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THE SEARCH FOR MUON-ELECTRON DIFFERENCES AND MUON-PROTON DEEP INELASTIC SCATTERING*

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I. The Muon-Electron Puzzle

One of the great puzzles of elementary particle physics is how to understand the relationships between the muon and the electron. These relationships are summarized in the table below.

Property	Comparison between muon and electron	If property is different: Muon	Electron
Intrinsic spin	both 1/2		
Statistics	both Fermi-Dirac		
Interact through the strong interactions	both no (within present experimental precision as discussed in the article)		
Interact through the electromagnetic interaction	both yes		
Magnitude of electric charge	same for both		
Sign of electric charge	both + or −, neither 0		
Gyromagnetic ratio	both given by quantum electrodynamics and particle's mass		
Interact through the weak interactions	both yes		
Magnitude of weak interaction coupling constant	same for both		
Associated neutrino	yes but different neutrinos	ν_μ	ν_e
Mass (MeV/c^2)		106	0.51

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The set of properties which the muon and electron possess in common, both qualitatively and quantitatively, are collectively described by the phrase "muon-electron universality". This set is large and comprises most of the properties of the muon and electron.

But in the few properties in which the muon and electron differ, they differ drastically. The muon mass is about 200 times larger than the electron mass. At present we have absolutely no understanding of how to explain this enormous mass ratio because we have no understanding of how to calculate the mass of the muon or of the electron. Another way in which these particles differ drastically has to do with their associated neutrinos. The neutrino associated with the muon is different from the neutrino associated with the electron. Although both neutrinos have zero electric charge, spin 1/2 and probably zero mass, these neutrinos cannot change into each other. Similarly a muon <u>cannot</u> change into an electron in reactions like

$$\mu + \text{proton} \rightarrow e + \text{proton}$$

even though charge, energy and three-momentum are conserved. These prohibitions are codified by the rule that the muon and its associated neutrino possess a property, conserved in all reaction, called the muon lepton[1] number (n_μ). n_μ is different from the lepton number (n_e) associated with the electron and its neutrino. These lepton numbers are separately conserved. The assignment of the lepton numbers is:

	μ^-	μ^+	ν_μ	$\bar{\nu}_\mu$	e^-	e^+	ν_e	$\bar{\nu}_e$	all other particles
n_μ	+1	-1	+1	-1	0	0	0	0	0
n_e	0	0	0	0	+1	-1	+1	-1	0

Here ν_ℓ ($\bar{\nu}_\ell$) is the neutrino (antineutrino) associated with the lepton ℓ. Thus decays

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu \tag{1}$$

and

$$\mu^- \rightarrow \nu_\mu + e^- + \bar{\nu}_e \tag{2}$$

are allowed, but the decay

$$\mu^- \rightarrow e^- + \gamma$$

is forbidden.

When two particles have so many properties in common, yet differ so drastically in a few properties, the physicist can't help speculating as to possible connections between the particles. Are the muon and electron manifestations of a single particle split into two mass levels by an unknown force? This is certainly an attractive speculation. But how then can one explain the very strict lepton number conservation rule which separates electrons from muons? Another speculation is that the electron and the muon are the lowest mass members of a larger family of leptons.

$$e, \mu, \mu', \mu'' \ldots$$

with associated neutrinos

$$\nu_e, \nu_\mu, \nu_{\mu'}, \nu_{\mu''} \ldots$$

If μ' and μ have the same lepton number then electromagnetic decays like

$$\mu' \rightarrow \mu + \gamma \tag{3}$$

can occur. If the μ' has a unique lepton number then there will be decay modes like

$$\begin{aligned} \mu'^- &\rightarrow \nu_{\mu'} + \mu^- + \bar{\nu}_\mu \\ \mu'^- &\rightarrow \nu_{\mu'} + e^- + \bar{\nu}_e \end{aligned} \tag{4}$$

If the mass of the μ' is greater than the mass of the pion, very interesting decay modes such as

$$\mu'^{-} \rightarrow \pi^{-} + \nu_{\mu'} \tag{5}$$

will occur.

At present we have absolutely no evidence as to the validity of any of these speculations. In fact we may also hypothesize that there is no connection between the muon and the electron. Perhaps there were two different Deities in two different universes who each constructed the simplest, spin 1/2, charged particle with nonzero mass. (As Dirac had shown the simplicity requirement means that the particle must obey the Dirac equation.) Then the two universes were mingled. The muon — the simplest, spin 1/2, charged, nonzero mass particle in one universe just turned out to have a larger mass than the electron — the corresponding particle in the other universe. The problem with this hypothesis is that we don't know why both Deities bothered to also construct neutrinos.

Until a young Einstein comes along to point out the simple truth that we hope lies behind the strange relationships between the muon and the electron, we have no choice but to proceed with further experiments. From these experiments we might hope to gain some clue, some new insight, into the connection between the particles. These experiments, which in their very nature must be speculative, have taken two directions. One direction consists of searches for unknown members of the electron-muon family. The other direction consists of comparative measurements of the properties of the muon and of the electron in the hope that hitherto unknown differences between the two particles will be discovered. Of course, for this second direction to be fruitful, one must measure known properties with greater precision or one must measure properties which have not been previously measured. The recent high precision measurements[2] of the gyromagnetic ratio of the muon are an illustration of the first type of measurement. The deep inelastic scattering experiment, which I will describe later, is an illustration of the second type of measurement. In discussing the comparative measurements of the properties of the muon and the electron, I will only briefly mention[2,3,4] most of the properties listed in the table. My emphasis will be on the third item in the table — the muon, the electron and the strong interactions. This discussion of the strong interactions will include consideration of the form factors of the muon and electron, an item not in the table.

II. Are the Muon and Electron Part of a Larger Family?

To the question, "Are the muon and the electron part of a larger family of charged leptons" we must give the unsatisfactory answer which follows. As far as we know there are no other charged leptons. But this knowledge does not go very far. This partial knowledge can be summarized easily.

1. Numerous experiments, many having to do with the decay of the K meson, have shown no additional leptons with masses below 0.5 GeV. The only exception to this statement is some surprising effects found by Ramm;[5] the nature of these effects is not clear.
2. No leptons with masses above 0.5 GeV have been found. But most searches[6] have required the formation of a beam of leptons and hence lepton lifetimes of greater than 10^{-8} or 10^{-9} seconds. Now when the lepton mass goes above 0.5 GeV the decay rate due to reactions like (4) and (5) rises rapidly, if first-order weak interaction theory is used. Thus by the time a mass of 1 GeV is reached the lifetime of the lepton may be as short as 10^{-11} seconds.

The lifetime will be even shorter if reaction (3) can occur. Therefore in these experiments if charged leptons with masses much above 0.5 GeV exist, they would not have been detected unless some additional and unknown conservation law closed their normal channels of decay. A few searches have been carried out to detect leptons with lifetimes shorter than 10^{-8} seconds. But all of these searches have had low overall sensitivity because either the rate of production of the hypothetical leptons was unknown or the hypothetical leptons could only be detected if they had special properties.

Thus there is a clear need to search directly for heavy leptons with lifetimes shorter than 10^{-9} seconds. Such a search should be sensitive to lifetimes as short as 10^{-13} or 10^{-14} seconds. Fortunately electron-positron colliding beam experiments are a just about perfect way to carry out such searches.[7] The reaction

$$e^+ + e^- \rightarrow \mu^+ + \mu^- \qquad (6)$$

is copious, has been studied and the measurements agree with the predictions of quantum electrodynamics.[8] When the colliding beams have sufficient energy, the reaction

$$e^+ + e^- \rightarrow \mu'^+ + \mu'^-$$

is almost as copious and the μ' s can be detected through decay modes like reactions (4) and (5). The search for higher mass leptons produced by electron-positron colliding beams has just begun.[7] Until such searches are completed we will not have the answer to the question I just raised — are the electron and the muon members of a larger family?

III. Comparison of Some Static Properties of the Muon and the Electron

One of the beautiful aspects of the search for muon-electron differences is the tremendous range of techniques which have been used. These techniques range from radio frequency measurements of the hyperfine structure of muonium[2,4] (4500 MHz equivalent to 1.9×10^{-14} GeV) to measurements of muon-proton elastic and inelastic scattering at energies above 10 GeV. In surveying the results from this range of techniques I will first discuss the static properties of the muon, then discuss muon-proton interactions at high energy and then return to low energies and the mu-mesic atom.

A. Electric charge

Four properties of the muon — the electric charge e_μ, the mass m_μ, the magnetic moment μ_μ and the gyromagnetic ratio g_μ — are connected by the relation

$$\mu_\mu = \left(\frac{g_\mu}{2}\right)\left(\frac{e_\mu \hbar}{2m_\mu c}\right) \tag{7}$$

g_μ and μ_μ have been determined with great precision;[2] .3 parts per million and 12 parts per million respectively. Therefore e_μ can be determined if m_μ is known from an independent measurement. Such a measurement is provided by the study of the mu-mesic atom, an atom in which a negative muon is captured in an atomic orbit.[9] Ignoring relativistic corrections, fine structure and hyperfine structure,the n^{th} energy level of such an atom is given by

$$E_n = \frac{-m_\mu e_\mu^2 (Ze_p)^2}{2n^2 \hbar^2} \tag{8}$$

which is just the formula first given by Bohr more than half a century ago. Here I have distinguished the muon charge e_μ from the charge on the nucleus Ze_p. By measuring the energy difference between levels, m_μ or more precisely the combination $m_\mu e_\mu^2$ can be determined. This measured value of $m_\mu e_\mu^2$ combined with Eq. (7) and the known values of μ_μ, g_μ, $\hbar$, c and e_e (the charge on the electron) yields[4]

$$e_\mu / e_e = 1 \pm 4 \times 10^{-5}$$

But a much lower limit can be obtained[4] by observing that if charge is conserved in the muon decay process

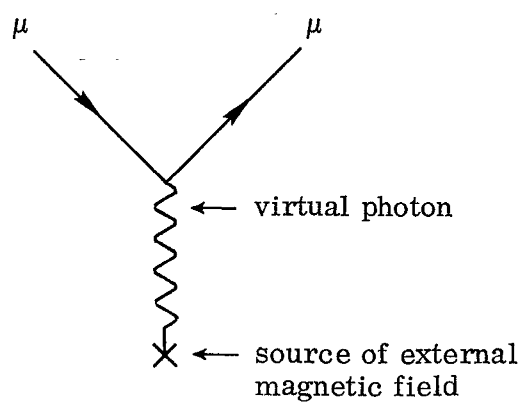
$$\mu \rightarrow e + \nu_\mu + \bar{\nu}_e$$

then one or both neutrinos will have a nonzero charge if $e_\mu \neq e_e$. However neutrinos could then be pair produced by low energy photons. This leads to an additional mechanism for energy loss in stars! Astrophysical considerations then set an upper limit on the charge that could be possessed by a neutrino. This limit leads to the conclusion that

$$e_\mu / e_e = 1 \pm 1 \times 10^{-13}$$

B. Gyromagnetic ratio

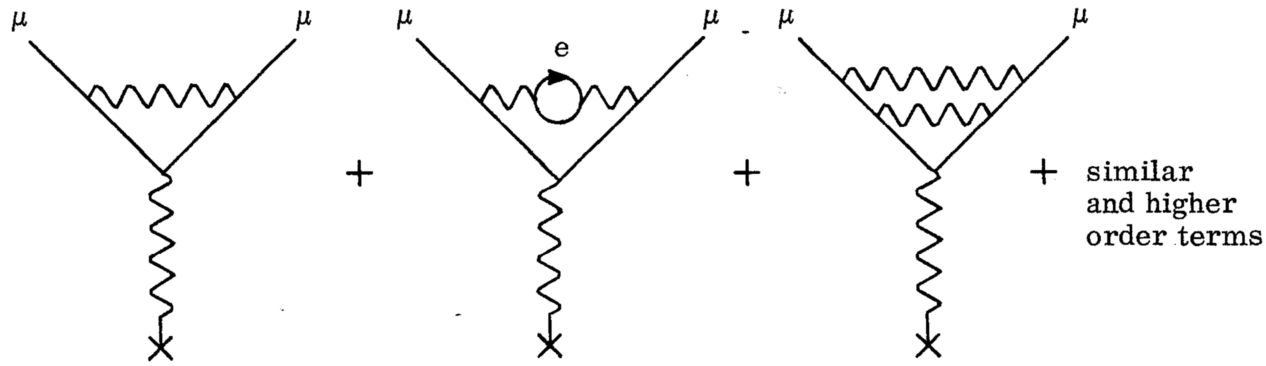
The gyromagnetic ratio, g_μ, can be calculated exactly from quantum electrodynamics, once the muon mass is known, if strong interactions are ignored. (Fortunately the influence of the strong interactions on g_μ is small; I will give the estimated size of the effect below.) The Dirac relativistic theory of the electron or the muon yields g=2. The Feynman diagram for the interaction of a muon with an external magnetic field (which yields g=2) is



But quantum electrodynamics shows that there is an anomalous magnetic moment so that g is not exactly 2. It is conventional to set

$$(g_\mu-2)/2 = a_\mu = \frac{1}{2}\left(\frac{\alpha}{\pi}\right) + A_2\left(\frac{\alpha}{\pi}\right)^2 + A_3\left(\frac{\alpha}{\pi}\right)^3 + \ldots$$

Here α is the fine structure constant ($e^2/\hbar c$) and equals approximately 1/137. The coefficients A_i are all of the order of magnitude of 10 or less so that α_μ is very small. Nevertheless it has been measured to great accuracy. The measurement of a_μ is a measurement of the combined effect of terms like



The most recent results of Farley, Picasso and their colleagues[2] at CERN yield

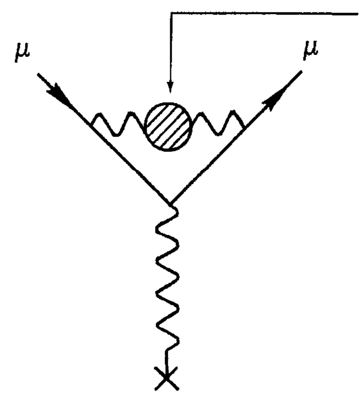
$$a_\mu^{exp} = (116616 \pm 31) \times 10^{-8}$$

and quantum electrodynamics yields

$$a_\mu^{theory} = (116581) \times 10^{-8}$$

Thus experiment and theory are in agreement. Even more precise agreement is found for the electron.[10]

Therefore with respect to the measurement of g-2, once the mass of the muon is taken into account, there is no observable difference between the muon and the electron. For future use I note that the strong interactions enter the g-2 calculation through the diagram



From electron-positron colliding beam measurements it is estimated[2] that the effect of this diagram should be

$$a_\mu^{\text{hadronic}} \approx 6\times 10^{-8} \approx 5\left(\frac{\alpha}{\pi}\right)^3$$

Therefore the precision of the existing g-2 measurement for the muon is not sufficient to detect strong interaction effects.

To summarize, the static properties of the muon (only some of which have been discussed here) compared to the static properties of the electron show no differences other than those explained by the mass difference.

IV. Muon-Proton Elastic Scattering

Although the static properties of the charged leptons show no unexplained differences, one might hope that differences will appear when the dynamic properties of the charged leptons are measured at high energy. For high energies were required to reveal the richness and complexities of the strong interactions. Might not high energies also reveal unsuspected complexities in muon and electron physics? The high energy reactions of the charged leptons may be divided into three classes.

1. One class consists of those reactions in which a neutrino is absorbed or produced.[11] Those reactions as presently measured show no violation of muon-electron universality.

2. Another class consists of purely electromagnetic reactions in which no hadron participates or in which the hadron has only an auxiliary role acting as an almost static source of electric charge. Examples are reaction (6) or muon bremsstrahlung

$$\mu + p \rightarrow \mu + \gamma + p$$

I do not have the space in this article to review the interesting upper limits that have been placed on possible muon-electron differences by this class of experiments.[3] But again, no violation of muon-electron universality has been found.

3. The third class of reactions, those which I shall emphasize in this article, consist of reactions in which hadrons play an intimate role. My interest in this class of reactions has two origins. First, as I shall discuss later, these reactions provide a way to search for spatial structure in the charged leptons; a way to test if the charged leptons are truly point Dirac particles. (Some Class (2) reactions of course also test for spatial structure.) Second, a speculation which particularly intriques me is that the leptons may in some very reduced manner take part directly in the strong interactions. After all, the mass difference between the muon and the electron is almost a pion mass and thus could be caused by the strong interactions. To see if the charged leptons in any way directly take part in the strong interactions, it is desirable to have hadrons present — hadrons act as a source for the strong interactions.

In the interaction of muons (or electrons) with protons we can consider two kinds of processes; <u>elastic scattering</u> where

$$\mu + p \rightarrow \mu + p$$

and <u>inelastic</u> <u>scattering</u> where

$$\mu + p \rightarrow \mu + \text{(any set of 2 or more hadrons)}$$

Examples of inelastic scattering are:

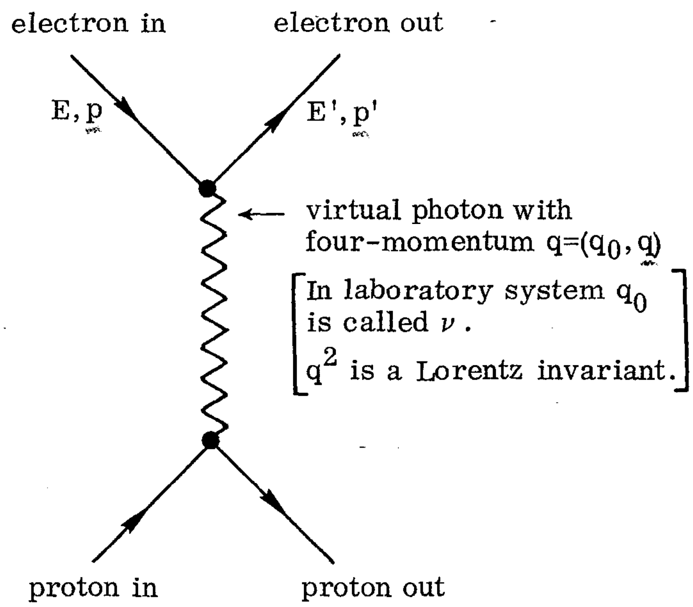
$$\mu + p \rightarrow \mu + p + \pi^{o}$$

$$\mu + p \rightarrow \mu + n + \pi^{+} + \pi^{o}$$

$$\mu + p \rightarrow \mu + \Sigma^{o} + K^{+}$$

In these elastic or inelastic scattering reactions, the charged lepton is not altered in the reaction. This distinguishes these processed from neutrino induced reactions (Class (1)).

I will consider first elastic scattering, and to set the stage I will discuss electron-proton elastic scattering. To a precision of a few percent all data on electron-proton elastic scattering is explained by the Feynman diagram



KINEMATICS
(in laboratory system)

$\nu = q_0 = E - E'$

$\underset{\sim}{q} = \underset{\sim}{p} - \underset{\sim}{p}'$

$q^2 = q_0^2 - (\underset{\sim}{q})^2 = -2M\nu$

M is proton mass

UNITS

energy-GeV

momentum-GeV/c

mass-GeV/c^2

$\hbar = 1$

$c = 1$

$e^2/\hbar c = \alpha \approx 1/137$

in which only one photon is exchanged. All experiments agree that the differential cross section for this elastic scattering process is described by the equation

$$\left(\frac{d\sigma}{dq^2}\right)_{ep,\,elas} = \left(\frac{d\sigma}{dq^2}\right)_{NS}\left[\frac{G_E^2(q^2) + \tau\, G_M^2(q^2)}{1+\tau} + 2\tau\, G_M^2(q^2)\tan^2\frac{\theta}{2}\right] \qquad (9)$$

where $G_E(0) = G_M(0) = 1$.

This equation, the Rosenbluth formula, assumes that the electron is a point Dirac particle with only electromagnetic and weak interactions. The equation is written for scattering in the laboratory system, θ is the electron scattering angle and $\tau = |q^2|/4M$, $(d\sigma/dq^2)_{NS}$ is the differential cross section for the scattering of an electron by a spin-zero point proton; NS denotes no spin. $(d\sigma/dq^2)_{NS}$ is a function only of the total energy of the system and θ; it is completely specified by quantum electrodynamics. $G_E(q^2)$ and $G_M(q^2)$ are the proton form factors. They take into account that the proton has nonzero spatial extent, and that the proton has strong interactions. If the proton were a point Dirac lepton G_E and G_M would both equal unity for all values of q^2.

The crucial variable is q^2, the square of the four-momentum transferred from the lepton vertex. In the center-of-mass system, $\sqrt{|q^2|}$ is just equal to the magnitude of the three-momentum transferred to the proton. Thus when $|q^2|$ is large, a large amount of three-momentum is transferred to the proton. By the uncertainty principle, $\Delta x \approx \hbar/(\sqrt{|q^2|})$. Therefore small spatial regions can be investigated at large $|q^2|$, and the internal structure of the proton can be studied. I remind you that it is found experimentally that

$$G_E(q^2) \approx 1/\left[1 + |q^2|/.71\right]^2 \quad \text{units are } (\text{GeV}/c)^2 \tag{10}$$

and

$$G_M(q^2) \approx 2.79\, G_E(q^2) \tag{11}$$

(In this article energy units will always be GeV, momentum units will be GeV/c and the units of q^2 will be $(\text{GeV}/c)^2$).

Now if the muon is a pure Dirac point particle we can use Eq. 9 for muon-proton elastic scattering. There are small effects due to the muon mass, which we have not exhibited explicitly; but these are known. Then

$$\left(\frac{d\sigma}{dq^2}\right)_{\mu p,\,elas} = \left(\frac{d\sigma}{dq^2}\right)_{ep,\,elas} \tag{12}$$

But suppose the muon is not a point particle; suppose the muon, like the proton, has a form factor $G_\mu(q^2)$. Then Eq. (13) becomes

$$\left(\frac{d}{dq^2}\right)_{\mu p,\,elas} = \left(\frac{d}{dq^2}\right)_{ep,\,elas} G_\mu^2(q^2)$$

and

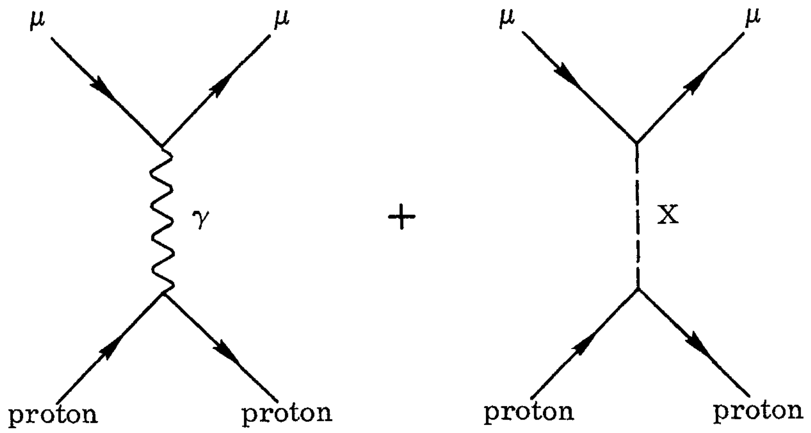
$$G_\mu^2(q^2) = \left(\frac{d\sigma}{dq^2}\right)_{\mu p,\,elas} \bigg/ \left(\frac{d\sigma}{dq^2}\right)_{ep,\,elas} \tag{13}$$

We have no theoretical guidance to what $G_\mu(q^2)$ might be, but at $q^2=0$ we must have[12] $G_\mu(0)=1$. Conventionally we take a form analogous to the proton form factor and write

$$G_\mu(q^2) = 1/\left[1 + |q^2|/\Lambda_\mu^2\right] \tag{14}$$

Note however that unlike Eq. (10), only the first power of $\left[1 + |q^2|/\Lambda_\mu^2\right]$ appears in the denominator. Λ_μ is a sort of inverse measure of the deviation of the muon from a point particle. The smaller Λ_μ, the greater the deviation.

But a muon might differ in other ways from and electron. There might be a special particle, the X particle, that couples to muons and hadrons but not to electrons. Then muon-proton elastic scattering would be the result of two amplitudes whose diagrams are



This would produce some deviation from Eq. (12), but the nature of the deviation cannot be determined because we do not know what X is. Therefore we continue to use $G_\mu(q^2)$ in Eq. (14) to express the deviation of muon-proton elastic scattering from electron-proton elastic scattering. In doing so we are making an assumption to which I shall return at the end of the article. We are assuming that the deviation between muon-proton and electron-proton elastic scattering will increase as $|q^2|$ increases.

In all of this we have assumed that the electron is a pure Dirac point particle. There is no need for this assumption. We can ascribe a form factor $G_e(q^2) = 1/\left[1 + |q^2|/\Lambda_e^2\right]$ to the electron. Then to order $|q^2|$

$$\frac{G_\mu(q^2)}{G_e(q^2)} = \frac{1 + |q^2|/\Lambda_e^2}{1 + |q^2|/\Lambda_\mu^2} \approx \frac{1}{1 + |q^2|/\Lambda_d^2} \tag{15}$$

where

$$\frac{1}{\Lambda_d^2} = \frac{1}{\Lambda_\mu^2} - \frac{1}{\Lambda_e^2}$$

Then Λ_d simply measures a difference in behavior between the electron and the muon. From now on I shall use Λ_d. Defining

$$\rho_{elastic}(q^2) = G_\mu^2(q^2)/G_E^2(q^2) = 1/\left[1 + |q^2|/\Lambda_d^2\right]^2 \tag{16}$$

Eq. (13) becomes

$$\rho_{elastic}(q^2) = (d\sigma/q^2)_{\mu p,\, elas}/(d\sigma/dq^2)_{ep,\, elas}$$

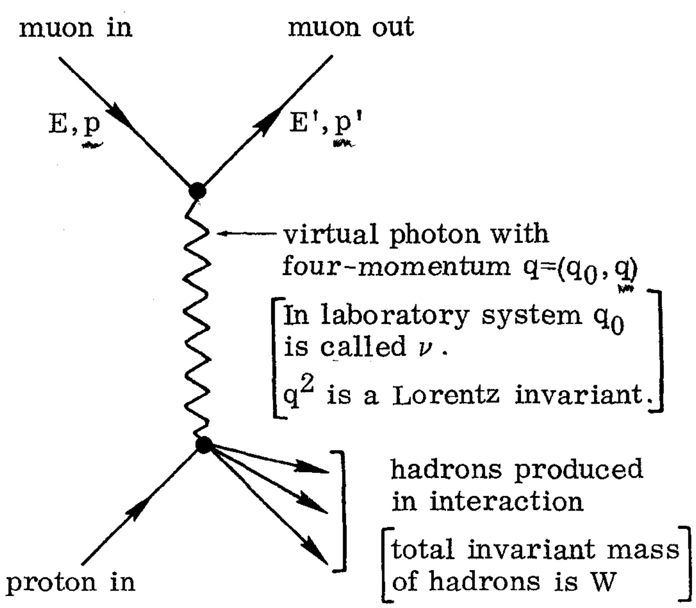
Enough speculation! Let us look at some data. Figure 1 shows the results from a muon-proton elastic scattering experiment of Camilleri et al.,[13] (assuming $G_M(q^2) = 2.79\, G_E(q^2)$. The figure shows that $\rho_{elastic}(q^2)$ is always close to 1, but is usually a little less than 1. Muon-electron universality demands of course that $\rho_{elastic}(q^2) = 1$. If we set $\rho_{elastic}(q^2) = .92$ we get a good fit. This looks like a normalization problem between the two experiments. To allow for this it is usual to rewrite Eq. (16) in the form

$$\rho_{\text{elastic}}(q^2) = \frac{N^2}{\left[1 + |q^2|/\Lambda_d^2\right]^2} \tag{17}$$

The authors of this experiment give as a best fit $N^2 = .95 \pm .035$ and $1/\Lambda_d^2 = .064 \pm .056\ (\text{GeV}/c)^2$. With 95% confidence, regardless of any normalization problem, $\Lambda_d > 2.4$ GeV/c.

V. Muon-Proton Inelastic Scattering — The Formalism and the Motive

I shall now describe a muon-proton inelastic scattering experiment[14] recently carried out at SLAC. Before telling you why I think inelastic scattering is such a good way to search for muon-electron differences, I will have to describe the formalism of inelastic scattering. The general diagram for charged lepton-proton inelastic scattering; with one photon exchange is



KINEMATICS
(in laboratory system)

$\nu = q_0 = E - E'$

$\mathbf{q} = \mathbf{p} - \mathbf{p}'$

$q^2 = q_0^2 - (\mathbf{q})^2$

$|q^2| < 2M\nu$

M is proton mass

$W^2 = 2M\nu + M^2 - |q^2|$

Muon-proton or electron-proton inelastic reactions comprise a vast field whose outlines are just now being experimentally determined. I shall describe results from the simplest experiment in that field — an experiment in which just the final scattered charged lepton is detected. No attempt is made to detect any of the hadrons produced. This inelastic scattering experiment then sums experimentally over the different hadronic states which can be produced.

At this point we must do a little kinematics. All the kinematic quantities are shown in the diagram and are in the laboratory system. The laboratory energy of the virtual photon is $\nu = E - E'$. Again q^2 is the square of the four-momentum transferred from the lepton vertex. If only the scattered lepton is detected, then the experiment automatically sums over all possible hadronic states which fit the lepton kinematics. To put this more explicitly, if only the scattered lepton is detected then the only kinematic variables which are determined are $\mathbf{p}'$ and $\mathbf{p}$, the final and initial lepton momenta. For fixed $\mathbf{p}'$ and $\mathbf{p}$, the reaction is then described by three independent kinematic quantities which we may take to be E (the initial lepton's energy), ν and q^2. It is important to observe that here, unlike elastic scattering, ν and q^2 are independent. The only restriction is $|q^2| < 2M\nu$. With E, ν and q^2 fixed, the only kinematic restriction on the produced hadrons is that their total invariant mass must be $W = 2M\nu + M^2 - |q^2|$. But we do not know what hadrons have contributed to that W, unless W is exactly the mass of a single proton — the elastic scattering case. If for example W=2 GeV, the produced hadronic system may consist of a nucleon and one pion, or a nucleon and four pions or a hyperon and a kaon.

Thus experimentally we only determine a double differential cross section $d^2\sigma/dq^2\, d\nu$, a function of E, ν and q^2. The one photon exchange property of the diagram above allows us to analyze the differential cross section $d^2\sigma/dq^2\, d\nu$ in more detail. Just as in elastic scattering (Eq. (9)) where the differential cross section depends on two independent experimentally determined quantities, $G_E(q^2)$ and $G_M(q^2)$, so the inelastic differential cross section $d^2\sigma/dq^2\, d\nu$ also depends on two independent quantities — quantities which must be experimentally determined. From the above diagram we see that inelastic scattering may be regarded as the production of a virtual photon by the charged lepton, and the subsequent reaction of that virtual photon with the proton leading to the production of all sorts of hadrons. Indeed one set of quantities $\sigma_T(q^2, K)$ and $\sigma_S(q^2, K)$, introduced by Hand,[15] may be thought of as the total cross section for the interaction of transverse and scalar virtual photons with protons. Here $K=\nu - |q^2|/2M$, and is the equivalent energy that a real photon must have to give the same total energy in the photon-proton center-of-mass system.[15] Also $(d^2\sigma/dq^2\, d\nu) = (d^2\sigma/dq^2\, dK)$ and I shall use the latter from now on. $\sigma_T(q^2, K)$ and $\sigma_S(q^2, K)$ are defined by

$$\begin{aligned} d^2\sigma/dq^2 dK &= \Gamma_T(q^2, K, p)\, \sigma_T(q^2, K) + \Gamma_S(q^2, K, p)\, \sigma_S(q^2, K) \\ &= \Gamma_T(q^2, K, p) \left[\sigma_T(q^2, K) + \epsilon(q^2, K, p)\, \sigma_S(q^2, K)\right] \end{aligned} \qquad (18)$$

Γ_T and Γ_S are the virtual photon fluxes for transverse and scalar photons, respectively, and ϵ is the ratio of these fluxes as shown in the next two equations.

$$\Gamma_T = \left(\frac{\alpha}{2\pi|q^2|}\right)\left(\frac{K}{p^2}\right)\left(1 - \frac{2m^2}{|q^2|} + \frac{2EE' - |q^2|/2}{(E-E')^2 + |q^2|}\right)$$

$$\epsilon = \Gamma_S/\Gamma_T = \left(\frac{2EE' - |q^2|/2}{(E-E')^2 + |q^2|}\right) \Big/ \left(1 - \frac{2m^2}{|q^2|} + \frac{2EE' - |q^2|/2}{(E-E')^2 + |q^2|}\right) \leq 1$$

Here $\mathbf{p}$ ($\mathbf{p}'$) and $E(E')$ are the momentum and energy in the laboratory system of the incident (scattered) lepton; m is the lepton mass and α is the fine structure constant. As q^2 goes to zero, $\sigma_S(q^2, K)$ goes to zero and $\sigma_T(q^2, K)$ goes to

$\sigma_{\gamma p}(K)$ — the total cross section for the interaction of a real photon of energy K with a proton. In our muon experiment we could not separate σ_T from σ_S; therefore we use only the combination

$$\sigma_{exp}(q^2, K) = \sigma_T(q^2, K) + \epsilon\, \sigma_S(q^2, K)$$

To get a feeling for the magnitudes of σ_T and σ_S, I remind you that $\sigma_{\gamma p}(K)$ is about 115 microbarns in the multi-GeV range and increases slowly as K decreases. Below 1 GeV $\sigma_{\gamma p}$ is several hundred microbarns. Now for small q^2 values, $\sigma_T(q^2, K)$ will have a similar behavior. I will show the larger q^2 behavior later. Electron-proton inelastic scattering experiments have shown that $\sigma_S(q^2, K)/\sigma_T(q^2, K)$ is definitely less than 1 and is usually as small as 0.2 or smaller. Since $\epsilon \leq 1$, the major contribution to σ_{exp} is σ_T.

The measurement of $\sigma_T(q^2, K)$ and $\sigma_S(q^2, K)$ as a function of q^2 and K provides information about the proton's structure and about the strong interactions. Just as in elastic scattering, large values of $|q^2|$, say larger than 1 $(\mathrm{GeV/c})^2$ are regarded as probing the fine spatial structure of the proton. This idea is behind the expressive phrase "deep inelastic scattering" which designates charged lepton-proton inelastic scattering at large $|q^2|$ values. But this is another subject and I must return to the search for muon-electron differences.

There are three reasons why lepton-proton inelastic scattering is a good way to search for muon-electron differences.

1. In elastic scattering, $\nu = |q^2|/2M$, whereas in inelastic scattering, ν and q^2 may be varied independently. Inelastic scattering therefore explores a much larger kinematic region.
2. Measurements of inelastic lepton scattering in which only the scattered lepton is detected, place no restrictions upon the nature of the final hadronic state. It is conceivable that a violation of muon-electron

universality involving hadrons other than the proton would more easily be seen in inelastic scattering than in elastic scattering.

3. It is possible that one or both of the charged leptons, like the proton, have vertex form factors which are decreasing functions of $|q^2|$. If the muon and electron indeed have different form factors, that difference may be most easily detectable at large $|q^2|$. One of the more unexpected results of inelastic muon and electron scattering was the large cross section, compared to elastic scattering, at large $|q^2|$. Hence inelastic scattering may provide greater sensitivity to form factor differences.

VII. Muon-Proton Inelastic Scattering — The Method and the Results

To measure the differential cross section $d^2\sigma/dq^2\,dK$ it is sufficient to just detect the scattered muon and to measure its vector momentum. Thus the experiment in principle is simple. In practice the experiment is difficult because it is necessary to make a muon beam with very low pion (or other hadron) contamination and with a reasonably small phase space. The first condition is required to prevent the contamination of the measurement of the small muon-proton inelastic cross sections by the much larger hadron-proton inelastic cross sections. The former is a few microbarns in magnitude while the latter is almost 10,000 times larger. The second condition is required so that the geometric acceptance of the apparatus can be accurately calculated.

The experiment was carried out at the Stanford Linear Accelerator Center using a 12 GeV/c, muon beam with a momentum resolution of ±1.5%. The apparatus, Fig. 2, consisted of a 198 cm long, liquid-hydrogen target, a large analyzing magnet, optical spark chambers and scintillation counters. The small momentum width and small phase space (3×10^{-3} cm^2 sr) of the incident muon beam allowed inelastic events to be defined by just measuring the scattering angle and final momentum of the muon. The spark chambers which provided this information were triggered whenever three banks of scintillation counters indicated a muon scattering angle greater than 30 mr. Thus irrespective of what hadrons were produced all these scattered muons were detected and their paths recorded. The beam, at the hydrogen target, contained less than 3×10^{-6} pions per muon. An additional pion-to-muon rejection factor of 50 was obtained through the requirement that the scattered muon pass through a series of iron plates and spark chambers without nuclear interaction.

To search for muon-electron differences we compared $\sigma_{\exp,\mu}(q^2, K)$ from our muon experiment[14] with $\sigma_{\exp,e}(q^2, K)$ from electron-proton inelastic scattering. We used the extensive and precise electron-proton inelastic scattering data obtained at the Stanford Linear Accelerator Center by the Stanford Linear Accelerator Center and Massachusetts Institute of Technology electron scattering groups.[16] We interpolated from their kinematic values to our kinematic values. This interpolation in principle depends upon knowing the ratio σ_S/σ_T. But we found that varying that ratio from 0 to 1 produced less than a 1% change in the comparison. This is so because our ϵ is close to 1. We used an average value of 0.18 for that ratio in making the comparison.

Figure 3 presents some of our data and comparable electron data. Our data are indicated by the circles, the electron data are indicated by the x's. The error bars show only the statistical errors. Usually the errors on the electron points are smaller than the x's and are not shown. Both sets of data have been corrected for radiative effects. I will digress from the comparison for a minute, to make a brief remark on the behavior of $\sigma_T(q^2, K)$ as a function of q^2 at fixed K. Since σ_S/σ_T is certainly less than 1, these plots of $\sigma_{\exp}$ are quite direct indications of how σ_T behaves. At small q^2, σ_{exp} and σ_T approach $\sigma_{\gamma p}$ and thus approach one or two hundred microbarns. As $|q^2|$ increases to 3 or 4 $(GeV/c)^2$, σ_{exp} and hence σ_T decreases by a factor of about ten.

In looking at these comparisons, we see that there is no obvious q^2 dependent difference between the muon data and the electron data. But on the average $\sigma_{exp,\mu}$ seems to be a little smaller than $\sigma_{exp,e}$. It is clear that we must use a more quantitative comparison method. Therefore we define

$$\rho_{inelastic}(q^2, K) = \sigma_{exp,\mu}(q^2, K)/\sigma_{exp,e}(q^2, K)$$

We see in Fig. 3 that within the errors the ratio ρ is always about 1.0, but on the average ρ seems to be a little less than one. This is simply another way to describe the observation I just made that σ_{exp} on the average is a little less for the muon than for the electron.

The errors shown in Fig. 3 are purely statistical, but in comparing two experiments we must also consider the possibility of relative overall normalization errors. We have done so, and we estimate that the overall relative normalization error due to systematic uncertainties may be as large as 7%. With this consideration we see that none of the individual deviations of $\rho_{inelastic}(q^2, K)$ from unity are significant.

To combine the data to search for less obvious differences and to quantify the observations I have just made, we need a model. As I said when discussing elastic scattering we have no profound theoretical guidance as to how to select such a model. For inelastic scattering, where the dynamics are even more complicated, we have even less guidance. Therefore I shall first follow convention and use the model used for elastic scattering. Assuming that the charged leptons have a form factor

$$G_\ell(q^2, K) = \left[1 / 1 + |q^2|/\Lambda_\ell^2\right] \qquad (19)$$

and allowing for a normalization problem we obtain

$$\rho_{\text{inelastic}}(q^2, K) = N^2 / \left[1 + |q^2| / \Lambda_d^2\right]^2 \tag{20}$$

just as in Eq. (17). But here we average over K to obtain N and Λ_d. Since N^2 and Λ_d^2, are correlated parameters, the best way to present the fit of the data to Eq. (20) is to use the error contour plot of Fig. 4. The ellipses show the one and two standard deviation contours. The center of the ellipse is at $N^2 = .95 \pm .04$ and $1/\Lambda_d^2 = .021 \pm .021\ (\text{GeV/c})^{-2}$. If just the statistical error are considered, the point $N^2 = 1$, $1/\Lambda_d^2 = 0$ which is demanded by muon-electron universality lies about three standard deviations from the center of the ellipse. But we must allow the overall normalization to have a systematic error as large as ±7%. This means that the origin can be shifted vertically by ±.07. If we allow a systematic error in the normalization to go to its lower limit of -7%, then the center of the ellipse is only about one standard deviation from the prediction of muon-electron universality. Obviously smaller vertical shifts of the origin will give almost as good agreement.

Now regardless of the normalization problem we can set a limit on how small Λ_d can be in Eq. (20). I remind you that a small Λ_d means a large muon-electron difference and a large Λ_d means a small muon-electron difference. No matter how we shift the origin vertically, the largest value of $1/\Lambda_d^2$, for two standard deviations, does not change. This means we can set a lower limit on the value of Λ_d. Therefore with 97.7% confidence $\Lambda_d > 4.1$ GeV/c. Thus we have found no strong, statistically significant, deviation from muon-electron universality in inelastic scattering. The comparison of the inelastic scattering results with the elastic scattering results of Camilleri et al. is shown below.

	Muon-Proton Inelastic Scattering	Muon Proton Elastic Scattering
Best fit $N^2 =$	$.95 \pm .04$	$.95 \pm .04$
Best fit $(1/\Lambda_d^2)(\text{GeV}/c)^{-2} =$	$.021 \pm .02$	$.064 \pm .06$
Λ_d (GeV/c) is greater than	4.1	2.4
with a confidence level of	97.7%	95%

VIII. Speculations on Muon Form Factors

Although no violations of muon-electron universality have been found in these scattering experiments, or for that matter in any experiment; I will use the results of the scattering experiments as a stimulus for some speculations on how such violations might be found. In our inelastic experiment and in the elastic experiment, there is no indication of any q^2 dependent difference between the muon and the proton. But in these experiments the muon cross sections turn out to be lower than the electron cross sections. I emphasized that in our experiment this difference is not significant because the overall normalization uncertainty is about 7%. In the elastic experiments the authors give a smaller normalization uncertainty for the muon data, but the combined overall normalization uncertainty of the muon and electron data might be as large as our 7%. Thus the low muon cross section in any one experiment is not significant. But perhaps, and this is a very weak perhaps, the two experiments together are telling us something. Perhaps they are saying that we have been looking for the wrong kind of muon-electron difference. We have used the form factor

$$G_\mu(q^2) = 1/\left[1 + |q^2|/\Lambda^2\right] \tag{21}$$

to describe the difference. As I have already stated, this form factor is in accord with the belief that any muon-electron difference will increase steadily with $|q^2|$.

But if we look at the muon-proton inelastic and elastic experiments, with no preconceived notions as to how the muon-electron difference might behave with q^2 we would not use that form factor. We would simply observe that all we have seen is a roughly q^2 independent difference in the cross sections. Thus experimentally we would choose the form factor

$$G_\mu(q^2) = N$$

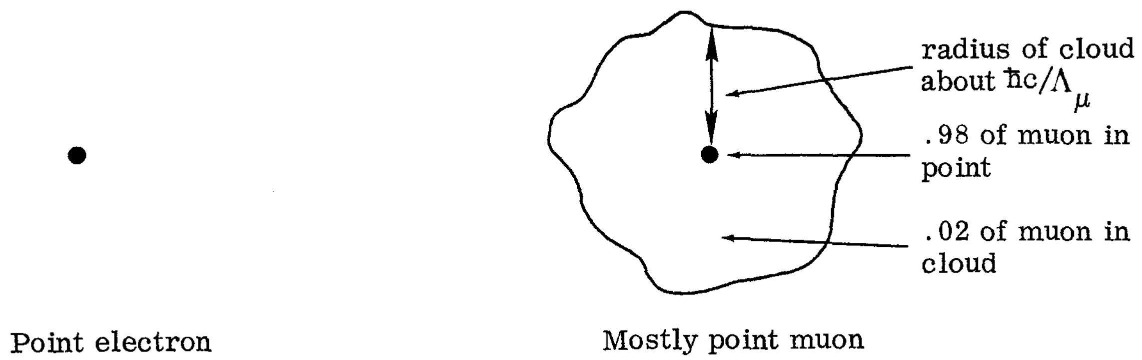
where N is a constant less than 1. But this contradicts $G_\mu(0) = 1$.

Thus it seems reasonable to select a form factor which is close to $G_\mu(q^2)=N$ but does not violate $G_\mu(0)=1$. Such a form factor is

$$\begin{aligned} G_\mu(q^2) &= (1-b) + b/(1 + |q^2|/\Lambda^2) \\ &= 1 - (b\,|q^2|)/(|q^2|+\Lambda^2) \qquad 0 \leq b \leq 1 \end{aligned} \tag{22}$$

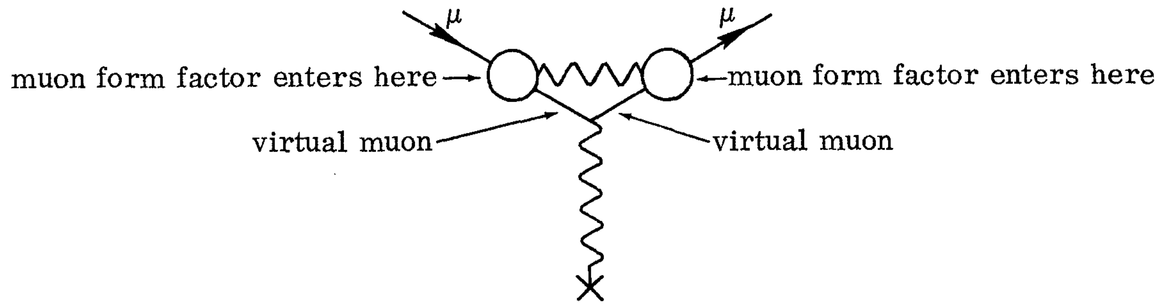
Then in the scattering experiments as $|q^2|$ increases $\rho_{inelastic}(q^2,K) = \rho_{elastic}(q^2) \rightarrow (1-b)^2$. If, for example, b=.02, then all that would be observed even at very high $|q^2|$ values would be a normalization difference of 4%. Such a difference in normalization would be masked by the systematic uncertainties of the scattering experiments under discussion. Thus b=.02 may be taken as an example of a possible muon-electron difference. The form factor of Eq. (22) could come from the following model. Take the electron to be a point charge. Take the muon to have 98% of its electric charge concentrated in a point and just 2% spread out in a halo whose average radius is given by

$r_\mu = \hbar c/\Lambda_\mu$. The picture is



I remind you that this is just speculation. At present all measurements agree, within their errors, with the assumption that the muon is a point particle. We cannot distinguish between the hypothesis of a diffuse, but very small, muon represented by Eq. (21) and the hypothesis of a mostly point particle muon represented by Eq. (22). But the planning of future high energy scattering experiments designed to search for muon-electron differences does depend upon which hypothesis lies closest to the heart of the experimenter. Thus if the experimenter believes in a mostly point particle muon, it would be best to plan an experiment at moderate q^2 values where high statistical precision and low systematic uncertainties can be most easily achieved.

Up to now we have been concerned with the limits set on possible muon form factors by the high energy elastic and inelastic experiments. But the measurement of the gyromagnetic ratio of the muon (g_μ) also sets limits on the muon form factor through the diagram

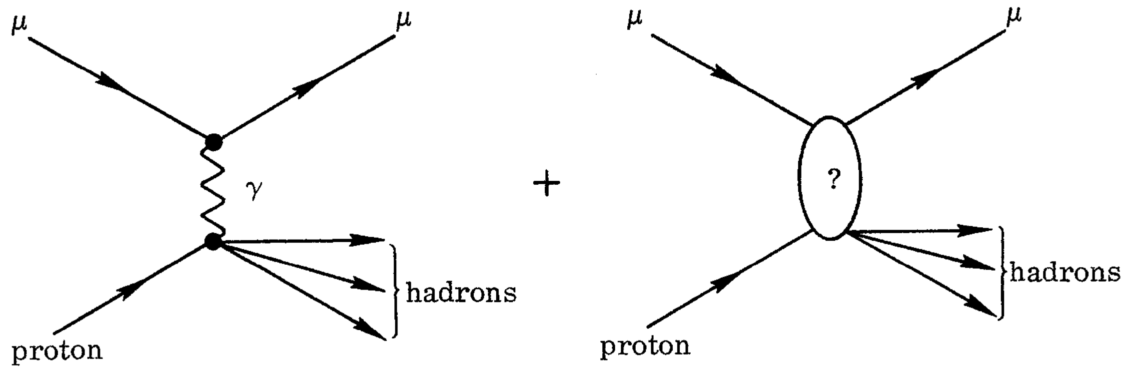


The q^2 which enters the muon form factor in this diagram comes from the virtual muons, and the important q^2 values are those whose magnitude is smaller than $m_\mu^2 c^2$; m_μ is the muon mass. Hence the measurement of g_μ determines the muon form factor with great precision for q^2 values less than .01 $(\mathrm{GeV}/c)^2$. Now the experimental value of g_μ is completely explained by quantum electrodynamics, as discussed earlier in this article. Therefore the muon form factor is unity at small values of q^2.

We can only compare the g_μ measurement of the muon form factor with the high energy scattering measurements of the muon form factor if we assume a specific function for the form factor. If we use Eq. (21), the g_μ measurement requires that $\Lambda_\mu > 7$ (GeV/c) with 95% confidence. This is a higher limit than that set by the inelastic experiment! The g_μ measurement attains this limit in spite of the very low q^2 values involved because of its enormous precision. If we assume some other function for $G_\mu(q^2)$ we will find some other limit. Therefore to connect the g_μ limit on the muon form factor with the high energy scattering limits on the muon form factor we must know the functional form of the form factor. But if we knew that we wouldn't be doing the experiments! Thus the g_μ limits on the muon form factor and the high energy scattering limits on the muon form factors should be regarded as independent limits.

IX. Speculation on Special Muon-Hadron Interactions

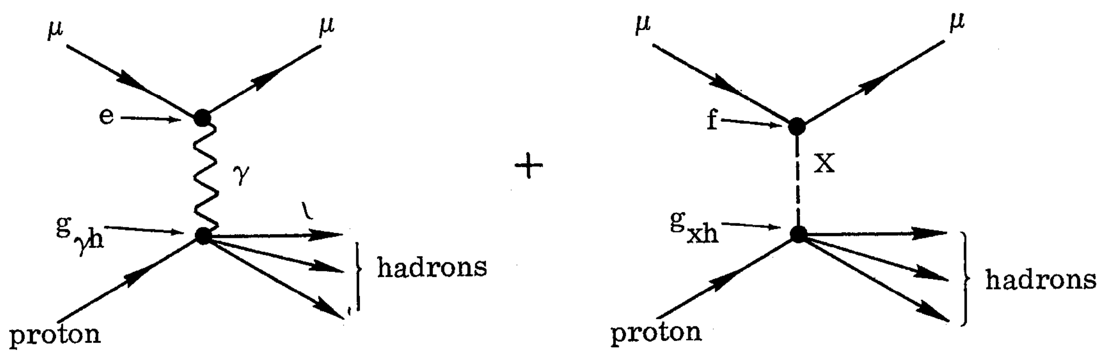
Another way to think about possible muon-electron differences is to speculate that the muon has a special interaction with the hadrons, an interaction not possessed by the electron. I first mentioned this speculation, in the elastic scattering section, in connection with the mass of the muon. Then muon-proton inelastic scattering would take place through the sum of two diagrams as follows.



One-photon exchange Special muon-hadron interaction

The second diagram would result in a difference between muon-proton and electron-proton inelastic cross sections, because only the first diagram would enter in electron-proton inelastic scattering. To determine the limits set on this special muon-hadron interaction by the high energy scattering experiments, a model is necessary.

As an example I shall assume that the muon interacts with the hadrons through the exchange of a particle X with spin 1 and mass M



One-photon exchange One-X exchange

The coupling constants are indicated in the diagrams; thus e is the electric charge. Those at the lower vertices are to be regarded only as very crude measures of the strength of the coupling of the virtual photon or the X particle to hadrons.

Then[17]

$$\rho_{\text{inelastic}}(q^2, K) \approx \left[1 + \left(\frac{f}{e}\right)\left(\frac{g_{xn}}{g_{\gamma n}}\right)\left(\frac{|q^2|}{|q^2| + M_x^2}\right)\right]^2 \tag{23}$$

There is of course a connection between the assumption of a special muon-hadron interaction and the assumption of a muon form factor. If, in Eq. (23) the product of the coupling constants is a negative real number, Eq. (22) is obtained; $M_x = \Lambda$ and $b=(f/e)(g_{xn}/g_{\gamma n})$. Thus the assumption of a special muon-hadron interaction which takes place through the exchange of a vector particle leads back to the model of a mostly point particle muon. The radius of the muon is given by the inverse mass of the exchanged particle.

A conventional speculation is that the X particle is some undiscovered heavy photon, but I prefer the speculation that the X particle is itself a hadron. More generally the X particle might be taken to represent the summation of the interaction of different kinds of hadrons with the muon. To estimate the present experimental limits on f, the coupling of the muon to the hadron X, I take $(g_{xn}/g_{\gamma n})^2$ to be the ratio of a typical hadron-hadron total cross section (30 mb) to the photon-proton total cross section (0.12 mb). As I discussed above, existing muon-proton scattering measurements easily allow b to be as large as .02. Then

$$f/e \approx .02/\sqrt{250} \approx 1/800$$

Thus in this "X=hadron" model, the coupling of the muon to the hadrons is much weaker than the electromagnetic coupling. If such a coupling does exist, it can most likely be found through the study of muon-hadron reactions. It will be difficult to find in purely electromagnetic experiments because the enhancement factor $(g_{xn}/g_{\gamma n})$ will not be available. In the g_μ measurement it will be masked by the strong interaction contribution $a_\mu \approx 5(\alpha/\pi)^3$, which I have previously mentioned.

In speculating on special interactions between the muon and the hadrons, the question arises as to the relation of such speculations to the mu-mesic atom. Will not such a special interaction perturb the energy levels of the mu-mesic atom from those given by Eq. (8)? The answer is yes. But the problem is how to estimate the perturbation. Once again we are faced with the problem of how to go from the high energy limit on a quantity to the low energy limit on that quantity. But here our ignorance is even greater than it was with respect to the question of the q^2 behavior of possible muon form factors. If the nature of the interaction is unknown, we cannot relate the high energy behavior of that interaction to the low energy behavior of that interaction. As an example, the proton-proton and pion-proton interactions have about the same strength at very high energy as measured by their respective total cross sections. But in the very low energy range the proton-proton interactions is several powers of ten stronger than the pion-proton interaction as measured by their respective scattering lengths. When we combine this ignorance with the realization that the mu-mesic atom's energy levels are strongly perturbed by the finite size of the nucleus[9] we conclude that present measurements on mu-mesic atoms do not set useful limits on the interactions of muons with hadrons at high energy.

Finally I must warn the reader that my speculations have been limited to muon-hadron interactions; they have not included muon-hadron-neutrino or neutrino-hadron interactions. There is not the space to discuss those interactions here. But studies of weak interactions at both high and low energy[4, 11] limit the strength of any anomalous neutrino-hadron interactions to less than 10% of the strength of the weak interaction itself. This is a much lower limit than the limits I have been discussing. Therefore the speculation on the muon which I have presented cannot be extended to interactions involving the muon neutrino. This is just as well. We have probably had enough speculations. What we really need before we can understand the muon-electron puzzle are more and better experiments.

FOOTNOTES AND REFERENCES

1. The term lepton describes those spin 1/2 particles which according to present knowledge do not directly take part in the strong interactions. The only known leptons are the e^-, μ^-, ν_e, ν_μ and the four associated antiparticles. The term hadron refers to those particles which do directly take part in the strong interactions. Excluding the leptons and the photon, all known particles are hadrons.

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17. For the application of this equation to muon-proton elastic scattering see D. Kiang and S. H. Ng, Phys. Rev. D2, 1964 (1970) and the references contained in that paper.

FIGURE CAPTIONS

1. $\rho_{elastic}(q^2)$ is the ratio of the muon-proton elastic differential cross section to the electron-proton elastic differential cross section. The principle of muon-electron universality requires $\rho_{elastic} = 1$ for all values of q^2. q^2 is the square of the four-momentum transferred from the lepton to the proton in the scattering process. The error bars represent only statistical errors; the systematic uncertainties are discussed in the test.

2. The apparatus for measuring muon-proton inelastic scattering.

3. For each K interval the upper plot gives the values of $\sigma_{exp,\mu}(q^2, K)$ denoted by a solid circle, $\sigma_{exp,e}(q^2, K)$ denoted by an x and $\sigma_{\gamma p}(K)$ denoted by a triangle. These quantities are defined in the text, but $\sigma_{exp,\mu}$ and $\sigma_{exp,e}$ may be thought of as measures of the respective magnitudes of muon-proton and electron-proton inelastic scattering. q^2 is the square of four-momentum transferred from the lepton. $K = \nu - |q^2|/2M$ where M is the proton mass and ν is the energy lost by the lepton in the laboratory system. The lower plot gives the values of $\rho_{inelastic}(q^2, K) = \sigma_{exp,\mu}(q^2, K)/\sigma_{exp,e}(q^2, K)$. The error bars represent only statistical errors. In most cases the errors in $\sigma_{exp,e}$ are too small to be displayed. The systematic uncertainties are discussed in the text. For the principle of muon-electron universality to be valid $\rho_{inelastic}$ should equal unity for all values of q^2 and K.

4. Contour plots for the parameters N^2 and Λ_d^{-2} obtained by fitting the experimental values of the ratio $\rho_{inelastic}(q^2, K)$ to the equation $\rho_{inelastic}(q^2, K) = N^2/(1.0 + |q^2|/\Lambda_d^2)^2$. The inner ellipse represents one standard deviation and the outer ellipse represents two standard deviations in the fit.

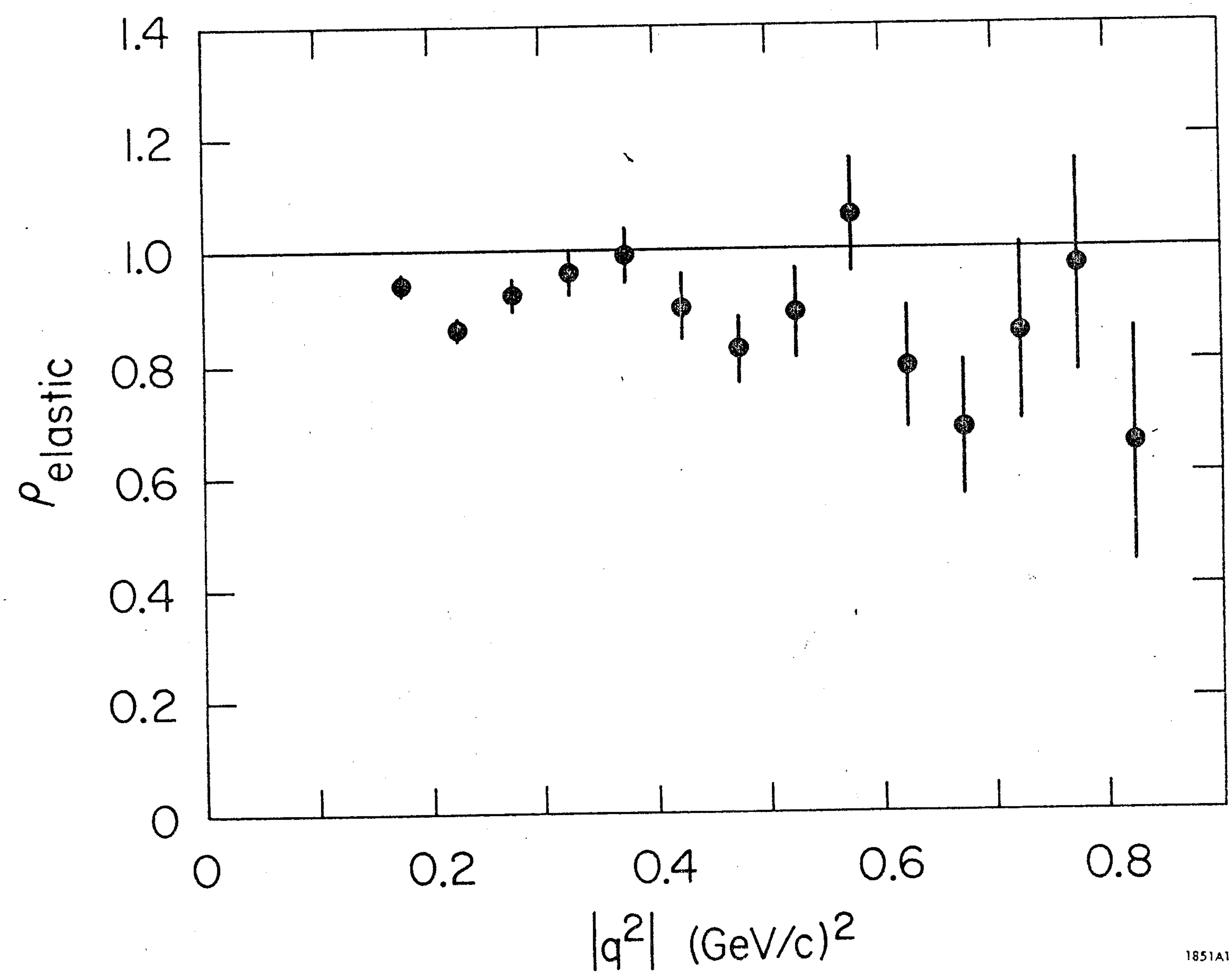
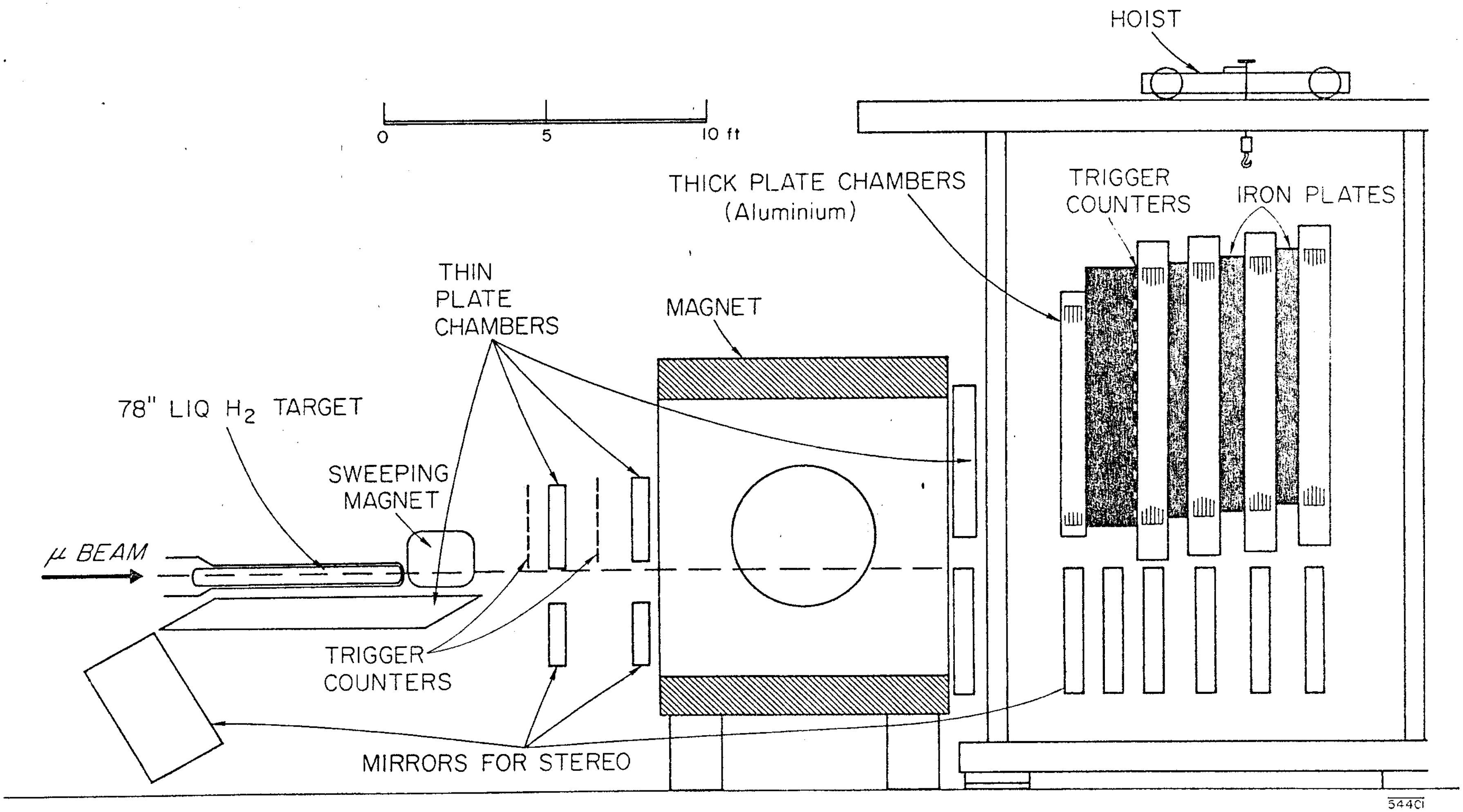


Fig. 1


HOIST
0
5
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COUNTERS
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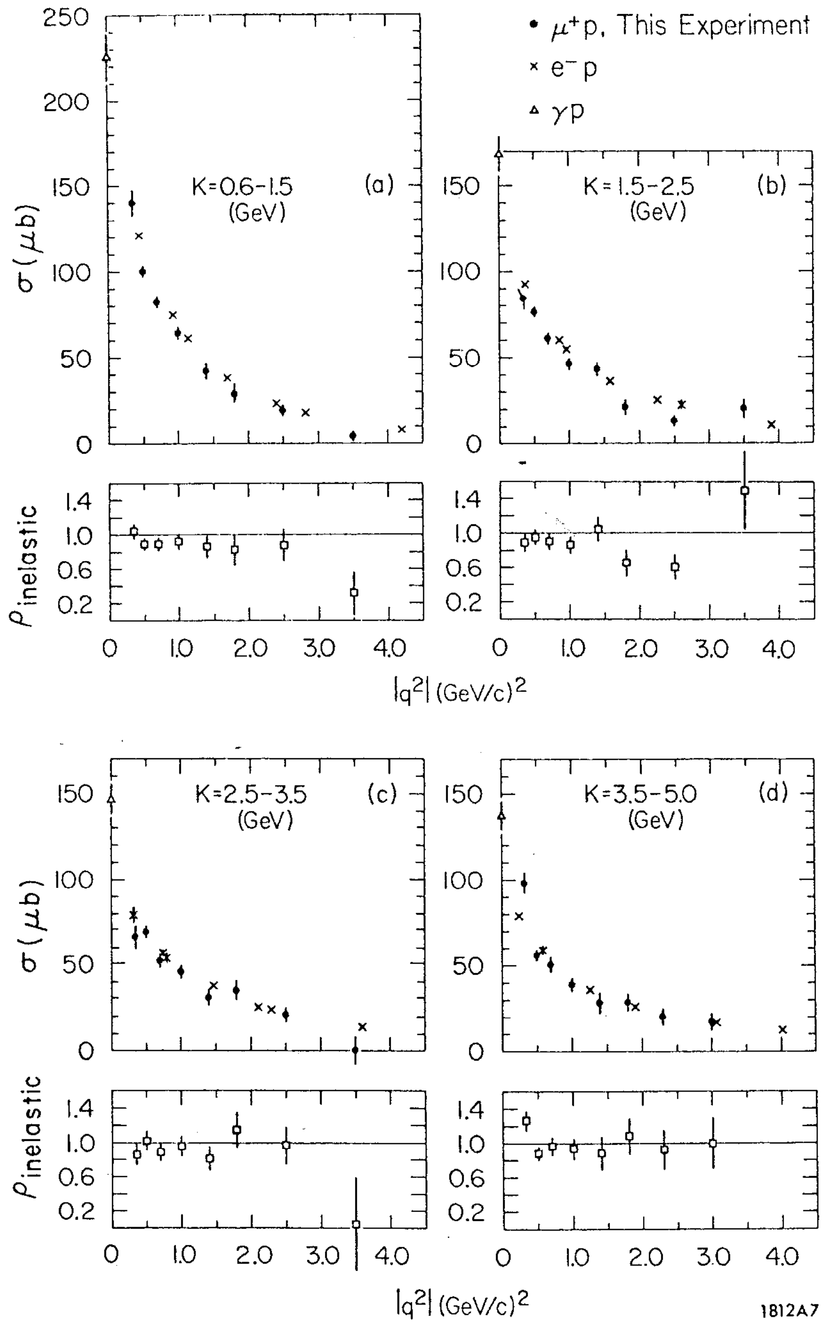
Fig. 2


• μ⁺p, This Experiment
× e⁻p
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σ (μb)
ρ inelastic
K=0.6-1.5 (GeV)
(a)
K=1.5-2.5 (GeV)
(b)
K=2.5-3.5 (GeV)
(c)
K=3.5-5.0 (GeV)
(d)
|q²| (GeV/c)²
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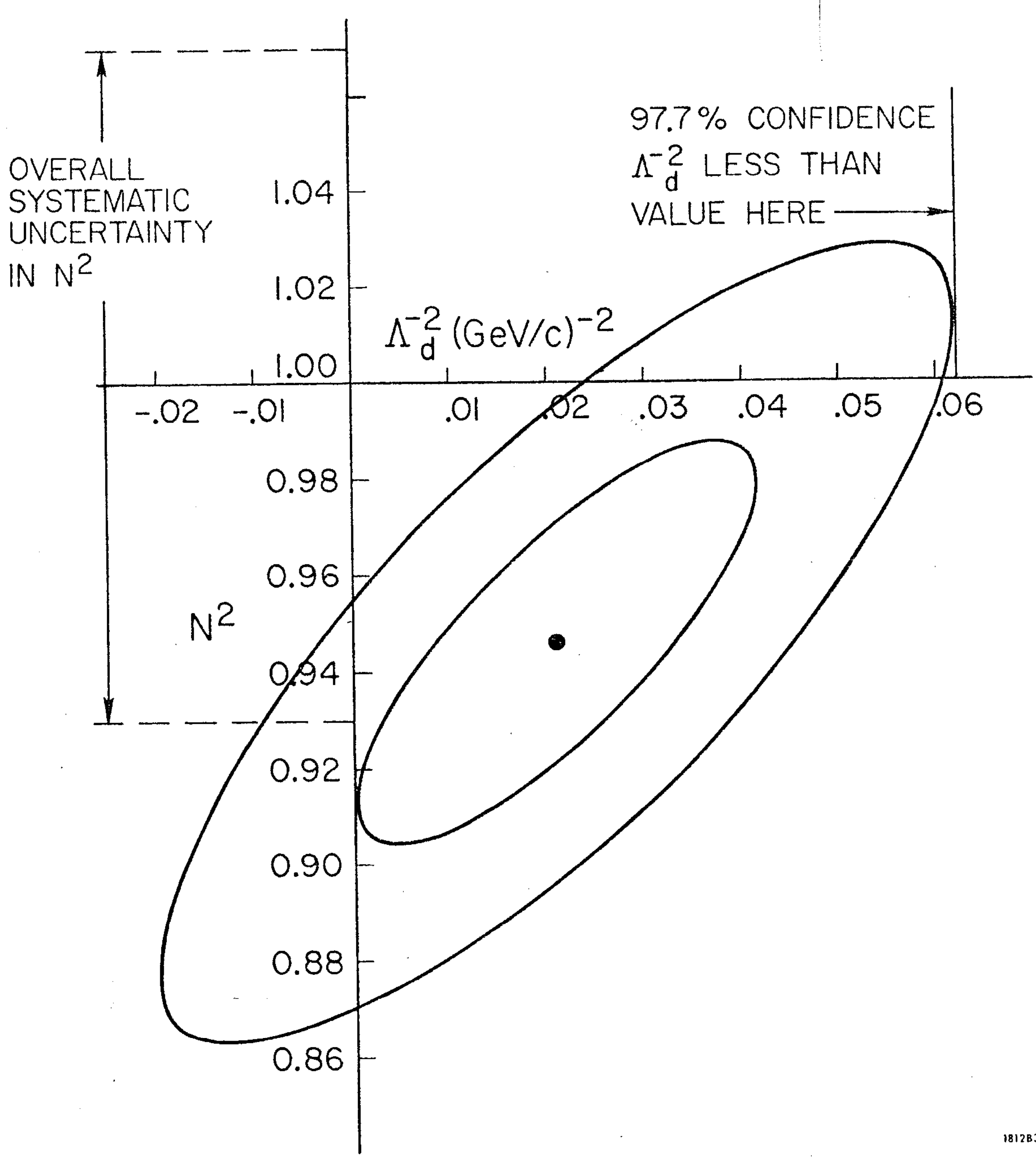
Fig. 3



OVERALL SYSTEMATIC UNCERTAINTY IN N^2
97.7% CONFIDENCE
Λ_d^{-2} LESS THAN VALUE HERE
Λ_d^{-2} (GeV/c)$^{-2}$
N^2
1.04
1.02
1.00
0.98
0.96
0.94
0.92
0.90
0.88
0.86
-.02
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Fig. 4