SLAC-PUB-961
(ACC)
September 1971

BEAM ENLARGEMENT WITH A HIGH FREQUENCY OSCILLATING QUADRUPOLE*

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Abstract

To achieve the maximum possible luminosity in electron or positron storage rings it may be necessary to increase the transverse emittance of the beam. In order not to swamp the transverse feedback system it is necessary to "blow up" the beam without affecting the position of the center of the beam.

For this purpose a quadrupole oscillating at approximately twice the betatron oscillation frequency is placed in the storage ring magnet lattice.

The particle dynamics with such a quadrupole in the lattice is discussed including the effects of fluctuations and nonlinearities and the possibility of modulating the strength or the oscillation frequency of the quadrupole field.

In order to achieve the maximum luminosity in SPEAR the beam size must be enlarged above the natural size that arises from the interplay between the radiation damping and quantum fluctuations. A method, suggested by Richter, is to insert a quadrupole magnetic field into the lattice with the field gradient oscillating at approximately twice the betatron oscillation frequency plus or minus some integral multiple of the revolution frequency, which has the effect of driving a half-integral resonance. The advantage of this method, over excitation by a dipole, is that the center of the charge in the beam would not be displaced, and the feedback system which controls the dipole instabilities would not be appreciably affected.

The fact that this oscillating quadrupole field can produce a linear resonance and increase the amplitude of the betatron motion has been well understood and is adequately described in the literature.[1,2] The problems considered here are: The influence of fluctuations and nonlinearities on the growth rate of beam size, and how to control the growth rate in order to obtain the desired beam size. One of the effects of the fluctuations and nonlinearities upon the particle motion is to change the natural betatron frequency so that the particle motion and the oscillating quadrupole field tend to become out of phase. Three possible ways of controlling the growth rate are: (1) Pick the quadrupole field strength so that the growth rate due to the resonance is just below the radiation damping rate and thus produce an effective damping rate which is much lower than the natural damping rate. (2) Introduce nonlinearities to detune the betatron oscillation frequency away from the resonance at the desired amplitude. (3) Modulate the frequency or strength of the oscillating quadrupole field so that motion is only growing part of the time.

The first of these methods which seems to be the most straightforward will not work for the following reason. Consider the case where the injection process is finished, and the oscillating quadrupole field is turned on to decrease the effective damping rate. In order to appreciably affect the beam size, the quadrupole field must produce a growth rate and resonance width which are the same order of magnitude as the radiation damping rate. However, the quantum fluctuations not only produce a growth in the betatron amplitude but also a change in the phase of the betatron motion. These fluctuations in betatron phase destroy the resonance condition and for this quadrupole field strength the effective damping rate is unchanged. If the quadrupole field strength is increased to the point where the phase fluctuation no longer destroys the resonance condition then the growth rate exceeds the radiation damping rate and the amplitude of the motion is unbounded.

Next we investigate the use of a nonlinear octupole field to limit the growth in the beam size. Consider the following equation of motion.

$$\ddot{x} + \left[\omega^2 + 2\omega\,\Delta\omega\,\sin 2(\omega t + \psi)\right]x = -\lambda x^3 . \qquad (1)$$

If we let the solution be given by

$$x = a(t)\sin(\omega t + \psi) + b(t)\cos(\omega t + \psi) , \qquad (2)$$

and assume that both a(t) and b(t) are slowly varying functions of time we obtain the following coupled equations for small $\Delta\omega$;

$$\begin{aligned} 2\omega\,\dot{a} + \frac{3}{4}\lambda\, b(a^2 + b^2) + \omega\,\Delta\omega\, a &= 0 \\ -2\omega\,\dot{b} + \frac{3}{4}\lambda\, a(a^2 + b^2) + \omega\,\Delta\omega\, b &= 0 . \end{aligned} \qquad (3)$$

The solutions for a and b lie on the curves shown in Fig. 1 where the arrows denote the direction of travel with time.

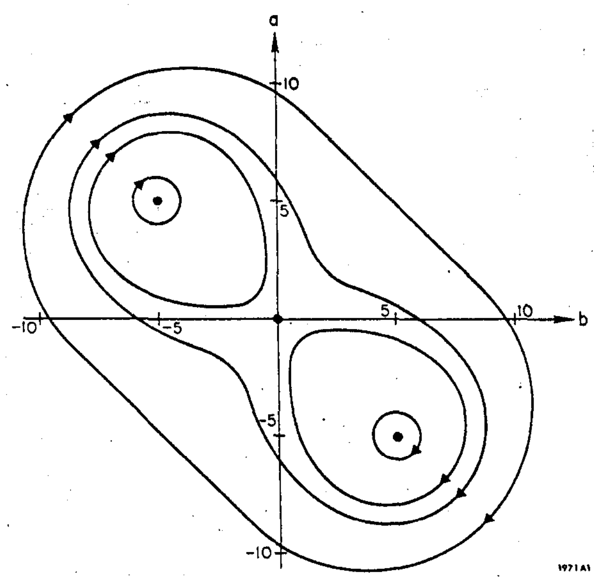

FIG. 1--Phase plots of a versus b for ω=5, $\Delta\omega$=.025, and λ=1/300.

*Work supported by U. S. Atomic Energy Commission.

(VIIIth International Conference on High Energy Accelerators, CERN, Geneva, September 20-24, 1971)

The unstable fix point is at a=b=0 and the stable fix points are at $a=-b=\pm\sqrt{2\omega\,\Delta\omega/3\lambda}$. If radiation damping and quantum fluctuations are included, the particles will achieve a Gaussian distribution about the two stable fix points. The distribution of the particle position at the interaction region is given by two Gaussians with a separation between them that oscillates at twice the betatron frequency. The effect of this throbbing motion of one beam upon the other beam, when the two beams are brought into interaction, has not been fully analyzed, and this effect may prove disastrous.

The third method is to produce a large growth rate during many short intervals by either turning the quadrupole field on and off or by sweeping the quadrupole excitation frequency. This method overcomes the shortcomings of the two previous methods. The growth rate and the stopband width during the short interval of growth are large compared to the damping rate so that the quantum fluctuations do not destroy the resonance condition. Between the times when the amplitude is being driven by the quadrupole field the motion is being damped and the motion becomes incoherent due to the quantum fluctuations. Thus the particle phase at the beginning of the growth periods is uncorrelated and the growth becomes stochastic. Consider the case where the quadrupole field of strength $\Delta\omega$ is turned on for short periods of time Δt at intervals T. The effective damping rate is given by

$$\alpha_{eff} = \alpha_{rad} - \frac{1}{T}\left(\frac{\Delta\omega\Delta t}{2}\right) , \qquad (4)$$

where α_{rad} is the natural radiation damping rate. In deriving the above formula we have assumed that the stopband width, $\Delta\omega$, is larger than the natural betatron frequency spread of the beam, the increment of growth $(\Delta\omega\Delta t/2)$ is small enough so that we do not drive the beam into the wall, and Δt is long compared to the betatron period.

For the case where the oscillation frequency of the quadrupole field is modulated with time as

$$\omega_{quad} = 2\,(\omega + A \sin \gamma t) , \qquad (5)$$

and ω is approximately equal to the natural betatron oscillation frequency, we have

$$\alpha_{eff} = \alpha_{rad} - \frac{\pi(\Delta\omega)^4}{64\,\gamma\,A^2} . \qquad (6)$$

We have assumed that A is much larger than both the stopband width and the natural betatron frequency spread. Also we have assumed the period during which the growth takes place $(\Delta\omega/\gamma A)$ is long compared to a betatron oscillation period, and the increment of growth $[\pi^2(\Delta\omega)^4/32\gamma^2A^2]$ is small.

In the case of "SPEAR" we assume a natural betatron frequency spread due to the beam-beam interaction of 1.65×10^4 rad/s $(\delta\nu = 0.025/\text{turn})$, a revolution frequency $\Omega = 6.6\times10^4$ rad/s, a natural betatron frequency $\omega = 3.5\times10^6$ rad/s $(\nu = 5.25/\text{turn})$, and at 2.5 GeV a radiation damping rate of 70 rad/s. A consistent set of parameters which satisfy the conditions stated above and reduce the effective damping rate to zero for the case where the quadrupole's strength is modulated are

$$\Delta\omega = 5\times10^4 \text{ rad/s } (\Delta\nu = 0.076/\text{turn}) ,$$
$$\Delta t = 12.6\,\mu s ,$$

and

$$T = 1.43 \text{ ms} .$$

For the frequency modulated case the consistent set of parameters that reduce the effective damping rate to zero are

$$\Delta\omega = 1.1\times10^4 \text{ rad/s } (\Delta\nu = 0.017/\text{turn}) ,$$
$$A = 10^5 \text{ rad/s } (0.15/\text{turn}) ,$$

and

$$\gamma = 2.2\times10^3 \text{ rad/s } (3.3\times10^{-3}/\text{turn}) .$$

Acknowledgements

It is a pleasure for the author to thank Drs. M. J. Lee, J. R. Rees and B. Richter for many stimulating discussions.

References

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2. N. W. McLachlan, Theory and Application of Mathieu Functions (Oxford University Press, Amen House, London, England, 1951).