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COMPTON SCATTERING AND FIXED POLES IN PARTON-FIELD THEORETIC MODELS†

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ABSTRACT

We extend a class of parton models to a fully gauge invariant theory for the full Compton amplitude. The existence of local electromagnetic interactions is shown to always give rise to a constant real part in the high energy behavior of the amplitude $T_1(\nu, q^2)$. In the language of Reggeisation this is interpreted as a fixed pole at $J = 0$ in T_1 and νT_2, with residue polynomial in the photon mass squared.

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Recent inelastic electroproduction experiments (which essentially measure the imaginary part of the forward off-shell Compton amplitude) hint at a composite nature for the nucleon. This has been represented by parton models involving point-like (possibly field theoretic) constituents, but up to the present time these concepts have only been applied to the scaling, incoherent impulse approximation, region. Gauge invariance and the low energy theorem place further restrictions upon such theories, and in this letter we report the extension of parton-field theoretic ideas to a discussion of the full Compton amplitude. In particular we shall see that such models always give rise to a real part at high energies additional to that expected from the Regge behavior of the imaginary part. This extra real part should be identified with the "fixed pole"[1] of conventional Regge analysis. Evidence for such a fixed pole for on-shell photons has been found phenomenologically from dispersion relations.[2] In addition we find that the "fixed pole" appears as a constant real part, C, in T_1 independent of q^2, and appears in νT_2 in the form $-Cq^2/\nu$.[3]

If the proton were as simple as the nucleus, then the high energy behavior of the Compton amplitude would follow directly from the coherent impulse approximation. At $\nu = 0$ the Compton amplitude on a nucleus is given by the Thomson limit[4] $f_1(0) = -(Z^2 \alpha/M_{nucleus})$ whereas at energies high compared to the binding energy, but below threshold for photoproduction of mesons, the forward amplitude is given by the coherent sum of the individual nucleon amplitudes, $f_1(\nu) = -\sum_{i=1}^{Z} \frac{\alpha}{\omega_i}$ $(\omega_i \simeq m_i)$. In fact, for the case of a composite proton the analogous high energy behavior would be given by the coherent sum of "seagull" terms for the individual proton constituents (quarks, bare hadrons) and the formulae (7), (11) we give later correspond to this picture.

Field theory gives us the clearest example of a fully covariant, gauge invariant, Compton amplitude which can also incorporate the composite nature of the nucleon.

As an example, consider a ϕ^3 field theory in which a scalar "proton" interacts with neutral scalar particles. We calculate the form factor $F_1(q^2)$ using old fashioned perturbation theory in an infinite momentum frame[5] by evaluating the matrix element of the electromagnetic current through second order from the time ordered contributions of the Feynman diagrams 1a-1c.

At $q^2 = 0$ we obtain

$$F_1(0) = \int_0^1 f(x)\, dx \tag{1}$$

where f(x) is the probability for the charged constituent to be in the one or two body state and to have fractional longitudinal momentum x (defined in the infinite momentum frame).[5] It is given by

$$f(x) = Z_2\, \delta(1-x) + \frac{g^2}{(2\pi)^3} \int d^2k_\perp \frac{x\,(1-x)}{D^2}$$

with $D = k_\perp^2 + x\mu^2 + (1-x)^2 M^2$ where $M(\mu)$ is the mass of the charged (uncharged) constituent and $Z_2 \equiv (1-B)^{-1} = 1 + B_{(2)} + O(g^4)$, the familiar wave function renormalization constant. From Figs. 1b, c one finds that

$$B_{(2)} = -L_{(2)} = -\frac{g^2}{(2\pi)^3} \int d^2k_\perp \int_0^1 dx\, \frac{x(1-x)}{D^2}$$

and so $F_1(0) = 1$ (a consequence of the Ward identity). One can similarly sum the contributions to Compton scattering (1d - 1h) and obtain

$$T_1(\nu, q^2) = -2 \left\{ Z_2 + \frac{g^2}{16\pi^3} \int d^2k_\perp \int_0^1 dx(1-x) \left[\frac{1}{D^2} - \sum_{\pm\nu} \frac{2k_i^2\,(1-x)}{D^2 \left[D' + x(1-x)(q_\perp^2 - 2M\nu) - i\epsilon \right]} \right] \right\} \tag{2}$$

where $D' = D(k'^2_\perp)$ with $\vec{k}'_\perp = \vec{k}_\perp - (1-x)\vec{q}_\perp$; k_i is the component of $k_\perp$ in the direction orthogonal to both $\vec{p}$ and $\vec{q}_\perp$. After integrating by parts on $dk_\perp^2$ and taking the limit $\nu \to 0$, (for q^2 $(= -q_\perp^2) = 0$), one recovers the Thomson limit,

$\lim_{\nu\to 0} T_1(\nu, 0) = -2 \equiv T_1^{Born}$. On the other hand, at large energies only the seagull terms (Figs. 1d, e, f) survive and

$$\lim_{\nu \to \infty} T_1(\nu, q^2) = -2 \left[Z_2 + \frac{g^2}{16\pi^3} \int d^2k_\perp \int_0^1 \frac{dx(1-x)}{D^2} \right] = T_1^{Born} \int_0^1 \frac{dx}{x} f(x) \tag{3}$$

We can also compute T_2 to the same order in perturbation theory and we find the following results

$$\text{(i)} \quad \lim_{\substack{\nu \to \infty \\ (-2M\nu/q^2) = \omega = \text{fixed}}} \nu W_2(\nu, q^2) \left\{ \equiv \frac{1}{2\pi M} \operatorname{Im} \nu T_2(\nu, q^2) \right\} = xf(x)\Big|_{x = 1/\omega} \tag{4}$$

$$\text{(ii)} \quad \lim_{\nu \to \infty} \nu T_2(\nu, q^2) = -\frac{q^2}{\nu} T_1^{Born} \int_0^1 \frac{dx}{x} f(x) \tag{5}$$

and the gauge invariance condition $\frac{q^2}{\nu^2} T_2 + T_1 = 0(q^2)$ is satisfied as $q^2 \to 0$.

Equations (1,3,4,5) in terms of $f(x)$ hold also for the case of spin 1/2 constituents interacting either by pseudoscalar or vector exchange, but the precise form of $f(x)$ is different for each case.[6]

These results can be generalized to all orders in perturbation theory as follows. We calculate the form factor (using the infinite momentum frame of footnote 5) by evaluating the matrix element of the j_0 current from the time ordered contributions of all Feynman graphs, through arbitrary order. The contributions can be classified according to the number of intermediate constituents present at the time the current acts (Fig. 2a) and the type of constituent, a, with charge $|e|\lambda_a$, upon which it acts. At $q^2 = 0$ we obtain[7]

$$F_1(0) = 1 = \int_0^1 dx \sum_a \lambda_a \sum_n N_a^n \, f_a^n(x) \equiv \int_0^1 dx \sum_a \lambda_a \, f_a(x) \tag{6}$$

where N_a^n is the multiplicity of parton type a in the n constituent state. Provided $Z_2 \neq 0$, $f_a^n(x)$ retains its interpretation as the probability for parton type a to have fractional longitudinal momentum x and to be in the n body state. The index a runs over both the partons and antipartons present. If $Z_2 = 0$ then $f_a^n(x)$ may lose its interpretation as a probability; $\sum_n \int f_a^n(x)\, dx$ can be infinite. The generalized seagull, or Z graph, contribution (Fig. 2b) yields a constant term in T_1 at high energy

$$T_1^{FP} = T_1^{Born} \sum_a \lambda_a^2 \int_0^1 \frac{f_a(x)}{x}\, dx = -\frac{\nu^2}{q^2} T_2^{FP} \tag{7}$$

analogous to the nuclear physics result. In the scaling limit we can identify

$$\nu W_2(x) = x \sum_a \lambda_a^2 \; f_a(x) \equiv x f(x) \quad \text{(Fig. 2c)} . \tag{8}$$

The question of convergence of Eq. (7) is very important and must be considered carefully. On the one hand it is possible that f(x) is well behaved and vanishes as $x \rightarrow 0$. In this case Regge behavior in the structure function νW_2 cannot result from the parton distribution. Eq. (7) then gives the exact parton contribution to the constant term in T_1 and $-(\nu^2/q^2)\, T_2$. On the other hand, the parton distribution for small x (i.e. wee partons) may give rise to leading Regge behavior as discussed in references (8) and (9). In such a case $f(x) \sim \sum_{\alpha > 0} x^{-\alpha} \gamma_\alpha$ for $x \sim 0$ and the integral in Eq. (7) diverges. (The integral for $F_1(0)$ does not diverge because of the cancellation of the Pomeranchon $\alpha = 1$ contribution, and in general all C = + exchanges, of a given parton with the analogous contribution of its anti-parton.) One can see from Eq. (2) that the non-seagull terms play an important role in removing the apparent divergence at x = 0. In fact, despite the presence of Regge terms, the fixed pole — constant real part — survives in a slightly altered form. To see this we now turn to the covariant, non-perturbative, parton model developed in reference (9) which allows one to incorporate leading Regge behavior in a natural way.

We restrict ourselves to spinless partons for which the required distribution function is

$$f^a(x) = \frac{Ax}{x-1} \int ds\, d^2K \,\mathrm{Im}\, T^a_R\,(s, \mu_s^2) \tag{9}$$

$\mu_s^2 = \frac{x(s-K^2)}{x-1} + xM^2 + K^2$, where K is spacelike, two dimensional and orthogonal to p and q; and μ_s^2 is the invariant four momentum squared of the interacting parton. The integral over s is over the right hand cut of the forward parton-proton scattering amplitude, $\mathrm{Im}\, T^a_R\,(s, \mu^2)$, (Fig. 3a) (which includes the propagators of the partons and is assumed to vanish as $\mu^2 \to \infty$). That f(x) is related to the forward parton-proton scattering amplitude is already apparent in the perturbation theory approach.[10] One notices that for small x $(\mu_s^2 \equiv -z + K^2)$

$$f^a(x) \to Ax^{-\alpha}\int_0^\infty dz \int d^2K z^\alpha \beta^a_\alpha(-z + K^2) = x^{-\alpha}\gamma^a_\alpha \qquad \text{if } \mathrm{Im}\, T^a_R(s, \mu^2) \sim s^\alpha \beta^a_\alpha(\mu^2).$$

One can calculate in a similar fashion the contribution for the anti-parton amplitude. Note the assumption that the parton-nucleon amplitude has normal Regge behavior. The resulting expressions for $F_1(0)$, fixed pole and νW_2 are formally the same as Eqs. (1,3,4). Note that no leading Regge exchanges in the parton-proton scattering amplitude survive in $F_1(0)$ after projection onto the J = 1, C = - quantum numbers of the photon channel. The seagull contribution (Fig. 3b) diverges as $x \to 0$ from $\alpha > 0$ contributions but there is a compensating divergence in the real parts from Figs. (3c). The compensating terms can be written (for all q^2 and ν)

$$A\int_0^1 dx \int dm^2 \rho_a(m^2) \int d^2K K^2 \int ds \left[\frac{1}{\mu_s^2 + 2xM\nu - m^2} + \frac{1}{\mu_s^2 - 2xM\nu - m^2}\right] \left\{\frac{1}{x-1}\,\mathrm{Im}\, T^a_R(s, \mu_s^2)\right\} \tag{10}$$

where $\rho_a(m^2)$ is the spectral function for the propagator of parton a. As in the second order perturbation theory case, a partial integration identity on Im T_R^a must be obeyed (in order that the low energy theorem be satisfied). One then finds that for large ν the sum of these terms and the seagull gives a constant real part to T_1 of the form

$$T_1^{FP} = \sum_a \lambda_a^2 \left\{ \int_0^1 \frac{\tilde{f}^a(x)}{x}\, dx - \sum_{\alpha > 0} \frac{1}{\alpha} \gamma_\alpha^a \right\} \times T_1^{Born} \tag{11}$$

where the $\tilde{f}^a$ are defined by $\tilde{f}^a(x) = f^a(x) - \sum_{\alpha>0} \gamma_\alpha^a x^{-\alpha}$. The remaining contributions from the above diagrams, and also the total contribution of diagram 3d, yield conventional leading Regge behavior with the expected phases. As before we also find a corresponding real part in $\nu T_2 = -q^2/\nu \; T_1^{FP}$. Thus a fixed pole arises in the Compton amplitude due to local photon interactions even though the parton-proton amplitude has no such term.

The q^2 independent value of the fixed pole in Eq. (11) is precisely the finite energy sum rule result which one obtains in the scaling limit. Explicitly the FESR (in the scaling limit) is

$$\int_0^1 \frac{\tilde{f}(x)}{x}\, dx - \sum_{\alpha > 0} \frac{1}{\alpha} \gamma_\alpha \equiv \int_0^\infty \tilde{F}_2(\omega)\, d\omega \overset{(FESR)}{=} \frac{\nu^2}{2q^2} T_2^{FP}$$

$$\tilde{f} = \sum_a \lambda_a^2 \tilde{f}_a \qquad \gamma_a \equiv \sum_a \lambda_a^2 \gamma_\alpha^a$$

which is precisely the result we obtain for all q^2, $\nu \to \infty$.[11]

Thus we find that fixed pole — real part contributions - are always associated with the existence of seagull or corresponding Z graph couplings to the charged constituents of the target. Apart from the term $\sum_\alpha \frac{1}{\alpha} \gamma_\alpha$ which results from the conventional Reggeization procedure, only $\tilde{f}(x)$, that part of the distribution function which does not contain the leading (i.e. divergent) x behavior $\sum_{\alpha} x^{-\alpha}\gamma_{\alpha}$, contributes. The remaining part is absorbed into the normal leading Regge behavior of the full Compton amplitude. The distribution function $\tilde{f}(x)$, which vanishes as $x \to 0$, may be associated with short range terms in the space-time structure of the current correlation function, as shown by suri and Yennie.[6] We note that the seagull (or corresponding Z graph) contribution to the real part of the general Compton amplitude $T_{\mu\nu}(q_1^2, q_2^2, s, t)$ is independent of the photon masses and depends only upon $t = (q_1 - q_2)^2$ in the form

$$g_{\mu\nu} \sum_a \lambda_a^2 \int_0^1 \frac{f_a(x,t)}{x} dx$$

and has dependence on t similar to that of the elastic form factor.[12]

$$\sum_a \lambda_a \int_0^1 f_a(x,t)\, dx = F_1(t).$$

As before leading Regge terms must be subtracted from the seagull contribution to obtain the fixed pole. This form can be tested in both non-forward (elastic, or inelastic) Compton scattering and photoproduction of lepton pairs, since the leading Regge contributions are expected to disappear much more rapidly than the fixed pole contribution as t grows. Our results also have interesting implications for the processes $\gamma + \gamma \to X$ which are accessible in $ee \to eeX$ measurements.

In the case of a simple three quark model of the nucleon with the same distribution function for p and n quarks, then $T_1^{FP}(n) = \frac{2}{3} T_1^{FP}(p)$.[13] In general a composite theory of the neutron with charged constituents leads to a non-zero fixed pole contribution; accordingly, direct measurements of the real part of the nucleon Compton amplitude from Bethe-Heitler interference experiments for both proton and deuteron targets will be very interesting.

We thank our colleagues at SLAC for interesting and helpful discusssions.

REFERENCES

1. The forward Compton amplitude is written $\epsilon^{\mu}\epsilon^{\nu}\,T_{\mu\nu}$ where

$$T_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)T_1(q^2,\nu) + \frac{1}{M^2}\left(P_\mu - \frac{P\cdot q}{q^2}q_\mu\right)\left(P_\nu - \frac{P\cdot q}{q^2}q_\nu\right)T_2(q^2,\nu)$$

Both $T_{1,2}$ are crossing symmetric so they Reggeise as

$$T_1 \sim (1 + e^{-i\pi\alpha})\,\nu^{\alpha} + C;\; T_2 \sim (1 + e^{-i\pi\alpha})\nu^{\alpha-2} - \frac{Cq^2}{\nu^2}$$

where the terms involving the real number C are possible fixed poles at $J = 0$. Note that the fixed pole is defined relative to Regge terms with the correct phase.

2. M. J. Creutz, S. D. Drell and E. A. Paschos, Phys. Rev. 178, 2300 (1969). M. Damashek and F. J. Gilman, Phys. Rev. D1, 1319 (1970). C. A. Dominguez, C. Ferro-Fontan and R. Z. Suaya, Phys. Letters 31B, 365 (1970). In the Damashek-Gilman analysis Re $T_1 \sim \nu^{1/2} + C$ where $C \sim T_1^{\text{Born}}$.

3. Polynomial behavior had been previously conjectured by T. P. Cheng and Wu-Ki Tung, Phys. Rev. Letters 24, 851 (1970) and J. M. Cornwall, D. Corrigan and R. Norton, Phys. Rev. Letters 24, 1141 (1970). The constraints of such behavior for the electroproduction data have been discussed by F. E. Close and J. F. Gunion, SLAC-PUB-892; 917 (Phys. Rev. D, Aug. 1; Aug. 15, 1970). It should be noted that polynomial residue plus finiteness as $q^2 \to \infty$ yields the form described in the text.

4. The normalization used is $f_1(\nu) = \frac{\alpha}{2M}\,T_1(\nu, 0)$.

5. The frame chosen is such that

$$P_\mu = (P + M^2/2P,\ \vec{0},\ P); \qquad q = \left(\frac{M\nu}{2P},\ \vec{q}_\perp,\ \frac{-M\nu}{2P}\right) \text{ and}$$

internal integration (on-mass shell) four-vectors are chosen in the form

$$k_\mu = \left(|1-x|\, P + \frac{k_\perp^2 + M^2}{2\,|1-x|\,P} , \; -\vec{k}_\perp , \; (1-x)\,P \right)$$

The only kinematic conditions imposed are $Q^2 = -q^2 > 0, p^2 = M^2$, $p \cdot q = M\nu$ and terms of order M^2/P^2, $M\nu/P^2$ are neglected as $P \to \infty$. Note that only one time ordering of the Feynman diagrams 1a-1h survives in ϕ^3 theory as $P \to \infty$. See S. Weinberg, Phys. Rev. 150, 1313 (1966). Many of the techniques used in this paper were developed by Drell, Levy and Yan, Ref. 7, 10.

6. For spin 1/2 particles, the "seagull" arises from a Z graph in the infinite momentum frame. (See Fig. 2b). Covariant regularization must be used in these theories to guarantee convergence, proper gauge invariance, and low energy theorems. Note also that to any finite order in the perturbation theory and for any regularized theory f(x) vanishes as $x \to 0$, and T_1 and νT_2 do not have leading Regge behavior. This exhibits the behavior of the short range component in νW_2 as discussed by A. suri and D.R. Yennie (SLAC-PUB-954). We thank Drs. suri and Yennie for helpful conversations.

7. In this sum of n-particle states, the n = 1 state is presumed absent in order to ensure that the elastic form factor vanishes as $q^2 \to \infty$. This is equivalent to assuming the dynamical requirement $Z_2 \to 0$ (i.e. dropping Born terms in the perturbation theory calculation — see S. Drell, D. Levy and T. M. Yan, Phys. Rev. 187, 2159 (1969), D1, 1035 (1970).

8. J.D. Bjorken, Phys. Rev. 179, 1547(1969). R. P. Feynman, Phys. Rev. Letters 23, 1415 (1969). J.D. Bjorken, E.A. Paschos, Phys. Rev. 185, 1975(1969). J. Kuti and V. F. Weisskopf, MIT preprint, May 1971.

9. P. V. Landshoff, J. C. Polkinghorne, and R. D. Short, Nucl. Phys. B28, 222 (1971). In the normalization of this reference $A = -\frac{2}{(2\pi)^3} Z_2^a$ where Z_2^a is the wave function renormalization constant for parton a. T is then a fully renormalized amplitude.

10. See S. J. Brodsky, F. E. Close and J. F. Gunion (in preparation), in which an exhaustive account of the results of this letter will be presented.

11. This gives a physical realization of the result obtained by Cornwall, Corrigan, and Norton, Ref. 3.

12. S. D. Drell, T. M. Yan, Phys. Rev. Letters 24, 181 (1970).

13. In such a simple three quark model $T_1^{FP} \sim 3T_1^{Born}$ (taking $<\frac{1}{x}> = 3$) in an energy region where the impulse approximation is relevant. This does not agree with the FESR dispersion analyses in Ref. 2, but it is quite possible that the assumptions on asymptotic behavior in these analyses are incorrect if a threshold for quark production exists which has yet to be reached. If this were the case then direct measurements of the real part of the Compton amplitude should show a slow transition to the ultimate asymptotic result. This of course would make the impulse approximation used to explain scaling hard to understand. Further discussion will be given elsewhere.

FIGURE CAPTIONS

1. (a,b,c) The vertex Feynman diagrams.

   (d,e,f,g,h) The Feynman diagrams contributing to T_1.

2. Generalized time ordered diagram for the (a) form factor; (b) "Seagull"; (c) νW_2. The matrix elements are proportional to $\sum_a \lambda_a N_a, \sum \lambda_a^2/x_a N_a$, $\sum \lambda_a^2 N_a x_a \delta(x_a - Q^2/2M\nu)$, respectively. N_a is the number operator for parton a.

3. (a) The parton proton scattering amplitude. (b) The generalized seagull contribution. (c) The freely propagating parton graphs. (The self energy modifications included in $\rho_a(m^2)$, Eq. (11), are not drawn.) (d) The fully connected diagram.

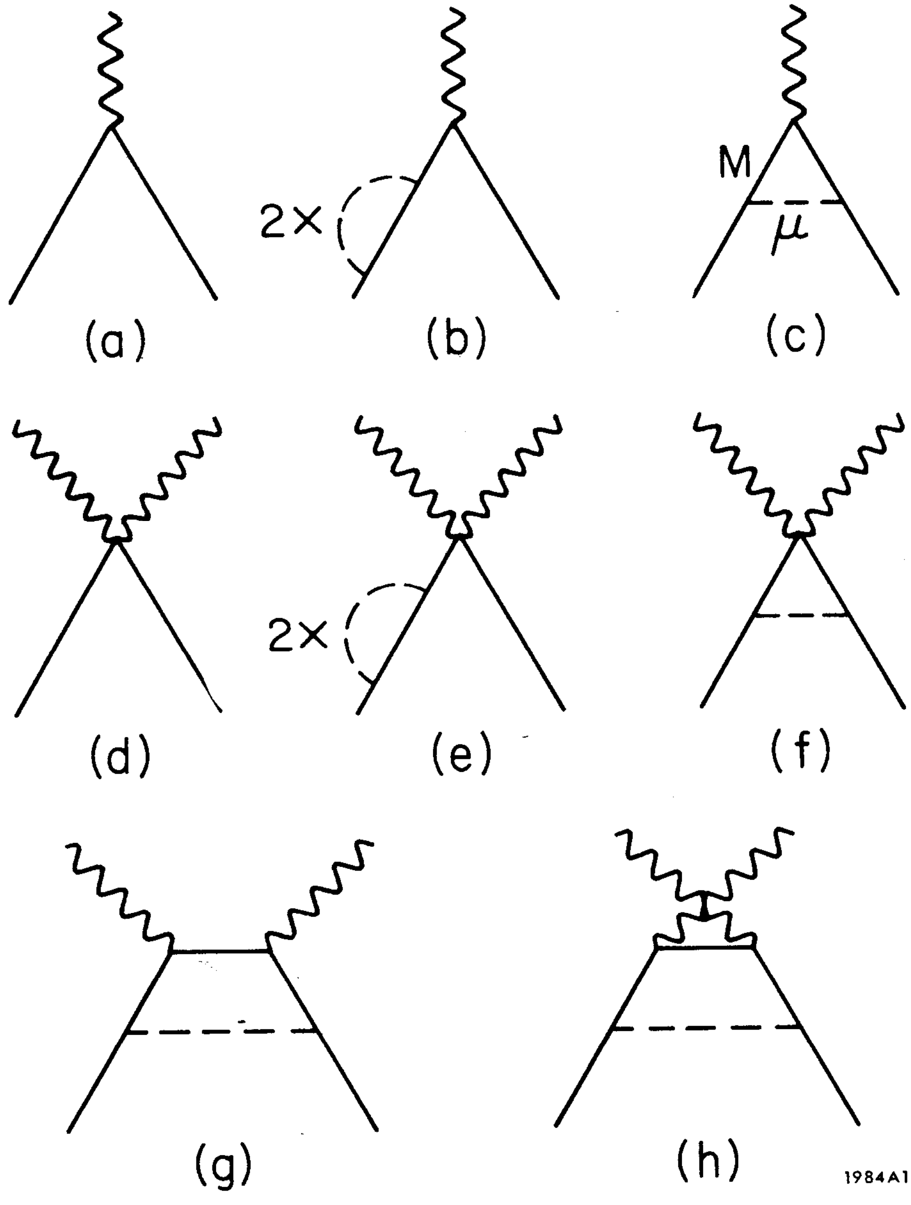

Fig. 1

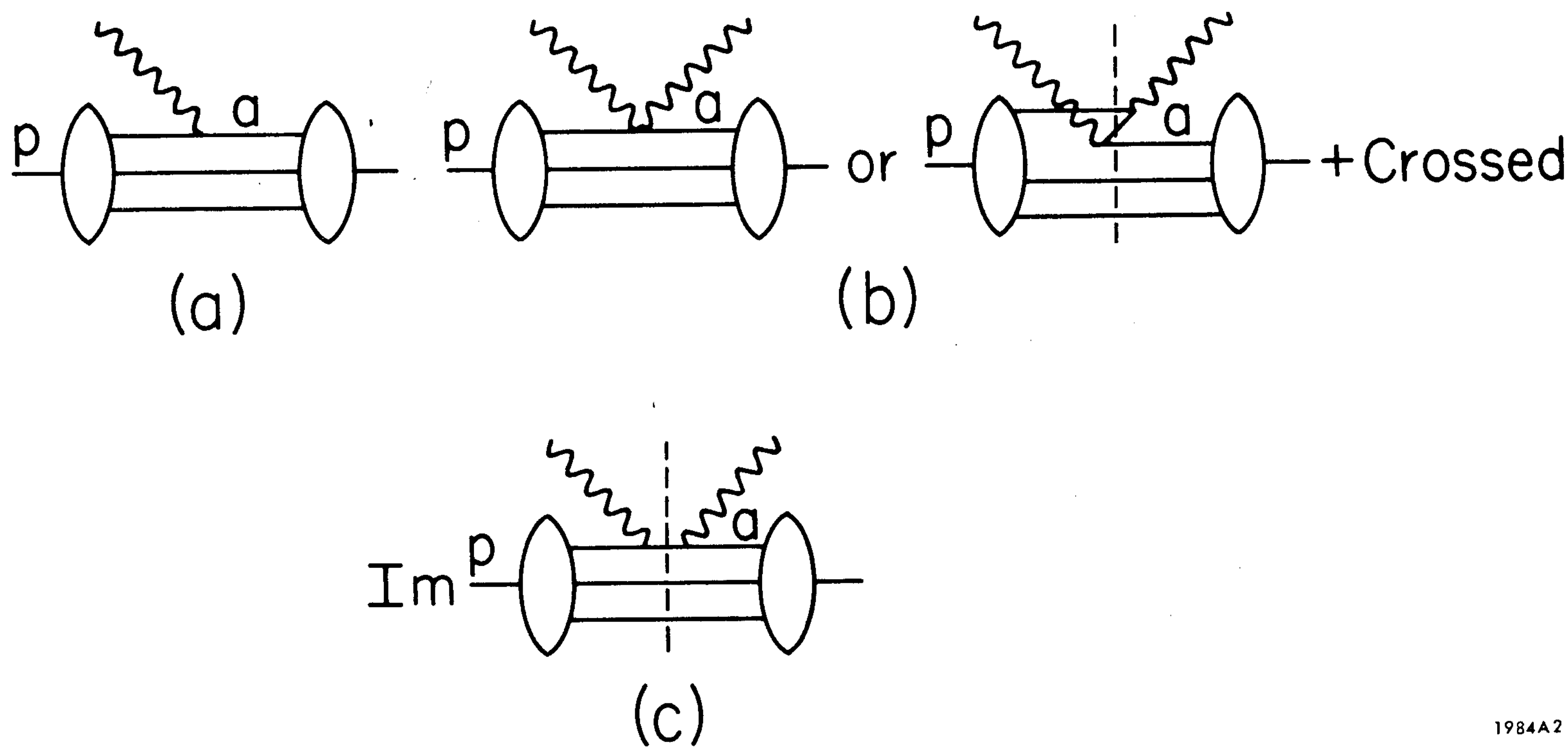

Fig. 2

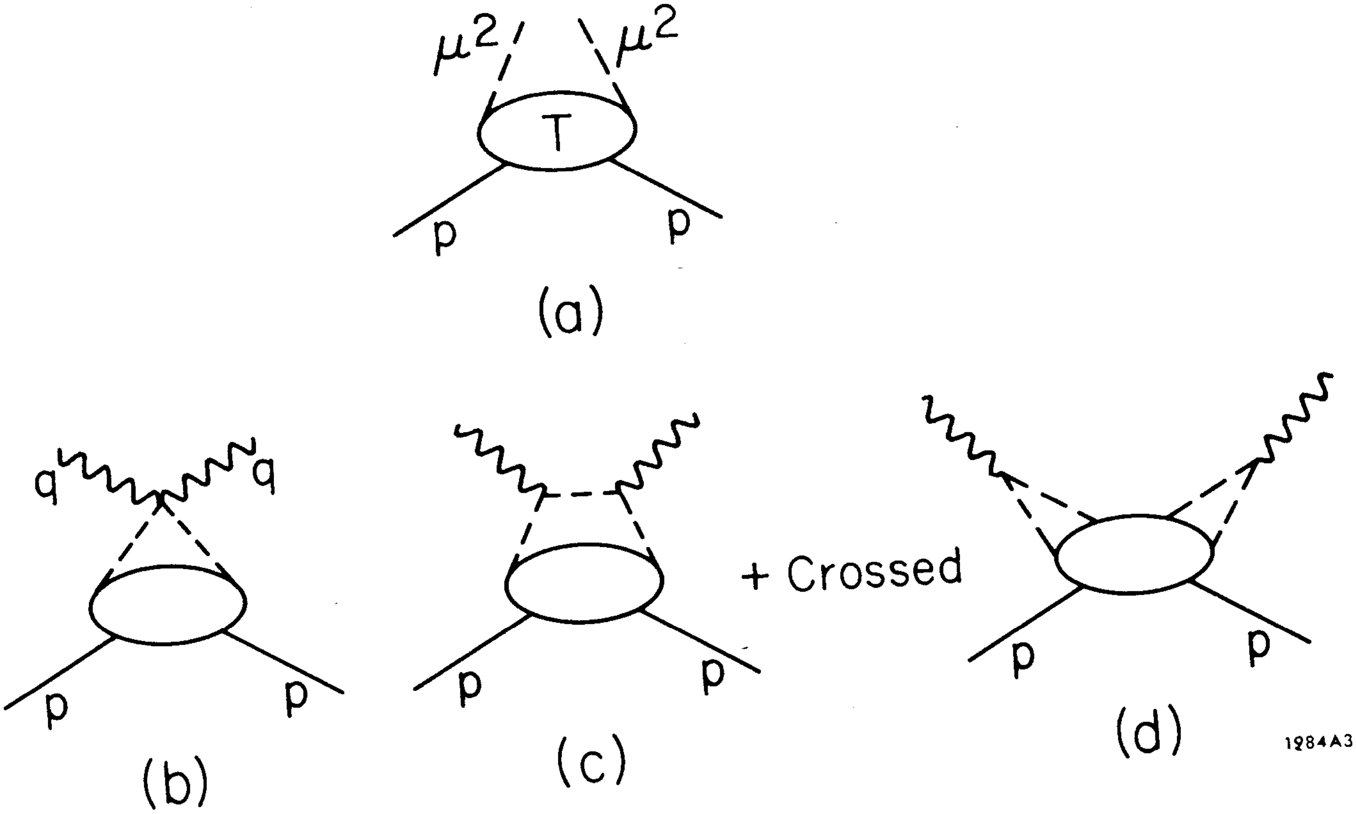

Fig. 3