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# POWER LOSSES IN RF SUPERCONDUCTING CAVITIES*

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## Abstract

Power losses in microwave superconducting cavities are not well understood at very low temperatures, and/or at very high field levels. Established theories of superconducting surface resistance do not work in these ranges. It is proposed that the discrepancy lies in the presence of non-superconducting power loss mechanisms. Various mechanisms are considered, such as dielectric loss due to surface contamination by oxides and adsorbed gas; field emission; normal regions; fluxoids; etc. Cavity Q ( $\sim 10^{11}$) is extremely sensitive to impurities and lattice defects on and near the surface. An oscillating fluxoid is most likely responsible for critical power loss at high field levels; and small stationary normal regions for residual power loss at low temperature.

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With the attainment of superconducting cavity Q's $> 10^{10}$, the cavity Q is extremely sensitive to various power losses which might otherwise be considered negligible. Whereas some of these losses will only limit the cavity Q and cause its gradual decrease as the power is increased, there are power loss mechanisms which may lead to thermal-magnetic instabilities with a concomitant runaway situation leading to either magnetic or electric breakdown in which the Q drops precipitously by orders of magnitude at certain power levels. One such mechanism which involves coupled electric and magnetic breakdown is related to field emission.

In addition to power dissipation when the field-emitted electrons are accelerated by the rf fields in the cavity and strike the cavity walls, even at $0^{\circ}$ K there is a power loss at a normal or superconducting microprotrusion when the electrons are field-emitted.[1-4] The latter is due to the fact that the electrons are field-emitted at an average energy below the Fermi energy, and are replaced in the metal by electrons at approximately the Fermi energy. This kind of effect was originally described by Richardson[5] for thermionic emission and later further analyzed by Nottingham[6] for thermionic and field emission for normal metals at much higher temperatures than we are here considering, for which Nottingham concluded ".... that very little heat loss is to be expected in the case of field emission, are well borne out by the experiments." This effect is now called the Nottingham effect. The temperature rise from this effect may be sufficient to cause the whisker to undergo transition to the normal state. When this occurs, there is additional Joule heating power loss, which is capable of causing a runaway instability.

Transition to the normal state may even be caused by the self-magnetic field of the field emission current, as is shown by the following calculation. For simplicity let us consider a hemispherically capped cylindrical microprotrusion of radius a, and height h on the inside cavity surface. From Maxwell's equation,

$$\oint \vec{H} \cdot \vec{d\ell} = \oint \left[ \vec{J} + \frac{\partial \vec{D}}{\partial t} \cdot d\vec{A} \right] \tag{1}$$

we have

$$I = 2\pi a H \tag{2}$$

as the relation between the emission current I and the self-magnetic field H it generates encircling the whisker at its surface. Equation (2) is a good approximation of the self-field at points far from either end of the whisker, such as the mid-plane. In addition to the self-field, there is the rf magnetic field at the whisker $H_p \cos \omega t$, plus the magnetic field $H_a$ due to the contribution from all other sources. For example, $H_a$ may be an applied dc field and/or the field penetration from a trapped fluxoid.

Let us consider two limiting cases. For the first case, assume that the vector sum $\vec{H}_a + \vec{H}_p \cos \omega t$ is essentially perpendicular to the whisker axis. In this case, the fields add on one side of the whisker and subtract with the self-field on the other side. Since we are interested in transition to the normal state, we need only consider the point of maximum field strength where the fields add together. The maximum field is

$$H_m = 2\left|\vec{H}_a + \vec{H}_p \cos \omega t\right| + I/2\pi a \ . \tag{3}$$

(The factor of 2 arises because of demagnetization of a long cylinder in a perpendicular field.) Part of the whisker first undergoes transition to the normal state when

$$H_m = H_0 \equiv \begin{cases} H_c(T) & \text{for type I} \\ \\ H_{c1}(T) & \text{for type II} \end{cases} \quad . \tag{4}$$

Therefore, in this case, transition occurs when the emission current I(t) exceeds the transition current $I_T(t)$, where

$$I(t) \geq I_T(t) = 2\pi a \left[ H_0 - 2 \left| \vec{H}_a + \vec{H}_p \cos \omega t \right| \right] \quad \text{(perpendicular case)} . \tag{5}$$

For the second case, assume that the vector sum $\vec{H}_a + \vec{H}_p \cos \omega t$ is essentially parallel to the whisker axis. The total field is

$$H_m = \left[ \left| \vec{H}_a + \vec{H}_p \cos \omega t \right|^2 + (I/2\pi a)^2 \right]^{1/2} \quad . \tag{6}$$

In this case, transition occurs when

$$I(t) \geq I_T(t) = 2\pi a \left[ H_0^2 - \left| \vec{H}_a + \vec{H}_p \cos \omega t \right|^2 \right]^{1/2} \quad \text{(parallel case)} \quad . \tag{7}$$

Of course, Nottingham effect heating, coupled with the decrease in critical field with elevated temperature, further decreases the necessary emission current to produce transition. It is interesting to note that even at $0^{\circ}$ K and neglecting $\vec{H}_p \cos \omega t$ and $\vec{H}_a$, the self-field generated by the emission current is sufficient to exceed the critical fields of Pb and Nb, for a whisker radius of

$3 \times 10^{-6}$ cm, and an enhancement factor $\doteq h/a = 300$. In the case of Pb, an emission current of $1.2 \times 10^{-2}$ A produced by a macroscopic electric field $\sim 2.7 \times 10^{5}$ V/cm generates a self-magnetic field in excess of the critical field of 803 gauss. In the case of Nb, an emission current of $2.9 \times 10^{-2}$ A produced by a macroscopic electric field $\sim 3.3 \times 10^{5}$ V/cm generates a field in excess of 1940 gauss. Smaller radius whiskers would require higher electric fields in this case.

The average dielectric power loss per unit volume is

$$\left\langle \frac{dP}{dV} \right\rangle = \frac{1}{2}\, \omega \kappa \epsilon_0 E_0^2 \,\mathrm{tn}\,\delta \; , \qquad (8)$$

where $\kappa$ is the dielectric constant, $\epsilon_0$ is the vacuum permittivity, $E_0$ is the peak electric field at the surface, and $\mathrm{tn}\,\delta$ is the loss tangent. From temperature and frequency dependence considerations,[2] it is unlikely that dielectric loss is responsible for residual power loss at levels thus far attained. As far as dielectric losses from the addition of oxides and/or adsorbed gas on the cavity surface go, it appears that they would not seriously degrade cavity Q for

$$\kappa \,\mathrm{tn}\,\delta < 10^{-4} \qquad (9)$$

at the operating temperature and frequency.

Small stationary normal regions[2] or strongly pinned fluxoids[7] have a surface resistance, R, independent of temperature, and

$$R \propto \omega^2 . \tag{10}$$

Szecsi[8] finds experimentally that the residual surface resistance varies approximately as $\omega^2$ and is independent of temperature. Therefore, it seems likely that small stationary normal regions or strongly pinned fluxoids are responsible for the residual power loss.

An oscillating fluxoid is most likely responsible for critical power loss at high field levels leading to magnetic breakdown.[7,9] In some cases, it may be possible for the critical power loss to be initiated by a non-magnetic, but quite thermally isolated, normal region.[7,9] As derived in references 7 and 9, the magnetic breakdown field is

$$H'_p = \frac{-\frac{k_1}{2B}\frac{T_c^2}{H_0} + \left[\left(\frac{k_1}{2B}\frac{T_c^2}{H_0}\right)^2 - 2N\,Ra\,\ell n\left(\frac{b'}{a}\right)\left\{\frac{1}{4}k_3\left(T_b^4 - h^4 T_c^4\right) + \frac{1}{2}k_1 T_c^2\left(g^2 - \frac{1}{B} + \frac{H_a}{BH_0}\right) - C_1\right\}\right]^{\frac{1}{2}}}{N F\,Ra\,\ell n\left(\frac{b'}{a}\right)} \tag{11}$$

Equation (11) relates the magnetic breakdown field to the various thermal conductivity parameters $k_i$, g, h, and $C_1$, the bath temperature $T_b$, the critical temperature $T_c$, the radius of the normal region a, its effective surface resistance R, etc. The surface resistance has a complicated frequency dependence[7] given by

$$R = \left[\frac{\omega^2 \phi^2 H H_0 \mu^2}{\rho_n^2\left(\omega^2 M - p\right)^2 + \omega^2 \phi^2 H_0^2 \mu^2}\right]\frac{\rho_n}{2\lambda} \tag{12}$$

where $\rho_n$ is the normal state resistivity, $\lambda$ is the penetration depth, $\phi$ is the trapped flux, M is the fluxoid mass/length, and p is the pinning constant/length.

There appears to be enough richness and self-consistency in this model to explain many, if not all, the hitherto unexplained aspects of power losses in rf superconducting cavities.

## References


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