

A HIGH ENERGY PROTON-ELECTRON-POSITRON COLLIDING BEAM SYSTEM*

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Abstract

A system of two intersecting storage rings of gross radius 260 m, one for protons and the other for electrons and positrons, is described. The maximum energy of the stored proton beam is 70 GeV, that of the electron and positron beams 15 GeV; so the center-of-mass energies are 65 GeV for e-p collisions and 30 GeV for e⁺-e⁻ collisions. The performance of the system is determined by the RF power available for the electron and positron beams and by the incoherent beam-beam limit on transverse beam density. With an available RF power for the beams of 2.7 MW, the system yields design luminosities in the vicinity of 10³² cm⁻² sec⁻¹ in both modes of operation. Some important physics experiments and their yields are discussed.

1. Introduction

Experiments using the beams from electron and proton accelerators have been the driving force in the pursuit of knowledge in elementary particle physics. Looking back over the last twenty years we see that what we know of particle structure comes principally (but not exclusively) from experiments done at the electron machines, while our knowledge of particle spectroscopy comes principally (but not exclusively) from experiments at proton machines, and our knowledge of particle dynamics comes from experiments at both kinds of machines. The conventional electron and proton machines — and now the electron-positron and proton-proton colliding beam machines — exist in a kind of symbiotic relationship which allows complementary experiments, and is crucial to the vitality of high energy physics. The maximum center-of-mass (c.m.) energies of the generation of electron storage rings now nearing completion lie in the vicinity of 6 GeV. This energy range matches the c.m. energy range of the present generation of conventional accelerators (BNL, CERN, SLAC) which lies around 7 GeV. However, with the advent of the NAL proton synchrotron soon to operate with a c.m. energy of 20 to 30 GeV (200 to 500 GeV proton beams), and with the CERN ISR operating at c.m. energies of up to 56 GeV, the proton machines have now far outstripped both the electron-positron storage rings and the conventional electron accelerators.

We have been studying the feasibility of greatly increasing the c.m. energy available for electron-positron colliding beams, and making use of the colliding-beam technique to study electron-proton interaction at c.m. energies which do not seem accessible with conventional accelerators. To this end, we have set our design goals in relation to the high energy physics experiments which seem most important, and we have designed a storage ring facility to meet those goals. We have chosen a c.m. energy of 30 GeV for e⁺-e⁻ interactions which matches the c.m. energy of NAL at 500 GeV, and represents a large extrapolation in energy from the present generation of e⁺-e⁻ rings. Storing the protons in a ring using conventional magnets and having the same circumference as

the electron ring so that it fits into the same housing results in an e-p c.m. energy of 65 GeV which is equivalent to 2100 GeV electrons striking stationary protons. The design luminosity is set at about 10³² cm⁻² sec⁻¹ for both e⁺-e⁻ and e-p collisions based on achieving a satisfactory counting rate in some of the more interesting experiments which can be done with the facility, and allowing an experimental program of extraordinary breadth.

This paper describes some of the physics which can be done with this double-ring system and presents the results of our efforts to design the rings. The experiments described below represent only a small fraction of the physics which can be done with the rings. The topics discussed were chosen because they are of particular interest now, and they give some idea of the scope of the possible experimental program. The resulting ring parameters show that the luminosity specified can be reached with conventional magnet, RF and injection techniques. We have not attempted to optimize the design for minimum cost, although we think the parameters are reasonable.

1.1 Physics with 65 GeV (c.m.) Proton-Electron Collisions

1.1.1 Inelastic electron scattering. Inelastic electron scattering experiments have generated a great deal of interest in the past few years. Recent experiments at SLAC have revealed a large inelastic cross section, giving support to the notion of a grainy substructure in the proton. With the electron-proton colliding beams, studies of the reaction e+p → e+X can be extended to momentum transfers and inelasticities far beyond anything available from other machines.

It is customary to describe the inelastic scattering in terms of electrons incident on stationary protons. In these terms, the e-p ring gives an equivalent electron laboratory energy of about 2100 GeV. The kinematic parameters of interest are q², the square of the invariant four-momentum transferred by the electron and ν, the energy loss of the electron in the system where the target proton is at rest. The maximum value of q² is 4000 (GeV/c)² and the maximum value of ν is 2100 GeV. This can be compared to a proposed μ-p inelastic scattering experiment at NAL, which gives a maximum practical q² of 40 (GeV/c)² and ν of 150 GeV with the apparatus planned.

To get an idea of the yields which might be expected in the e-p ring, we will extrapolate from SLAC energies, employing the most generally used theoretical model (scaling). In bins of dq²/q² and dν/ν of ±10%, we find a yield of about 5 events/day at the maximum momentum transfer of 4000 (GeV/c)². For ν=1000 GeV in the proton rest system, and q²=1000 (GeV/c)², we get about 20 events per day. These events are quite easy to detect. For example, the case of q²=1000 (GeV/c)² and ν=1000 GeV gives a final electron of about 20 GeV energy coming out at nearly 90° to the storage ring beams, and there are few background sources which can give such large transverse momenta.

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1.1.2 Photoproduction. Strictly speaking, there is no pure photoproduction in electron-proton collisions. However, the special case of inelastic electron scattering with $q^2 \ll m_e^2$ can be looked at as a two-step process — first, the radiation of a real photon, and second, the interaction of this photon to produce hadrons. The Weizsacker-Williams approximation gives the equivalent photon-beam intensity coming from this very low q^2 inelastic scattering, and the effective luminosity for photoproduction reaction is $\mathcal{L}(k) = 0.07 \times 10^{32} (\Delta k/k) \text{ cm}^{-2} \text{ sec}^{-1}$, where k is the photon energy. In terms of photons incident on protons at rest, the maximum photon energy is 2100 GeV.

Counting rates for photoproduction processes range from very large to very small. For example, the total photon cross section for hadron production is estimated to give a yield of 700 $(\Delta k/k)$ per second independent of k , while a single ρ -meson photoproduction gives a yield of 70 $(\Delta k/k)$, also independent of k . A typical small photoproduction cross section would be that for single-pion production ($\gamma P \rightarrow \pi^+ N$). Measurements made up to 20 GeV indicate the energy dependence for this cross section to be given by k^{-2} , where k is the photon energy in the proton rest system. In the range of equivalent photon energy from 200 GeV to 500 GeV, where photoproduction experiments will complement experiments done with hadron beams at NAL, the rate for even this small cross section varies from 15 $(\Delta k/k)$ to 2.5 $(\Delta k/k)$ events per hour. Thus, the e-p ring can span most of the spectrum of photon experiments that have been done with conventional accelerators at lower energy.

1.1.3 Weak interactions. The basic Fermi interaction has a total cross section which rises as the square of the c.m. energy until damped by an intermediate boson (W) or by some other, as yet, unknown process. This dependence has been confirmed to energies as high as several GeV in the c.m. by means of neutrino experiments at CERN and Brookhaven. If we extrapolate the measured cross section to the 65-GeV c.m. energy of the e-p rings, we find it to be about $5 \times 10^{-35} \text{ cm}^2$. With a luminosity of $10^{32} \text{ cm}^{-2} \text{ sec}^{-1}$, this would imply 500 weak interactions per day. It appears quite feasible to separate these weak interactions from the general background of highly inelastic electromagnetic interactions.

If the vector boson (W), which has been hypothesized to mediate the weak interaction, exists with a mass less than 50 - 60 GeV, it should be produced with a cross section in the region of 10^{-36} cm^2 . This would give about 10 production events per day, a very detectable number.

1.2 Physics with 30 GeV (c.m.) e^+e^- Colliding Beams

1.2.1 The total hadronic cross section. Experiments in the 1 - 2.5 GeV region in the c.m. have shown that both the energy dependence and the magnitude of the total hadronic production cross section are very roughly the same as those for mu-pair production. Such large cross sections were not expected at the time of the first attempts to build colliding electron and positron beams, but this behavior is what we have learned to expect now on the basis of constituent models of the hadron such as those used successfully in explaining deep inelastic electron-proton scattering. Should the hadronic cross section be comparable to the mu-pair cross section at very high energies, we can expect hadronic event rates of order 0.01/sec at 30 GeV in the c.m. We must, however, be prepared for major surprises at these energies.

Indeed, there are models which predict even larger than pointlike cross sections at high energies.

1.2.2 Inclusive hadronic reactions. With 30 GeV colliding beams it is possible to study the inclusive reaction $e^+ + e^- \rightarrow h + \text{"anything"}$ for large energy and transverse momentum deposited on the one detected hadron h . Some of the fundamental questions to be studied in this process are the energy dependence and yields for different hadrons such as π , K, N, Σ , Λ , etc., and the relation of the structure functions to the analogous structure functions for deep inelastic scattering from protons. Besides these general studies, specific tests of C invariance, SU_3 invariance, etc. are possible. Estimates all suggest large cross sections and comfortably observable event rates for these processes. For example some parton or point-constituent models suggest event rates of from 1 to 25 events/hr for a luminosity of $10^{32}/\text{cm}^2 \text{ sec}$.

1.2.3 Two-body final states. Present experiments on two-body (exclusive) hadron production are not sufficiently accurate and do not cover a wide enough range in energy to permit a reliable extrapolation to the storage ring energies considered here. On theoretical grounds, however, we would expect hadron form factors falling at least as fast as $1/E^2$, which would give unobservable counting rates at $E = 15$ GeV with planned luminosities for pi pairs, nucleon pairs, etc. However, many other two-body channels are of great interest and should have large counting rates. For example, there may exist pointlike heavy leptons (μ^*) in the mass range $3 \leq M_{\mu^*} \leq 15$ GeV which are similar to muons in that they have no strong interactions and pointlike electromagnetic couplings. These will be pair-produced just as ordinary muons are and, for a mass of the μ^* up to 13 GeV, the expected counting rate is of order 0.01 per second. The W meson discussed previously in connection with the weak interactions will be copiously produced electromagnetically if its mass is less than 15 GeV. For a W with magnetic moment and quadrupole moment of zero, we expect a rate of pair production proportional to m_W^{-2} and about equal to 0.01 per second for $m_W \approx 14$ GeV.

Three other processes of particular interest are μ -pair production, e^+e^- scattering, and two-quantum annihilation. Existing colliding-beam experiments at Orsay, Novosibirsk, and Frascati indicate that QED is valid up to a cutoff of 2 or 3 GeV. By increasing the value of the c.m. energy of the incident pair to 30 GeV, the large ring will allow precision tests to probe for possible modifications resulting from cutoffs in the 50 - 100 GeV region.

2. Storage Rings

2.1 General Considerations

The device we consider is shown in Fig. 1. It consists of two storage rings which intersect each other in four places. We consider the case of one bunch in each beam which provides for delivering the maximum possible luminosity at either pair of diametrically opposite interaction regions. We have only a single tunnel which has the consequence that the protons have, for a reasonable design, several times higher maximum energy than the electrons. For the reasons summarized in Section 1, we have chosen 15 GeV for the energy of the stored electron beam and hence take 72 GeV for that of the proton beam, assuming that conventional magnets would be used throughout. With superconducting magnets, proton energies up to 150 GeV could be reached.

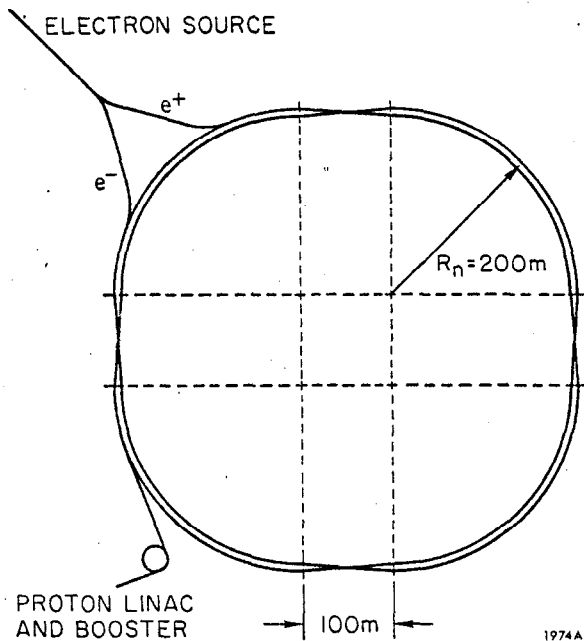


FIG. 1--Schematic drawing of the geometric configuration of the rings. The electron and positron beams cross vertically at an angle of 10 mrad, and the proton and electron beams cross horizontally at an angle of 10 mrad.

In the configuration of Fig. 1 the two rings intersect at a small angle (10 mrad) in the horizontal plane. At the intersection points, the beams in each ring are focused in both dimensions to a tiny size by low beta optical systems.¹ One of the rings is designed primarily for the storage of protons, having variable-frequency radiofrequency systems and high magnetic field capability. The other ring is designed to store electrons, having a fixed-frequency RF system and smaller magnets. For electron-positron collisions, both beams are stored in the electron ring and intersect vertically at a small angle (also 10 mrad). It is possible also to store electrons in the proton ring in order to effect e-e collisions; and to store protons in the electron ring to study p-p collisions. However, in the present study, we ignore these possibilities.

The fundamental limitations on the $e^+ - e^-$ colliding-beam performance of electron rings in the energy regime under consideration are imposed by the power radiated from the stored beams as synchrotron radiation, which must be supplied by the RF accelerating system and absorbed by the vacuum-chamber walls; and by the limit on tolerable transverse space-charge density beyond which one beam disrupts the other (the incoherent beam-beam limit). When the two rings function together for electron-proton collisions, the performance of the system again depends on the RF power available for the electron beam but not on the electron beam density, assuming the beams are adjusted to the same effective size; because the proton beam, having greater transverse mass, is not disrupted by the electron beam. The performance of the electron-proton system also depends on the achievable proton transverse space-charge density at the interaction regions. We have chosen the parameters of the proton ring to achieve a transverse

charge density at the interaction region just sufficient to bring the colliding electrons to their incoherent limit, or in other words, the same density as that at which the electron-positron system operates. We find that we can in fact achieve this high proton density despite space-charge and phase-space density limits. The protons are confined in a single short bunch, as are the electrons, so that all the protons collide with the entire electron swarm and vice versa at each of the two interaction regions. The machine circumference is more than 1000 meters and, in each ring, there is only a single bunch of a few centimeters length.

The luminosity for each interaction region for either kind of collision is

$$\mathcal{L} = \frac{f}{4} \frac{N_1 N_2}{A_{\text{int}}}, \quad (1)$$

where f is the revolution frequency, A_{int} is the effective interaction area and N_1 and N_2 refer to the total numbers of stored particles in the colliding beams. The revolution frequencies must of course be the same in both rings. The effective interaction area depends on the heights, widths and lengths of the bunches and the crossing angles.

As argued above, in all cases, the beam limit is the limit determined by the electron incoherent beam-beam instability which may be expressed

$$\frac{N_p}{A_{\text{int}}} = \frac{N_{e^+}}{A_{\text{int}}} = \frac{2\Delta\nu_0 \gamma_e}{r_e \beta_e^*}, \quad (2)$$

where r_e is the classical electron radius, γ_e is the electron energy in rest mass units, β_e^* is the vertical beta-function at the interaction region in the electron ring and $\Delta\nu_0$ is a parameter which is experimentally — and theoretically — found to be about 0.025 in both $e^+ - e^-$ and $e^- - e^-$ storage rings. Now we may write the luminosities for the two cases as follows:

$$\mathcal{L} = \frac{f}{2} \left(\frac{\Delta\nu_0 \gamma_e}{r_e \beta_e^*} \right) [N_{ee} \text{ or } N_{ep}], \quad (3)$$

where N_{ee} is the number of electrons stored in the case of electron-positron collisions and N_{ep} is that in the case of electron-proton collisions. The luminosity in either case depends on the parameters of the electron ring alone, providing Eq. (2) can be satisfied for both rings.

The number of electrons stored is limited by RF power considerations. Let P_b be the total RF power available to be supplied to the beam or beams in the electron ring.

$$N_{ee} = P_b / 2fU, \quad (4)$$

and

$$N_{ep} = P_b / fU. \quad (5)$$

The radiation loss per turn U is

$$U = 4\pi r_e m_e c^2 \gamma_e^4 / \rho_e, \quad (6)$$

where ρ_e is the bending radius of the electron beam and $m_e c^2$ is the electron rest energy. The factor of two appears in the denominator of Eq. (4) because the RF power must be divided between the electrons and the

positrons when both are stored. In terms of P_b ,

$$\mathcal{L}_{ep} = 2\mathcal{L}_{ee} = \frac{3}{8\pi} \left(\frac{\Delta\nu_0 \rho_e P_b}{r_e^2 m_e c^2 \gamma_e^3 \beta_e^*} \right) \quad (7)$$

We proceed first to the design of the electron ring.

2.2 Electron Ring

For the lattice we assume a simple separated-function structure which ensures an appropriate distribution of radiation damping rates. We must choose the gross radius of the arcs R_n , the bending radius ρ_e and the beam power P_b so as to achieve the desired luminosity.

The principal question of feasibility is that of the radiofrequency system. The radiation loss per turn U and therefore the peak accelerating voltage required will be much higher than those of any existing storage ring or synchrotron. The peak accelerating voltage requirement is set by the need to achieve a quantum lifetime which is long compared to the storage time. The requirement is a function of various parameters determined by the lattice design and of the chosen radiofrequency. Higher voltages are required at high frequencies. We will need peak voltages of the order of 100 MV. Presently operating systems on the highest energy synchrotrons produce peak voltages of the order of 10 MV using copper cavities with total lengths of the order of 10 meters and with instantaneous power inputs of the order of several hundreds of kilowatts. Thus, we can employ a conventional RF system.

In selecting the radiofrequency we make a compromise. The shunt impedance per unit length increases with frequency, but the peak voltage also increases. For example, the voltage required at a radiofrequency of 300 MHz is twice the radiation loss. A reasonable compromise is 250 MHz.

As explained in Section 1, physics requirements dictate a design luminosity in the range of $10^{32} \text{ cm}^{-2} \text{ sec}^{-1}$. We take this as the luminosity for the electron-proton system and so adopt $\mathcal{L}_{ee} = 0.5 \times 10^{32} \text{ cm}^{-2} \text{ sec}^{-1}$. (The total luminosity, summed over all interaction regions, is $10^{32} \text{ cm}^{-2} \text{ sec}^{-1}$.) We choose $R_n = 200 \text{ m}$ and four 100-meter-long straight sections in view of the requirement for long RF accelerating structures. The parameters of the electron ring are summarized in Table 1.

Table 1: Electron Ring Parameters

Maximum Beam Energy	15 GeV
Lattice	
Gross Radius of Circular Arcs (R_n)	200 m
Length of Interaction Regions	100 m
Gross Radius (R)	260 m
Beta-function at Interaction Point ($\beta_x = \beta_y = \beta_e^*$)	0.05 m
Effective Aspect Ratio of Beam at Interaction Point	0.5
Typical Beta-function in Cells	25.1 m
Betatron Wave Number in Arcs	12.5
Typical Local Dispersion in Cells	3.2 m
Momentum Compaction Coefficient	0.012
Bending Radius (ρ_e)	100 m
Bending Field	5.0 kG

Radiofrequency System

Radiofrequency	250 MHz
Harmonic Number	1370
Total Available Power	5.3 MW
Total Accelerating Cavity Length	200 m
Total Shunt Impedance	$3.2 \times 10^9 \text{ ohms}$

Performance at 15 GeV ($e^+ - e^-$ collisions)

Number of Stored Particles (each beam)	1.1×10^{12}
Average Circulating Current (each beam)	30 mA
Radiation Loss (U)	45 MeV
Peak RF Voltage	90 MV
Power Radiated by Beam	2.5 MW
Power Dissipated in Cavities	2.8 MW
Energy Spread ($2 \times \text{rms}$)	38 MeV
Longitudinal Damping Time	1.8 ms
Beam Width ($2 \times \text{rms}$): Typical	1.4 cm
Interaction Region	0.06 cm
Crossing Angle (2δ)	10 mrad
Luminosity (\mathcal{L}_{ee})	$0.5 \times 10^{32} \text{ cm}^{-2} \text{ sec}^{-1}$

2.3 The Proton Ring

The proton ring parameters must be chosen to satisfy Eq. (2) which may be written

$$\frac{N_p}{\frac{\pi}{4} h^* (w^{*2} + l^{*2} \theta^2)^{1/2}} = \frac{2\Delta\nu_0 \gamma_e}{r_e \beta_e^*} \quad (8)$$

in terms of the proton beam parameters, h^* , w^* and l^* which are twice the rms height, width and length respectively, and θ which is half the crossing angle. The crossing angle is determined by the requirement that the proton and electron beams be separable at the ends of the interaction regions which sets a minimum on θ . Since, for given θ , it is clear that w^* can be allowed to be as large as $l^* \theta$ before it seriously cuts into the density, and since, for a variety of reasons, it is desirable to make w^* as large as practical, we choose $w^* = l^* \theta$.

The most important constraint on the choice of beam parameters is an upper limit on the six-dimensional phase-space density of the protons σ : Under ideal conditions, σ is preserved from the ion source to the final colliding beam condition, but in practice it is reduced by various mishandlings of beams and space-charge blowups.

We define normalized emittances by

$$\tilde{\epsilon}_L = \Delta(\beta\gamma)_f l^* \quad (9)$$

$$\tilde{\epsilon}_H = \pi(\beta\gamma)_f w^{*2} / 4\beta_p^* \quad (10)$$

$$\tilde{\epsilon}_V = \pi(\beta\gamma)_f h^{*2} / 4\beta_p^* \quad (11)$$

where β_p^* is the proton lattice function at the crossing point (taken the same for horizontal and vertical), β and γ are proton relativistic factors and the subscript f refers to the final (colliding) energy. Implicit here is the assumption that the local dispersion at the interaction region is small so that the beam width due to momentum spread is negligible. With these definitions,

$$N_p = \sigma \tilde{\epsilon}_L \tilde{\epsilon}_H \tilde{\epsilon}_V \quad (12)$$

and we have from Eq. (8)

$$\sigma \tilde{\epsilon}_L (\tilde{\epsilon}_H \tilde{\epsilon}_V)^{1/2} = 2^{3/2} \Delta\nu_0 \gamma_e \beta_p^* / r_e \gamma_f \beta_e^*$$

as a design restraint.

We insert a numerical value of σ deduced by examination of existing machines. We take $\sigma = 3 \times 10^{15} \text{ cm}^{-3}$, a value equal to that corresponding to 95% of the beam in the CERN PS just after capture at 50 MeV and six times higher than that presently achieved at 19 GeV under the best conditions with Q-jump at transition. Note that σ is typically 3 times higher in the core of a beam (65% rather than 95% of the particles), so our choice is — perhaps — not too optimistic.

The RF voltage which is required to make a proton bunch of length l^* and energy width $\Delta(\beta\gamma)_f$ is a function of the RF harmonic number k_f . The minimum voltage V_f , is obtained when $k_f = 2\pi R/l^*$, and is given by

$$eV_f = \frac{\pi}{2} \left(\frac{2\pi R}{l^*} \right) \frac{\eta_f}{\gamma_f} \Delta(\beta\gamma)_f^2 m_p c^2, \quad (14)$$

where η_f is given by

$$\eta_f = |\gamma_f^{-2} - \gamma_T^{-2}|,$$

with γ_T the transition energy of the proton ring (in units of $m_p c^2$).

For the proton ring we take a filling factor of 0.6 in the bending sections and a peak field of 20 kG, and hence we obtain $\gamma_f = 72$. For β_p^* we adopt 15 cm in both transverse dimensions on the assumption that the first quadrupoles at the ends of the crossing region will be employed to make the very low β_p^* . We adopt a rather strong focusing structure so that η_f is small; namely we take $\gamma_T = 20$, and an average β , $\langle \beta \rangle = 13$ meters. We take the RF voltage, eV_f , to be 10 MeV since, in a practical design, the actual voltage required to obtain a short bunch is approximately four times this value, and hence the actual eV_f is similar to that in the electron ring. The resulting parameters of the proton ring are given in Table 2.

Table 2: Proton Ring Parameters

Maximum Beam Energy	72 GeV
<u>Lattice</u>	
Gross Radius (R)	260 m
Beta-function at Interaction Point (β_p^*)	0.15 m
Typical Beta-function in Cells	13 m
Transition Gamma (γ_T)	20
Bending Radius (ρ_p)	120 m
Bending Field	20 kG
<u>Radiofrequency System (final system)</u>	
Radiofrequency	250 MHz
Harmonic Number (k_f)	1260
Peak RF Voltage	40 MV
<u>Performance at 72 GeV</u>	
Number of Stored Protons (N_p)	7.5×10^{12}
Beam Width ($2 \times \text{rms}$) (w^*)	0.12 cm
Beam Height ($2 \times \text{rms}$) (h^*)	0.06 cm
Bunch Length ($2 \times \text{rms}$) (l^*)	23 cm
Momentum Spread [$\Delta(\beta\gamma)_f$]	0.17
<u>Injection Parameters</u>	
Injection Kinetic Energy	1 GeV
Radiofrequency	1 MHz
Harmonic Number	6
Beam Width ($2 \times \text{rms}$)	4.8 cm
Beam Height ($2 \times \text{rms}$)	2.2 cm
Bunch Length ($2 \times \text{rms}$)	270 m
Momentum Spread [$\Delta(\beta\gamma)$]	1.0×10^{-4}

We now employ the beam parameters, as determined in the previous sections, to determine the injection energy into the main ring. We adopt an injection kinetic energy of 1 GeV, which value affords a reasonable — but by no means optimized — compromise to the various competing considerations. In computing beam parameters at injection we have, guided by experience, assumed a factor of ten degradation of phase density in the main ring. Thus we employ $\sigma = 30 \times 10^{15} \text{ cm}^{-3}$ for injection considerations.

The space-charge limit at injection is closely related to the choice of the RF system, as the RF-harmonic number will determine the initial bunching factor. We choose an RF system that will initially work at a frequency around 1 MHz ($k_1 = 6$) and fill one bucket of the six available. We evaluate the space-charge limit taking the bunching factor $B_1 = (2k_1)^{-1}$ and find $N_{SC} = 8.5 \times 10^{12}$ which is just adequately larger than N_p . We note that, despite the very tight bunching at top energy, the space-charge limit is adequate also at top energy.

The primary beam manipulation which is required in the main ring, other than acceleration from the injection kinetic energy of 1 GeV to top energy of 72 GeV, is a terrific longitudinal beam compression from a bunch length of $2.7 \times 10^4 \text{ cm}$ to one of 24 cm. Basically the method to be employed is to use the initial RF system, operating on harmonic $k_1 = 6$, to bunch the beam both by accelerating it (adiabatic damping) and by subjecting it to a much larger voltage than is employed when the beam is first captured. The first system is then turned off non-adiabatically and a second system operating on a much higher harmonic k_2 (which has been chosen just to tightly fit the bunch) is turned on at a low voltage and raised to a high voltage thus further compressing the bunch. The process is repeated as often as necessary (two or three times), finally employing a system which requires approximately 40 MV per turn. Only the final system has to work at this very high voltage level. Space-charge effects at transition also require a large RF voltage (30 MV).

Reference

1. See the following for discussions of the concepts employed in Section 2: M. Sands, "The Physics of Electron Storage Rings, an Introduction," in Physics with Intersecting Storage Rings, edited by B. Touschek (Academic Press, New York and London, 1971). (This work is also available as Stanford Linear Accelerator Center Report No. SLAC-121, November, 1970.) H. Bruck, Accélérateurs Circulaires de Particules (Presses Universitaires de France, Paris, 1966).