But it seems to me that the virtues in making model calculations are largely lost when the theoretical multiparticle cross sections are then mapped by the projection-operator $\mathcal{S}$ into the function S(x) in order that you and I can compare theory with the data. It would be better to see the full prediction of the theory directly compared with the data, without any such human element intervening. One can dream of a procedure in which there is first generated from the theory a large set of events which are then directly compared by the computer with the observed events. The goal would be to determine where in the multidimensional phase-space theory and experiment disagree most violently. The computer would then optimize the choice of a projection-operator $\mathcal{S}$ whose purpose would be to most dramatically display to us limited humans the nature of the disagreement. In this way we might discover useful projection-operators $\mathcal{S}$ as well as the necessary revisions of our theoretical ideas. While this dream is probably unrealistic, something much better than the present one-dimensional projections should be possible.

With this general prologue to set the tone, it is time to get down to specifics. They will be: (1) A choice of variables, (2) A way of looking at multibody events, and (3) an analogy with ordinary many-body systems. The presentation is based mainly on work of Feynman,[2] Wilson,[2] and Mueller.[3] We shall first discuss light-cone variables and a variant of the now-familiar rapidity variable, along with a suggestion of how to build 3-dimensional models of individual multibody events.[4] Then we shall go on to discuss the concepts of short-range correlation and short-range mobility and the analogy between multiparticle populations in momentum space and populations of ordinary atoms in configuration space. This is the Feynman liquid,[2] and it suggests a variety of ways of looking at multibody phenomena. In this connection we shall discuss a formalism for inclusive distribution functions developed by Mueller[3] which points toward a thermodynamics of hadron phenomena.

I. LIGHT-CONE VARIABLES AND RAPIDITY

The trajectory of a high energy particle lies very near the light-cone, and it is natural to consider a rotation of coordinates to the light-cone. This idea occurred to Dirac long ago and has been found convenient many times in the interim for many theoretical reasons (Table I). But there may

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Table I Some theoretical uses of light-cone variables[5]

P.A.M. Dirac	Classical equations of motion
S. Fubini	Multiperipheral-model calculations
V. Sudakov	Calculation of Feynman diagrams
V. Gribov and A. Migdal	"Reggeon calculus"; properties of Pomeranchuk poles and cuts
H. Leutwyler J. Jersak and J. Stern	Derivation of Adler sum rule
L. Susskind K. Bardacki and M. Halpern	Representations of local algebra of charge densities at infinite momentum
H. Cheng and T. T. Wu S. Chang and S. Ma J. Kogut and D. Soper F. Rohrlich	Quantum electrodynamics at high energies
R. Brandt J. Cornwall and R. Jackiw D. Gross and S. Treiman	Light-cone current algebra

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be a few practical reasons as well[6] for using them. In momentum space, write

$$\eta = \frac{E+p_z}{2}\ , \qquad H = E-p_z = \frac{p_\perp^2+m^2}{2\eta} \tag{3}$$

These rotated coordinates have nice properties:

1. Under longitudinal boosts

$$\eta \to \eta e^{\omega} \qquad H \to He^{-\omega} \qquad p_\perp \to p_\perp \tag{4}$$

2. Covariant calculations can still be used. The metric tensor is off-diagonal; if

$$P_\mu = (H, p_x, p_y, \eta)$$

then

$$P^\mu = (\eta, -p_x, -p_y, H) \qquad P_\mu P^\mu = M^2 \tag{5}$$

3. There is an analogy to a nonrelativistic system in two dimensions:

$$\begin{aligned} p_\perp &\longleftrightarrow \text{2-dimensional momentum} \\ H &\longleftrightarrow \text{energy} \\ \eta &\longleftrightarrow \text{mass} \\ \frac{m^2}{2\eta} &\longleftrightarrow \text{constant potential energy} \end{aligned} \tag{6}$$

4. The analogy goes deeper; Lorentz-invariance alone implies Galilean invariance in these variables; i.e., invariance under the infinitesimal transformation

$$\begin{aligned} p_\perp &\to p_\perp + \eta\, \delta \underset{\sim}{v} \\ \eta &\to \eta \\ H &\to H + \underset{\sim}{p_\perp} \cdot \delta \underset{\sim}{v} \end{aligned} \tag{7}$$

Aside from being a possible source of theoretical ideas based on analogy with 2-dimensional nonrelativistic quantum mechanics, there are some minor kinematical advantages. For example the invariant mass of a particle pair with coordinates $(H_1, \underset{\sim}{p_1}, \eta_1)$ and $(H_2, \underset{\sim}{p_2}, \eta_2)$ is, from the analogy

$$H = \frac{P^2+M^2}{2\eta} = \underbrace{\frac{(p_1+p_2)^2}{2(\eta_1+\eta_2)}}_{\substack{\text{kinetic}\\ \text{energy of}\\ \text{center-of-}\\ \text{mass motion}}} + \underbrace{\left(\frac{m_1^2}{2\eta_1} + \frac{m_2^2}{2\eta_2}\right)}_{\substack{\text{"potential}\\ \text{energy"}}} + \underbrace{\frac{1}{2} \underbrace{\left(\frac{\eta_1 \eta_2}{\eta_1+\eta_2}\right)}_{\substack{\text{reduced}\\ \text{"mass"}}} \underbrace{\left[\frac{p_1}{\eta_1} - \frac{p_2}{\eta_2}\right]^2}_{\substack{\text{relative}\\ \text{velocity}}}}_{\text{internal kinetic energy}} \tag{8}$$

For pions, the "potential energy" can usually be neglected, and the mass of the pair M is

$$M = \left| \mathbf{p}_1 \sqrt{\frac{\eta_2}{\eta_1}} - \mathbf{p}_2 \sqrt{\frac{\eta_1}{\eta_2}} \right| \tag{9}$$

Just multiply $\mathbf{p}_1$ by the scale factor $\sqrt{\eta_2/\eta_1}$, divide $\mathbf{p}_2$ by the same factor, and take the vector difference.

5. The phase-space element is, without approximation

$$\frac{d^3p}{E} = d^2p_\perp \frac{d\eta}{\eta} = d^2p_\perp \, dR \tag{10}$$

We take

$$R = \log \eta \qquad \eta \text{ in GeV} \tag{11}$$

It is closely related to conventional rapidity[7]

$$y = \frac{1}{2} \log \frac{\eta}{H} = \log \frac{\eta}{\sqrt{p_\perp^2 + m^2}} = R - \frac{1}{2} \log \left(p_\perp^2 + m^2\right) \tag{12}$$

and has the same virtue of undergoing a simple displacement under longitudinal boosts

$$R \rightarrow R + \omega \tag{13}$$

The phase-space volume available to secondary particles is shown in Fig. 1.

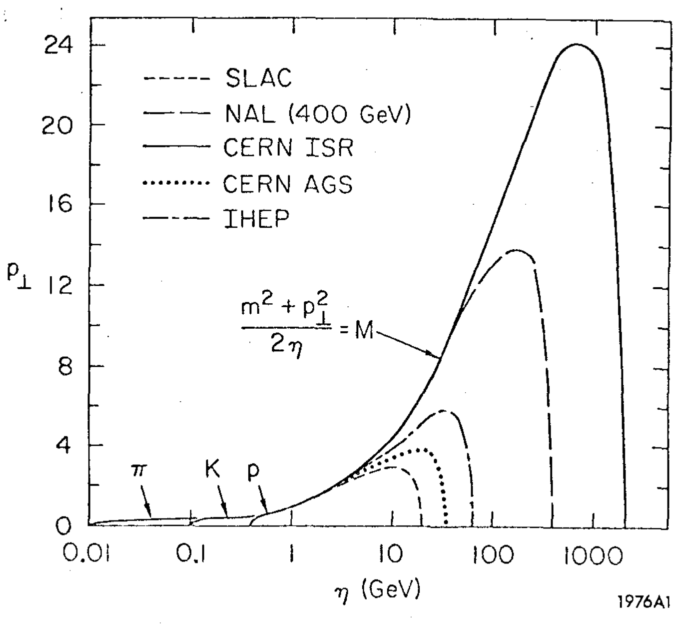


Fig. 1. Phase-space volume provided by various accelerators.

6. R is possibly convenient in that only p_{lab} and β determines R, while computation of y involves m_t; this may make the fast π^+/K^+ problems slightly easier.

7. Particle distributions are reasonably uniform in R; there is no crowding.

8. Conservation laws at high energy become

$$\sum_i H_i = M \qquad \text{(contributed mainly by target fragments)}$$
$$\sum_i \eta_i = \eta_{inc} \qquad \text{(contributed mainly by projectile fragments)} \tag{14}$$

The target and projectile kinematics are largely decoupled.

These last comments are very minor considerations. Perhaps the most major consideration of all this is to think of longitudinal momentum in multiplicative terms. The natural unit is the decibel, or possible the octave.[8]

II. THREE-DIMENSIONAL MODELS

Erwin[4] tried to visualize multiprong events by making models of them out of pipe-cleaners jabbed into little balls. I think he has abandoned that. But the light-cone variables suggest a variation of his model which I think is an improvement.

The idea, of course, is that our human powers of visualization and pattern-recognition in three dimensions are very good. So maybe we can discern regularities which might otherwise be missed, and by this subjective method be led to ask (not answer) objective questions. Such questions could lead to invention of other projection-operators $\mathcal{S}$ to reveal the content of the data.

I first tried to make the models out of drinking straws with toothpicks jabbed into them (toothpick length = $p_\perp$; position of the jab = rapidity). But it seems to be better to make a two dimensional picture of such a model (see Figs. 2-4). Just lay out an η-axis with a slide rule on a linear grid (for $p_\perp$), and put a vector $p_\perp$ with base at its η for each secondary particle. Be sure to color protons green.

A bonus is that because of the Galilean invariance, masses of pairs of pions can be simply estimated graphically, using Eq. (9), and read off the picture. Just

a. Measure half the ΔR with a slide rule. That gives the scale factor $\sqrt{\eta_1/\eta_2}$ in Eq. (9).

b. Scale up the lesser vector p_2 (with smallest η).

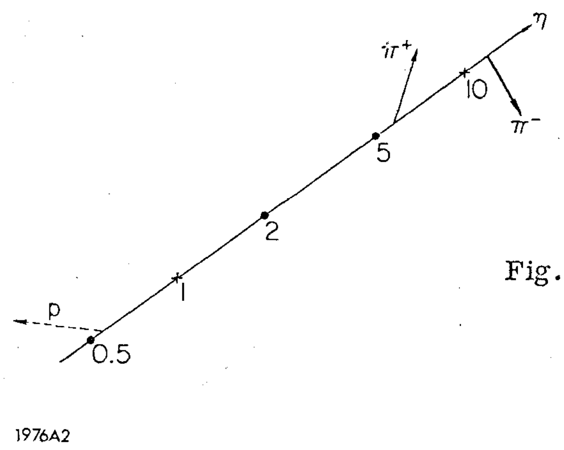
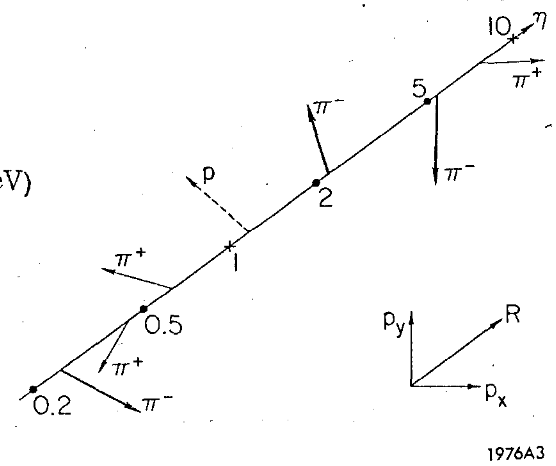


Fig. 2. $\gamma p \rightarrow \rho^o p \rightarrow \pi^+\pi^- p$ $(E_\gamma = 18\ \text{GeV})$

Fig. 3. $\gamma p \rightarrow (3\pi^+)(3\pi^-)p$ $(E_\gamma = 18\ \text{GeV})$



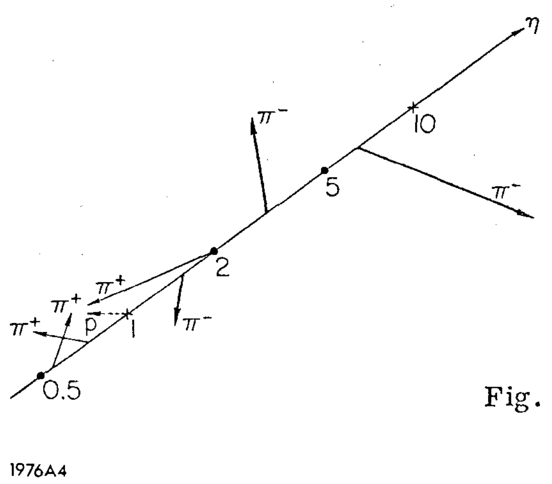


Fig. 4. $\gamma p \rightarrow (3\pi^+)(3\pi^-)p$ $(E_\gamma = 14.2\ \text{GeV})$

c. Scale down the greater vector $\underset{\sim}{p_1}$.

d. Take the vector difference and measure. That's the mass.

Without much effort, John Kogut, Dave Soper and I make about 100 such pictures of the reaction $\gamma p \to (3\pi^+)(3\pi^-)p$ supplied by the SLAC streamer chamber group.[9] While I know I'm wiser for the experience, I cannot express why.

III. CORRELATIONS, MOBILITY, AND FEYNMAN'S LIQUID

The problem of understanding the functions $\rho_n(p_1 \ldots p_n)$ is the problem of studying populations of phase-points in the momentum-space. A crucial issue is the nature of the correlations; if a secondary particle is known to be at (p_1, η_1), how does that influence the probability of finding another at (p_2, η_2)? The presently popular hypothesis of short-range correlation is suggested by Mueller's work[10] and by the notion that resonant two-body interactions are most prominent in the low energy region and may be the dominant mechanism of providing correlation. Duality would imply non-Pomeron t-channel exchanges play the same role as well, emphasizing low subenergies.

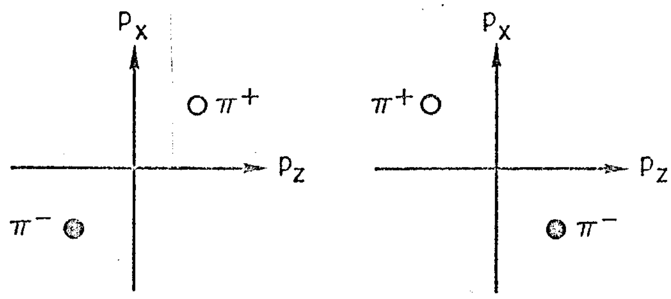
When the Δy or ΔR between two phase-points is small, the invariant mass or subenergy of the pair is generally small because of the observed bounded $\langle p_\perp \rangle$. For pions one can estimate that if $\Delta y \lesssim 2$ it is very probable that this subenergy lies below 1 GeV; $\Delta y = 2$ may be a good measure of correlation length (provided the concept is right). Notice also that $\Delta y = 2.303$ is 10 db, or an order of magnitude in the ratios of η's. To get one more correlation-length of longitudinal phase-space we need to increase the beam energy about 1 order of magnitude. Another way of estimating this number is from the Regge estimate $\sim s^{-1/2} \sim e^{-1/2\, \Delta y}$. The short-range order idea can also be expressed in terms of diagrams (dispersion, Feynman, fishnet or whatever). In most diagrams (excluding Pomeron exchange), if lines are close together they have low subenergy; if they are far away (many vertices in between) they have large subenergy. If very many diagrams are contributing, the information that a particle was emitted at some distant point gets lost in all the confusion in between. Thus when the separation Δy is large compared to 2 (or 10 db), there are so many paths composed of successions of low energy interactions that connect them it is reasonable to assume any correlations get washed out.[11]

Within a correlation-length, there are mechanisms for easily moving the phase-points around. For example, a low energy scattering of two pions originally at 45° in their center-of-mass to 135° (Fig. 5) implies a change in their y or R of ≈ 2 units. Likewise decay pions from ρ's of a given momentum spread over two units of R or y. The isotropic decay of a "fireball" into pions of a few hundred MeV produces an approximately Gaussian distribution[12] of ≈ 10 db or 2.2 units of y. We can call all this short-range mobility.

The idea of short-range correlation already leads to the hypothesis of limiting fragmentation and the existence of the central plateau in rapidity space.[13] Far from the boundaries of rapidity space the particle density

$$\Delta y = \log \frac{1+\cos \pi/4}{1-\cos \pi/4} \sim 2$$

Fig. 5. Example of short-range mobility in rapidity: low energy $\pi\pi$ rescattering.



should become constant; near the boundary of phase-space the density should depend only on the distance to the boundary. Indeed the picture looks quite analogous to that of a liquid or gas in ordinary configuration space, an analogy which can be precisely formulated.

The existence of short-range mobility leads to the conclusion that rapidity-distributions for pions should be smooth within the 10 db correlation length, except possibly near phase-space boundaries. This should be true even if the hypothesis of short-range correlation is not exact.

IV. MUELLER'S OTHER PAPER[3]

Mueller precisely defines a hypothesis of short-range correlation. He starts with the general inclusive distribution functions and proposes they have the property they factorize

$$\frac{d\sigma}{dy_1 \ldots dy_m} = F_a(y_1, \ldots y_j)\, F_b(y_{j+1}, \ldots y_m) \qquad \{y_i\}_a \gg \{y_j\}_b \tag{15}$$

when the group $\{y_1, \ldots y_j\}$ is far away (much farther than the correlation length) from the group $\{y_{j+1}, \ldots y_m\}$ in rapidity-space. This leads to cluster-decomposition properties familiar from the theory of many-body systems[14]:

$$\frac{d\sigma}{dy_1} = \sigma_{tot}\, f(y_1) \qquad \frac{d\sigma}{dy_1 dy_2} = \sigma_{tot}\left[f(y_1)\, f(y_2) - f(y_1, y_2)\right]$$

$$\frac{d\sigma}{dy_1 dy_2 dy_3} = \sigma_{tot} \left\{ \begin{matrix} f(y_1)\, f(y_2)\, f(y_3) + f(y_1, y_2)\, f(y_3) + \text{perm.} \\ + f(y_1, y_2, y_3) \end{matrix} \right\}$$

etc. (16)

The cluster functions $f(y_1, \ldots y_m)$ have the property that they vanish rapidly if any two phase-points y_i and y_j are separated by a distance large compared to the correlation length.

This looks very similar to statistical mechanics, with y-$p_\perp$ space replacing ordinary configuration space (not momentum space). And it is; when traced through, $\sigma(n, \log s)$ plays the role of partition function in the canonical ensemble. It is an integral over phase-space of the square of a matrix element, which plays the role of $e^{-\beta H}$. As usual it is better to consider the grand canonical ensemble and define the grand partition function[15]

$$Q(z, Y) = \sum_n z^n \sigma(n, Y) \tag{17}$$

where $Y = \log s$ plays the role of volume and z is the fugacity (logarithm of the chemical potential). Evidently

$$\begin{aligned} Q(1, Y) &= \sigma_{tot} \\ \left.\frac{\partial Q}{\partial z}\right|_{z=1} &= \langle n \rangle \sigma_{tot} = \int dy_1 \frac{d\sigma}{dy_1} \\ \left.\frac{\partial^2 Q}{\partial z^2}\right|_{z=1} &= \langle n(n-1) \rangle \sigma_{tot} = \int dy_1 dy_2 \frac{d\sigma}{dy_1 dy_2} \\ &\text{etc.} \end{aligned} \tag{18}$$

Thus it is clear from (16) that Q can be determined from the cluster functions f. Doing the calculation gives

$$\begin{aligned} \log Q(z, Y) &= \log \sigma_{tot} + (z-1) \int dy_1 f(y_1) + \frac{(z-1)^2}{2!} \int dy_1 dy_2 f(y_1, y_2) \ldots \\ &\overset{?}{=} W(z) + Y p(z) \end{aligned} \tag{19}$$

because all the integrals over cluster functions should individually have this structure. So there could very well be[16] a thermodynamic limit:

$$p(z) \overset{?}{=} \lim_{s \to \infty} \frac{1}{Y} \log Q \tag{20}$$

and an equation of state, obtained by eliminating z in favor of density ρ via

$$\rho(z) = \lim_{s \to \infty} z \frac{\partial}{\partial z} \frac{1}{Y} \log Q \tag{21}$$

Notice that if $\sigma_{tot} \sim$ const or $(\log s)^m$, then at $z=1$

$$p = 0 \qquad \rho = \langle \rho \rangle \qquad (z = 1) \tag{22}$$

We now can compute equations of state for popular models (see Figs. 6-9).

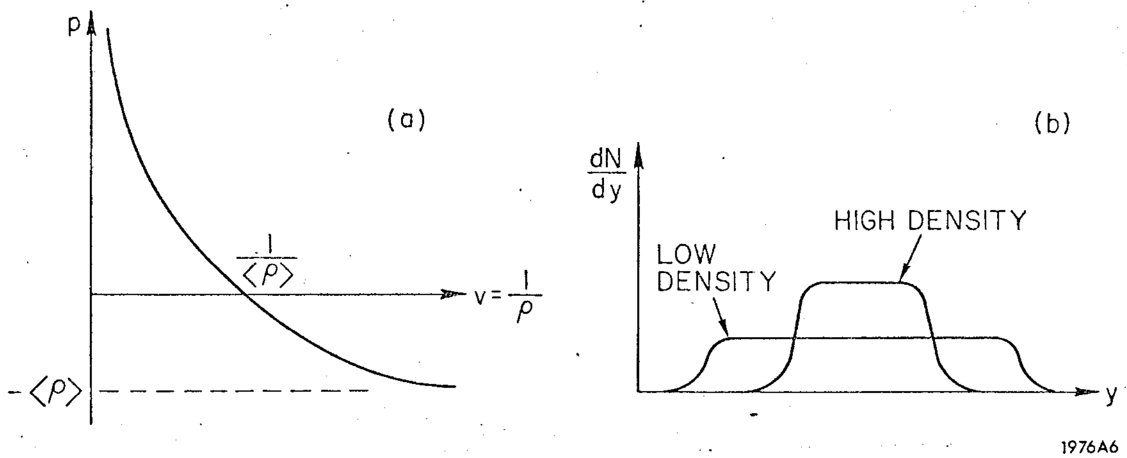


Fig. 6. (a) Equation of state for "multiperipheral" (Chew-Pignotti) cross sections:

$$\sigma(n, Y) = (gY)^n/n! \qquad\qquad p = \rho - g = \rho - \langle\rho\rangle$$

(b) Single-particle distribution in that model.

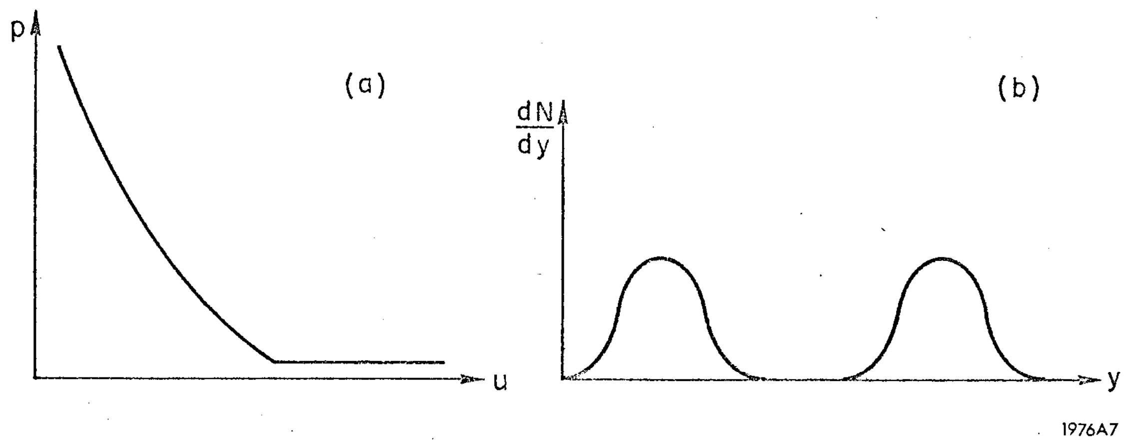


Fig. 7. (a) Equation of state for "multiperipheral" plus diffraction-dissociation:

$$\sigma(n, Y) = (gY)^n/n! + f(n)$$

In the pure diffraction-dissociation phase, $p=0; \rho=0$, corresponding to the liquid sticking to the walls; the particle distribution in that case is shown in (b).

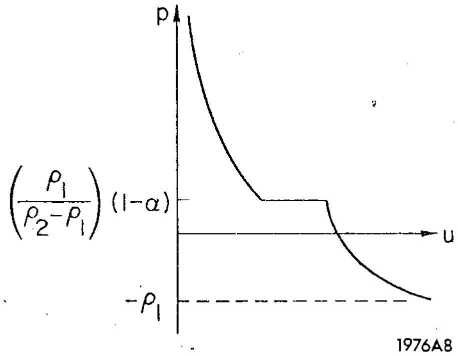


Fig. 8. Two-trajectory multiperipheral model:

$$\sigma(n, Y) = (\rho_1 Y)^n \, e^{-\rho_1 Y}/n! + \left(\frac{s}{s_0}\right)^{\alpha-1} (\rho_2 Y)^n \, e^{-\rho_2 Y}/n!$$

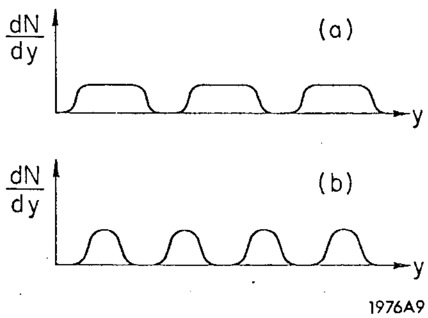


Fig. 9. (a) Can there be bubbles in the liquid? (b) Or droplets? This is multiple-Pomeron exchange.

I have my doubts about the usefulness of such equations of state, at least at present energies and multiplicities and the present level of understanding. Indeed the whole idea of short-range correlation, limiting distributions, central plateaus, etc. may be utterly wrong. But even if that is the case, the additional long-range correlations (which go as powers of rapidity instead of exponentials, and are, at least, related to Pomeron cuts, elastic scattering, and diffraction-dissociation) might be treated by analogy to effects of long-range forces in liquids (e.g., electrostatic or gravitational).

Therefore we can perhaps use the picture at least for intuition to start us in thinking up new questions to ask the data, and to invent new projection operators $\mathcal{S}$. It may also be a compact way to codify the information contained in the energy dependence of channel and topological cross sections. So what is this liquid like? Is it like water? A superfluid? Crude oil? Tar? Or something like rubber? What happens when we compress the system (decrease the energy s) at fixed particle number? Is it possible to make a crystalline solid?? That sounds utterly ridiculous and probably is. But it looks slightly better when one considers the example shown in Fig. 10. For each π in the configuration, there are seven other π's, all approximately resonating in the ρ, ϵ mass region $\approx 750 \pm 100$ MeV. It is a super-overlap of resonance-bands in a multidimensional Dalitz plot. I hesitate in even mentioning this because it is so unlikely. I have done so because even were someone to bother to have a look (and no doubt not find anything) he would have the chance of stumbling across something else interesting along the way. After all multibody physics will always be an exploratory venture. Like any good exploration, it should be entered into with a spirit of adventure, tempered by the discipline the scientific method imposes. All we can do is hope that nature rewards our efforts.

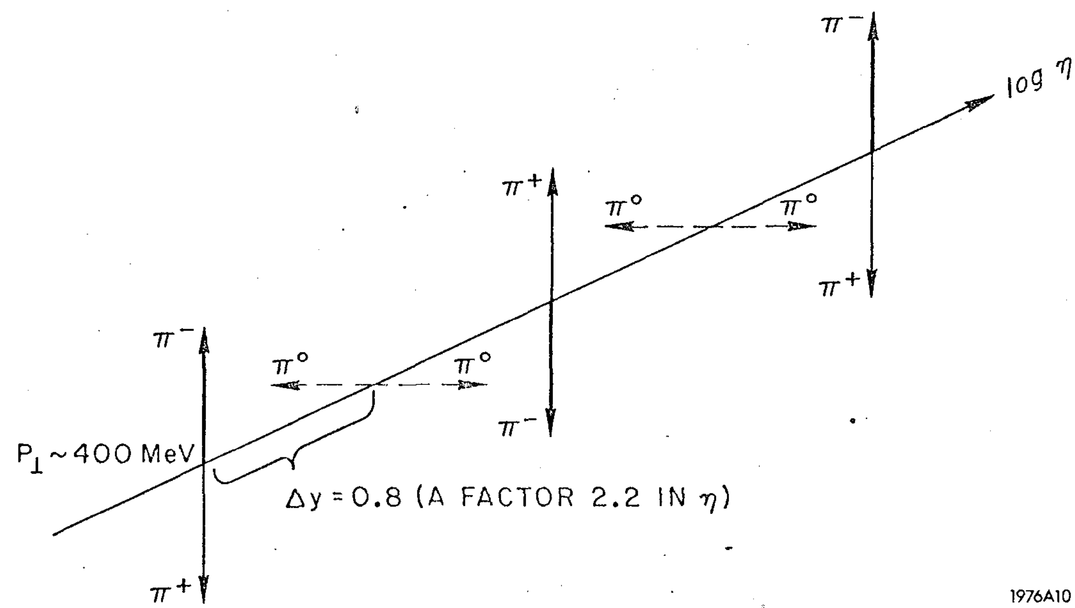


Fig. 10. Model of a possible crystalline configuration. Each π resonates, or nearly so, with seven neighbors.

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