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ERRATA

p. 9 First line of Eq. (II.10): Add a minus sign immediately after the equal sign.

p. 25 Line 6: Change "giving as significant a contribution" to read "as giving a significant contribution."

p. 29 Last line of Eq. (IV.9): "$\nu_{min} = 3.7\ \mathrm{GeV}^2$" should read "$\nu_{min} = 3.7\ \mathrm{GeV}$."

p. 34 The last term of Eq. (IV.19c): "$\ldots + Cx'\}^{-1}$" should be changed to read "$\ldots + Cx'^2\}^{-1}$."

p. 37 Line 7: After the word "are," add ($\underline{a}$=2, β=4).

p. 41 Equation (V.5a): Add a minus sign immediately after the equal sign.

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THE SPACE-TIME PHENOMENOLOGY OF PHOTON ABSORPTION AND INELASTIC ELECTRON SCATTERING*

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ABSTRACT

We discuss the space-time phenomenology of real and virtual photon absorption by nucleons, with the aim of developing intuition about these processes. We argue that they may be understood in terms of two principal types of interactions, a short-range one which we refer to as a bare photon interaction, and a long-range one which represents the photon's hadronic constituent. This hadronic constituent is not restricted to the surface of the light cone. We give a tentative interpretation of the data in terms of these two types of interaction. The principal feature of our data analysis is that the slow approach to scaling at small x is due to the long-range terms. We find that in general we should expect scaling to set in when $\nu R_p \gg 1$ and $Q^2/m^2 \gg 1$. Here R_p is a size, characteristic of the structure of the target and m is a mass, characteristic of the hadronic constituents of the incident photon.

We assume scaling and a power law fall off at large longitudinal distances for the absorptive part of the forward Compton amplitude. We are then lead to a Regge-like behavior for the total cross section and for the scaling structure functions at small x. The intercept of the Regge trajectory turns out to be intimately related to the power of this large distance fall off.

I. INTRODUCTION

Several years ago, Gribov, Ioffe, and Pomeranchuk (1) suggested that the qualitative features of the energy-momentum dependence of high energy amplitudes could be related to the space-time properties of their Fourier transforms. In photon initiated reactions, this Fourier transform is a matrix element of the retarded commutator of the electromagnetic currents. They were particularly interested in studying the longitudinal range of the interactions. Ioffe (2) subsequently showed that certain features of the experimental results from inelastic scattering of electrons by protons (3) imply that the longitudinal range increases with energy. This result is associated with the small photon mass (q^2), large photon energy (ν), region. Recently, several authors (4) have undertaken the study in the scaling limit (5) (ν and q^2 tending to infinity for finite values of the ratio q^2/ν) to correlate the behavior of the current commutator on the surface of the light cone to that of the inelastic structure functions of nucleons in momentum space. It turns out that the structure functions are something like Fourier transforms of functions measured along the light cone. An analysis by Pestieau, Roy, and Terazawa (6) shows that these functions have a long range (power law) (7) component, tending to confirm the general picture obtained by Ioffe. We will see later how these results are related.

It seems apparent from these results that a study of the space-time behavior of high energy processes is likely to provide a new intuitive way of understanding them. Such a study will encompass more than the behavior at the light cone, determined solely by the scaling behavior. In the present paper, we study inelastic electron scattering at less than asymptotic energies in order to develop this intuition and get some understanding of how the transition to asymptotic behavior takes place and what energies might be relevant in this transition. It will become clear that in addition to the behavior on the light cone ($y^2=0$), the behavior inside the light cone ($y^2 > 0$) is very important for the understanding of the transition. It will turn out that there are two rather separate contributions to the processes under consideration. The first is a short-range contribution which is close to the light cone and which we might term a "bare photon" type of interaction. The other is a long range contribution which has propagation characteristics. We imagine that this corresponds to the "hadronic cloud" of the physical photon; i.e., we imagine that the incoming photon is composed of hadrons a small fraction of the time and the long-range is due to the long time such virtual states may exist at very high energies.

The restrictions on the behavior of the current commutator which lead to scaling in the Bjorken limit (for short, the Bj limit: $\nu \to \infty$, $q^2 \to -\infty$, $x = -q^2/(2M\nu)$ fixed) have been adequately investigated by other authors (4). It turns out that scaling restricts the nature of the singularities on the surface of the light cone to a "canonical" form which is also obtainable from free field theory. We shall accept these restrictions and make the further assumption that the functions are sufficiently smooth in the interior of the light cone that the standard methods (8, 9) may be used to study the asymptotic behavior of the electron scattering structure functions. If necessary, we interpret certain apparently divergent integrals in terms of distribution theory (9). The current commutator can be expressed in terms of two invariant functions of the invariant distance (y^2) and the longitudinal distance ($y \cdot P = y_0 M$ in the laboratory frame). The way scaling sets in as a function of energy and x, and the behavior of the scaling functions depend sensitively on the longitudinal structure of the commutator, as will be seen. The experimental evidence indicates that there are contributions at large longitudinal distances which fall with power law behavior

$(1/y \cdot P)^{\underline{a}}$, with $\underline{a} \geq 1$ (not necessarily integral). This power law behavior for $3 > \underline{a} \geq 1$ leads to a Regge-like cross section $\sigma(q^2, \nu) \sim \nu^{\alpha(0)-1}$ at any fixed q^2, with $\alpha(0) = 2-\underline{a}$. For large $|q^2|$, it gives a power law in q^2 which leads to a scaling function of form (10) $\nu W_2(x) \sim |x|^{\underline{a}-1}$. In particular, diffraction corresponds to a very long range with $\underline{a}=1$. Shorter range terms (including those with $\underline{a} \geq 3$) scale at large energy without regard to whether $|q^2|$ is large or not. Returning to the idea of two contributions mentioned earlier, we think the short range terms correspond to the "bare photon" type of interaction and the $\underline{a}=1$ terms to the "hadronic cloud" type of interaction. An intermediate contribution with $\underline{a}=2$ also seems to be important and we believe it may correspond to an interference between the other two contributions. Whether this is the correct interpretation or not we shall refer to it frequently as the interference contribution. Terms with $\underline{a}=3/2$, corresponding to expected Regge behavior could be present, but do not seem to be exceptionally important. Although such long range behavior can be related to Regge behavior, e.g., through the DGS (11) or JLD (11) representation (12), it is hard to understand intuitively the significance of the particular powers obtained.

We try to apply these ideas to the interpretation of present data on total photon absorption (13) and inelastic electron scattering (3). For this, we need a more specific model of the large distance behavior. Since there is evidence that the vector mesons play an important role in photon interactions, we take the "ρ-dominance" contribution as a model (14) for this large distance behavior. This model seems to give an adequate account of the variation of the structure functions with energy in the small x region. The short distance contributions represent a breakdown of this VMD term as an explanation of all the data. As data is refined and higher energies are obtained, the small x region will give us more information on the hadronic constituents of the photon. To the extent that these terms can be separated out, the remainder will contain information about the interference and short range terms. It is interesting to note that these terms vanish as $|x| \to 0$, in linear and quadratic manners respectively. In the real photon total absorption, the leading hadronic contribution gives an energy independent cross section, the interference decreases like $1/\nu$ and the short range terms as $1/\nu^2$.

II. KINEMATICS AND DEFINITIONS

In this section, we compile the main kinematic formulas for inelastic electron scattering which will be used in the remainder of this paper. Refer to Fig. 1 for the definition of some of the variables; we define in addition (metric (1, -1, -1, -1))

$$
\begin{aligned}
Q^2 &= -q^2 = 4EE' \sin^2(\theta/2) \qquad \text{when } m_e \doteq 0 \\
\nu &= E - E' \\
s &= (q+P)^2 = -Q^2 + M^2 + 2M\nu \qquad\qquad (\mathrm{II.1}) \\
x &= Q^2/2q \cdot P = Q^2/2M\nu \\
x' &= Q^2/(s+Q^2) = Q^2/(M^2+2M\nu)
\end{aligned}
$$

The kinematic domain for inelastic electron scattering is $0 \leq x \leq 1$. The cross section for inelastic electron scattering may be expressed in the laboratory frame ($\vec{p}=0$) as

$$
\begin{aligned}
\frac{d^2\sigma}{d\Omega dE'} &= \sigma_{NS}\left[W_2(Q^2,\nu) + 2\tan^2(\theta/2)\, W_1(Q^2,\nu)\right] \\
&= \left[\Gamma_t/(s-M^2)\right] X(Q^2, s, \theta) \qquad\qquad (\mathrm{II.2})
\end{aligned}
$$

where the dimensionless "experimenter's structure function" is defined as

$$\begin{aligned}
X &= (s-M^2)\left\{\sigma_T(Q^2,s) + \epsilon(Q^2,s,\theta)\,\sigma_L(Q^2,s)\right\} \\
\epsilon(Q^2,s,\theta) &= \left\{1+2(1+\nu^2/Q^2)\tan^2(\theta/2)\right\}^{-1} \\
\Gamma_t(Q^2,s,\theta) &= (\alpha/(4M\pi^2))\,((s-M^2)/Q^2)\,(E'/E)(1-\epsilon)^{-1} \\
\sigma_{NS}(Q^2,E',\theta) &= \left[4\alpha^2 E'^2\cos^2(\theta/2)\right]/Q^4
\end{aligned} \tag{II.3}$$

where α is the fine structure constant.

The usual theoretical structure functions W_i are related to the transverse (σ_T) and the longitudinal (σ_L) virtual photon absorption cross sections by

$$\begin{aligned}
MW_1 &= \frac{(s-M^2)}{8\pi^2\alpha}\,\sigma_T \\
\nu W_2 &= \frac{2x}{\left\{1+\frac{2Mx}{\nu}\right\}}\cdot\frac{(s-M^2)}{8\pi^2\alpha}\cdot(\sigma_T+\sigma_L)
\end{aligned} \tag{II.4}$$

and their ratio is

$$R(x,\nu) = \frac{\sigma_L}{\sigma_T} = \frac{\left\{\left(1+\frac{2Mx}{\nu}\right)\frac{\nu W_2}{2x} - MW_1\right\}}{MW_1} \tag{II.5}$$

These cross sections are positive definite in the physical region $\nu > 0$, $0 \leq x \leq 1$.

These structure functions occur in a gauge invariant expression obtained by summing over all final states of the hadronic system

$$\begin{aligned}
W_{\mu\nu} &= (2\pi)^2\,(P^o/M)\int_{-\infty}^{\infty} d^4y\, e^{iq\cdot y}\langle P|\left[J_\mu(y), J_\nu(0)\right]|P\rangle_{csa} \\
&= -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)W_1(q^2,q\cdot P) + (1/M^2)\left(P_\mu - \frac{q\cdot P}{q^2}q_\mu\right)\left(P_\nu - \frac{q\cdot P}{q^2}q_\nu\right)W_2(q^2,q\cdot P)
\end{aligned} \tag{II.6}$$

where csa means connected and spin averaged. It is sometimes convenient to define another set which is free of kinematic singularities

$$W_{\mu\nu} = -(q^2 g_{\mu\nu} - q_\mu q_\nu)\, C_1(q^2, q\cdot P)$$
$$+\left\{-q^2 P_\mu P_\nu + (q\cdot P)(q_\mu P_\nu + q_\nu P_\mu) - g_{\mu\nu}(q\cdot P)^2\right\} C_2(q^2, q\cdot P) \qquad \text{(II. 7)}$$

clearly the relationship between the two sets is given by

$$\begin{aligned} MW_1 &= -MQ^2 C_1 + M^3 \nu^2 C_2 \\ \nu W_2 &= M^2 Q^2 \nu C_2 \end{aligned} \qquad \text{(II. 8)}$$

Another set which is convenient in connection with the separation into transverse and longitudinal cross sections is

$$\begin{aligned} MW_1 &= M^3 \nu^2 C_T \\ \nu W_2 &= M^2 Q^2 \nu \left[C_T + Q^2 C_L\right] \end{aligned} \qquad \text{(II. 9)}$$

We may transcribe these expressions to configuration space; for example, if $\partial_\mu = \partial/\partial y^\mu$

$$C_1(y^2, y\cdot P) = \left(P_\sigma \partial^\sigma\right)^2 C_L(y^2, y\cdot P)$$

$$C_2(y^2, y\cdot P) = C_T(y^2, y\cdot P) + \partial_\mu \partial^\mu C_L(y^2, y\cdot P)$$

$$C_{\mu\nu} = (2\pi)^2 (P^o/M) \left\langle P\left|\left[J_\mu(y), J_\nu(0)\right]\right|P\right\rangle_{csa} \qquad \text{(II. 10)}$$

$$= (g_{\mu\nu}\partial_\sigma\partial^\sigma - \partial_\mu\partial_\nu)\, C_1 + \left\{P_\mu P_\nu \partial_\sigma \partial^\sigma - P_\sigma \partial^\sigma (P_\mu \partial_\nu + P_\nu \partial_\mu) + g_{\mu\nu}(P_\sigma \partial^\sigma)^2\right\} C_2$$

where $C_i(y^2, y\cdot P)$ so defined are odd and causal (local) functions which are free from kinematic singularities. We will call y^2 the "invariant" distance between the points of absorption and re-emission of the virtual photon and $y\cdot P$ the "longitudinal" distance (also called the range). This is illustrated in Fig. 2. Note: y is not the difference between the coordinate of the photon and the nucleon. Because of this interpretation, the points of absorption and emission can also be represented in a space-time diagram as shown in Fig. 3.

III. RESULTS OF THE SPACE-TIME ANALYSIS

Before giving a more thorough treatment of the space-time approach, we will present a heuristic derivation and discussion of the principal results. It should always be kept in mind that we are interested in large, but not necessarily asymptotic, values of the energy. In particular, we shall frequently use the approximation $\nu^2 \gg Q^2$ to neglect certain terms. Within present experimental accuracy, the data can be interpreted in a reasonable way in terms of our assumption that the asymptotically leading terms do dominate the behavior of deep inelastic electron scattering. There is admittedly the possibility that this could be misleading. We also keep in mind that our aim is to obtain an intuitive understanding of the experimental results in terms of the space-time behavior rather than to compute the actual functions from first principles. We feel that the intuition thus gained will be helpful in evaluating other more fundamental work, which however always involves approximations of one sort or another.

We shall illustrate the ideas by discussing νW_2; the treatment of MW_1 is similar in principle, but involves some additional details. Using the symmetry properties of $W_{\mu\nu}$, we may write

$$iC_2(y^2, y\cdot P) = \epsilon(y\cdot P)\ \theta(y^2)\ f_2(y^2, y\cdot P) \qquad \text{(III. 1)}$$

where f_2 is an even function of $y\cdot P$. It has been shown by other authors (4) that f_2 should be regular near the light cone (and presumably inside as well) in order that νW_2 possess a scaling limit. Using this canonical definition, the expression for νW_2 may be written

$$\nu W_2 = 2Q^2 \nu M^2 \int_{-\infty}^{\infty} d^4y \sin(q\cdot y)\, \theta(y\cdot P)\, \theta(y^2)\, f_2(y^2, y\cdot P) \qquad (III.2)$$

To investigate the integral, we take the 3-axis along $\vec{q}$ in the laboratory frame. Then

$$\begin{aligned} q\cdot y &= q^o y^o - |\vec{q}|\, y^3 \\ &= \frac{1}{2}(q^o + |\vec{q}|)(y^o - y^3) + \frac{1}{2}(q^o - |\vec{q}|)(y^o + y^3) \end{aligned} \qquad (III.3a)$$

Frequently, we use the proton mass to provide a scale for momenta and lengths; i.e., we use units in which M=1. Then we define

$$\begin{aligned} \bar{\nu} &= \frac{1}{2}(q^o + |\vec{q}|) \simeq \nu\left(1 + \frac{x}{2\nu}\right) \\ \bar{x} &= (|\vec{q}| - q^o) = \frac{Q^2}{2\bar{\nu}} \sim x\left(1 - \frac{x}{2\nu}\right) \end{aligned} \qquad (III.4)$$

It is then natural to introduce light cone variables (laboratory frame)

$$\begin{aligned} y_+ &= \frac{1}{2}(y^o + y^3) \\ y_- &= (y^o - y^3) \end{aligned} \qquad (III.5)$$

so that

$$q\cdot y = \bar{\nu} y_- - \bar{x} y_+ \qquad (III.3b)$$

Roughly speaking, y_+ measures the longitudinal range when y is on the light cone, and y_- gives a measure of the separation of y from the light cone. This is illustrated in Fig. 3. Because y_- is multiplied by the large argument ν in (III.3b), the main contributions to the integral are expected to come from small values of y_-; more precisely, the behavior at small values of y_- is supposed to control the integral. This follows from the Riemann-Lebesgue lemma for any fixed y_+ as ν tends to infinity; but, as we shall see, for fixed ν, there may be situations where for large enough y_+, the integral depends on the integrand at finite values of y_-. However, it is generally true that the most important region has $y_- \ll y_+$.

We note that

$$y^2 = 2y_+ y_- - \vec{y}_\perp^{\,2} \tag{III.6}$$

so that (III.2) may be written

$$\nu W_2 = 2\pi Q^2 \nu \iint_0^\infty dy_+ \, dy_- \, \sin(\bar{\nu} y_- - \bar{x} y_+) \int_0^{2y_+ y_-} f_2(\lambda, y_+ + \tfrac{1}{2} y_-) d\lambda \tag{III.7}$$

We shall make the approximation of neglecting y_- compared to y_+ in the second argument of f_2. This approximation leads to contributions which will be shown in Section V to be of order $1/\nu$ compared to terms which are kept. The leading such contribution is of the same form as one of the short range contributions which will be discussed immediately.

Short Range Contributions

Suppose, for the moment, that f_2 contains only short range contributions, i.e., contributions which drop off rapidly when y_+ exceeds a distance of order the proton radius, R_p. Then we can integrate (III.7) by parts twice to obtain the leading term of a series in $1/\nu$

$$\begin{aligned} \nu W_2^{(SR)} &= 2\pi Q^2 \iint_0^\infty dy_+ dy_- \cos(\bar{\nu} y_- - \bar{x} y_+) \, 2y_+ \, f_2^{(SR)} \, (2y_+ y_-, y_+) \\ &= 8\pi x^2 \int_0^\infty dy_+ \, y_+ \left(\frac{\sin(\bar{x} y_+)}{x} \right) f_2^{(SR)} \, (0, y_+) \\ &\quad - 8\pi x \iint_0^\infty dy_+ dy_- \sin(\bar{\nu} y_- - \bar{x} y_+) \, y_+ \, \frac{\partial f_2^{(SR)}}{\partial y_-} \end{aligned} \tag{III.8}$$

The first term is well known from the works of several authors. However, it has not been previously emphasized that it vanishes quadratically in x as $x \to 0$. The second term can be integrated by parts once again and be shown to have at least one more inverse power of ν and to vanish linearly in x as $x \to 0$ (except by accident). Thus, the short range contribution is of the form

$$\nu W_2^{(SR)} \simeq x^2 F_2^{(SR)}(x) + \frac{x}{\nu} G_2^{(SR)}(x) \tag{III.9}$$

Generally we neglect the term $G_2^{(SR)}$. At this point, we include it only because it is the only part of $\nu W_2^{(SR)}$ which contributes to the real photon cross section, which is given by the coefficient of Q^2 as $Q^2 \to 0$. The correction mentioned in connection with (III.7) has the same form as this term, unfortunately making it impossible to give a unique interpretation to the part of the real photon contribution which is of this form $(1/\nu^2)$. This is clearly seen in Eq. (VI.4).

If this were the only contribution, it would disagree with the data on two counts. (i) It vanishes strongly as $x \to 0$ in the scaling limit. The data appear to remain finite in this limit. (ii) The real photon cross section decreases like $1/\nu^2$ at high energies, in disagreement with the observed constant cross section. This disagreement was anticipated, in effect, by the analysis of Roy, Pestieau, and Terazawa. They tried to invert the first term of (III.8) in order to obtain the function f_2 at the light cone. As the data suggest that the scaling function attains a nonzero constant for small x, they were led to the conclusion that in addition to a short range component, $f_2(0, y^o)$ has a long range part.

$$f_2(0, y^o) \sim 1/|y^o| \tag{III.10}$$

The integral in (III.8) is then interpreted in a distribution sense

$$\int_0^\infty dy_+ \sin(\bar{x} y_+) = 1/\bar{x} \tag{III.11a}$$

For later use, we note the other distribution result

$$\int_0^\infty dy_+ \cos(\bar{x} y_+) = \pi\,\delta(\bar{x}) = 0 \quad \text{if } \bar{x} \neq 0 \tag{III.11b}$$

These results can most easily be obtained by taking the real and imaginary parts of

$$\lim_{\epsilon \to 0_+} \left\{ \int_0^\infty dy_+ \, e^{i(\bar{x}+i\epsilon) y_+} = \frac{i}{\bar{x} + i\epsilon} \right\} \tag{III.11c}$$

Since (III.10) is not a short range contribution, it is necessary to consider long range contributions to νW_2.

Long Range Contributions

The simplest type of long range contribution is an inverse power law, as already suggested by (III.10), and as first used by Okubo (7). In the following, we assume such behavior. It is trivial to add in terms which are powers of logarithms times inverse powers, but they do not change the essential features of the analysis. Accordingly, we study contributions to f_2 of the form

$$f_2^{(a)} = h_2^{(a)}(y^2)/|y \cdot P|^{\underline{a}} \tag{III.12}$$

Although this is the asymptotic form for large $y \cdot P$, we extend it, where possible, to the origin, as illustrated in Fig. 4. It will be shown in Section V that the corrections are equivalent to short range terms. The analysis of such terms is now quite straightforward. We make the following transformation of variables

$$\begin{aligned} \beta &= \bar{\nu}\, y_- \\ \eta &= y_+/\bar{\nu} \end{aligned} \tag{III.13}$$

The contribution from (III.12) then becomes

$$\nu W_2^{(\underline{a})} \simeq 2\pi \frac{Q^2}{\nu^{\underline{a}-1}} \iint \frac{d\eta}{\eta^{\underline{a}}}\, d\beta \sin\left(\beta - \frac{1}{2} Q^2 \eta\right) \int_0^{2\beta\eta} h_2^{(\underline{a})}(\lambda)\, d\lambda$$

$$\simeq \frac{Q^2}{\nu^{\underline{a}-1}} G_2^{(\underline{a})}(Q^2) \tag{III.14}$$

Careful examination in Section V of the integral in (III.14) shows that it converges properly only if $\underline{a} < 3$; for higher values of a the asymptotic form cannot be extended to the origin and the contributions are essentially of short range character. We note that the result is not in scaling form. However, we shall see shortly that when Q^2 becomes large, the asymptotic form of $G_2^{(\underline{a})}$ will take a simple power form such that a scaling function results. To show this, we imagine that the β-integration has been carried out to give a result

$$\nu W_2^{(\underline{a})} \simeq \frac{Q^2}{\nu^{\underline{a}-1}} \int_0^{\infty} \frac{d\eta}{\eta^{\underline{a}}} \left\{ \cos\left(\frac{1}{2} Q^2 \eta\right) S_2^{(\underline{a})}(\eta) - \sin\left(\frac{1}{2} Q^2 \eta\right) C_2^{(\underline{a})}(\eta) \right\} \tag{III.15}$$

where Riemann-Lebesgue analysis of the β-integral shows that for small η, the two functions have the asymptotic behavior

$$C_2^{(\underline{a})}(\eta) \simeq b_2^{(\underline{a})} \eta$$

$$S_2^{(\underline{a})}(\eta) \simeq d_2^{(\underline{a})} \eta^2 \tag{III.16}$$

The value of $b_2^{(\underline{a})}$ depends on $h_2^{(\underline{a})}(0)$, while $d_2^{(\underline{a})}$ depends on $h_2^{(\underline{a})\prime}(0)$. For large Q^2, we can examine the asymptotic behavior of the integral and find that only the $C_2^{(\underline{a})}$ term contributes and it leads to the proper power of Q^2 such that

$$\nu W_2^{(\underline{a})} \simeq L_2^{(\underline{a})} |x|^{\underline{a}-1} \tag{III.17}$$

The contribution to the real photon cross section comes from the $S_2^{(\underline{a})}$ term and has the power law

$$\sigma_\gamma \sim 1/\nu^{\underline{a}-1} \tag{III.18}$$

The Combined Result

It will be seen later that separate contributions of the form (III.14) are meaningful in the small x region (say when $|x| \lesssim 0.2$). The reason for this is that for larger x, the contribution is affected by the approximation of extending the large y_+ behavior to the origin. Of course the corrections are again of the short range form, so it is justified to write the total contribution as a sum of terms like

$$\nu W_2 \simeq x^2 F_2^{(SR)}(x) + \frac{x}{\nu} G_2^{(SR)}(x) + \sum_{\underline{a}<3} \frac{Q^2}{\nu^{\underline{a}-1}} G_2^{(\underline{a})}(Q^2) \tag{III.19}$$

but our discussion indicates that such a decomposition is not unique and certainly has no great physical significance for values of $|x|$ larger than about 0.2. One could, for example, redefine the terms in the sum by subtracting their Bj limit values and adding those limits to the short range term to obtain

$$\nu W_2 = \mathscr{F}_2(x) + \sum_{\underline{a}<3} \frac{Q^2}{\nu^{\underline{a}-1}} \hat{G}_2^{(\underline{a})}(Q^2) \tag{III.20}$$

where now

$$\lim_{Q^2 \to \infty} \left\{ \frac{Q^2}{\nu^{\underline{a}-1}} \hat{G}_2^{(\underline{a})}(Q^2) \right\} = 0 \tag{III.21}$$

Taking the Bjorken limit, we find that (III.19) reduces to

$$\nu W_2 \sim x^2 F_2^{(SR)}(x) + \sum_{\underline{a}<3} L_2^{(\underline{a})} |x|^{\underline{a}-1} \tag{III.22}$$

where we have also dropped the asymptotically small nonscaling contribution. The corresponding real photon cross section takes the form

$$\sigma_\gamma \sim \frac{1}{\nu^2} G_2^{(SR)}(0) + \sum_{\underline{a}<3} \frac{1}{\nu^{\underline{a}-1}} G_2^{(\underline{a})}(0) + 0\left(\frac{1}{\nu^2}\right) \tag{III.23}$$

What Regions of Space-Time are Important?

It is interesting to identify the important space-time region for each type of contribution. One cannot strictly say that a given integral comes primarily from a certain region since the magnitude of the integrand may actually be larger in some other region. Integrals of the type we are considering often do have their asymptotic behavior controlled by the properties of the integrand in a given region as long as the modulating functions are sufficiently smooth in other regions and the range of integration includes very many oscillations. For example, the short range result depends only on the value of f_2 on the light cone. This is a good approximation if $f_2(y^2, y \cdot P)$ does not vary significantly as a function of y_- over regions of order $1/\nu$. In common parlance, one says that the integral comes mainly from within $1/\nu$. This is somewhat misleading, since if f_2 varies rapidly within a distance $1/\nu$ anywhere inside the region of integration, the correction terms will be very significant. What we have is a situation where the contributions from two successive half waves in the integral nearly cancel, but leave a small residue. If the modulating function has derivatives of sufficiently high order and ν is large enough, the sum of these small residues is independent of the detailed behavior of the function away from the end points of integration, and can be expressed in terms of the properties of the modulating function at the end points. We assume that the short range contribution has some characteristic distance R_p, which we associate with the proton radius. In a rough sense, we imagine each successive derivative to be of order $1/R_p$ times the preceding one. Then we expect the asymptotic approximation to be good when $\nu R_p \gg 1$.

The longer range terms are a bit trickier. One might incorrectly infer from (III.7) that the main region of y_- is again of order $1/\nu$. However, as mentioned in the preceding paragraph, this could only be true if $h_2^{(a)}(2y_+y_-)$ were a slowly varying function of y_-. Since we expect that $h_2^{(a)}$ is some sort of an oscillatory function, associated perhaps with vector mesons or other particle propagators, we see that for very large y_+ the oscillations in y_- will come at very small distances. This is illustrated in Fig. 3. Thus as y_+ becomes large, the most important region of integration will no longer be near the light cone. It will either spread to the entire region inside the light cone, or more likely, to the stationary phase region where the oscillations in the sine function are matched by those of $h_2^{(a)}$. This is also indicated schematically in Fig. 3. It is for this reason that the result of the y_- (or β) integration in (III. 14) does not give the result (III. 16) for all values of y_+ (or η). The power law growth of (III. 16) must ultimately stop, and perhaps go over to an exponential decrease. Now when we consider the behavior of the η integration as a function of Q^2, we expect that for very small Q^2 the behavior of $C_2^{(a)}$ and $S_2^{(a)}$ at very large values of η will be important. However, as Q^2 increases, the behavior of these functions at small values of η will dominate the result. This is the region usually referred to as

$$\begin{aligned} y_- &\sim 1/\nu \\ y_+ &\sim 2\nu/Q^2 \end{aligned} \tag{III. 24}$$

One should not attempt to give too precise a meaning to relations such as (III. 24). The actual domains are not all well defined, as should be evident from the discussion. These relations indicate only the typical values of these variables within a factor of two or three.

We may now ask the question: At what value of Q^2 can we expect the function $G_2^{(a)}$ to acquire its asymtotic form such that (III. 17) becomes valid? This depends on the scale of the function $h_2^{(a)}$. For example, if the ρ-meson is very important, we would expect the natural scale of y^2 to be $1/m_\rho^2$. The behavior (III.16) should be valid then for

$$\eta \lesssim 1/m_\rho^2$$

or (III.25)

$$y_+ \lesssim \nu/m_\rho^2$$

Then if Q^2 is comparable to m_ρ^2 the contribution from inside the light cone will be important and we obtain (III.14). When $Q^2 \gg m_\rho^2$, the asymptotic result (III.17) is expected. However, if Q^2 is too large, the power law behavior becomes a poor approximation since the value of y_+ in (III.24) may lie inside the nucleon. Thus we expect this contribution to be lost as a meaningful separate term if

$$2\nu/Q^2 \lesssim R_p$$

or (III.26)

$$|x| > 1/MR_p \cong 0.2$$

The virtual photon also suffers a transverse displacement in forward scattering. This can be estimated from $\vec{y}_\perp^{\,2} \le 2y_+y_-$ by using the typical values of y_+ and y_- in the various cases. Thus in the short range case

$$\vec{y}_\perp^{\,2} \sim \mathrm{Min}\,(R_p/\nu\,,\ 1/Q^2) \qquad \text{(III.27a)}$$

This is always much smaller than the size of the proton. On the other hand, the transverse size can be rather large in the long-range case where

$$\vec{y}_\perp^{\,2} \sim \mathrm{Min}\,(1/m^2,\ 1/Q^2) \qquad \text{(III.27b)}$$

Here m is the mass of the intermediate hadronic state. In the region $Q^2 \lesssim m^2$ this is, if anything, an underestimate of the transverse size since we have used the typical range (ν/m^2) at which we expect the contributing region to begin to veer inside the light cone. To the extent that this inside region is important, much larger values of $\vec{y}_\perp^2$ may be attained. The physical meaning of this transverse spreading and its consequences for momentum transfer distributions has been discussed by several authors (15). They argue that the width of momentum transfer distribution and transverse momentum distribution depend on the effective size of the colliding objects. In particular (15) the momentum transfer distribution in a two-body collision is very narrow for a pair of extended colliding objects, while it is broader if one or both the objects are point like. Similar behavior is expected, by them, for the distribution of transverse momentum of the produced particle P in multiparticle production processes, like $B + T \rightarrow P +$ anything. This is not surprising to the extent that uncertainty principle arguments would lead (15c) us to expect that one needs large transverse momentum components to confine the interaction to a small transverse region. And in turn this transverse size of interaction is directly related to the transverse size of the colliding objects and the nature of the collision.

Returning to the short range, or large Q^2, case the entire disturbance created by the photon is contained in a small transverse region, and we think of it as a "bare photon" interaction. For this case, the preceding arguments would lead us to expect a relatively broad momentum transfer distribution. This must be the 'parton' domain from the space-time point of view since if the proton were merely a cloud of charge it would be soft to this localized type of interaction. If $|x|$ was also large we could think of the photon as actually hitting a bit of matter (parton) in the proton. The long range interaction (with small Q^2) is rather different in nature. Because the energy dependence can be scaled out by (III.13) ($2y_+y_- = 2\eta\beta$), the transverse behavior becomes frozen in at high energies. This constancy of the transverse size together with strong absorption leads naturally to the constancy of the total cross section at high energies and narrow momentum transfer distributions. It would also imply a nonshrinking diffraction peak for the $\underline{a}=1$ term. However, we cannot completely rule out a shrinking diffraction peak since our analysis of the $\underline{a}=1$ term implies only that the total cross section is constant and the "characteristic" transverse size is of order $1/\underline{m}$. Therefore it cannot forbid a logarithmic increase in transverse size together with a suitable decrease in absorption to keep a constant cross section and still give a shrinking diffraction peak. We may view this long range contribution in the following way: The incoming photon makes a transition over some distance to its hadronic constituents. These constituents spread transversely, but are limited to stay inside the light cone. If this transition occurs mainly inside the characteristic region ($y_+ \sim \nu/m^2$, $y_- \sim 1/\nu$); the transverse size of the disturbance when it hits the proton will be of order $1/m$. To the extent that it comes from inside the light cone, it may be larger and may correspond to an extended particle hitting the proton. One then has the picture of one extended object hitting another, very much like the usual hadronic interactions.

The $\underline{a}$=1 term

From (III.18), this is the most important term at large energies for real photon absorption; it is of course the term associated with diffraction. It also appears to be the dominant scale violating term at small x. Its contribution to νW_2 is

$$\nu W_2^{(1)} = Q^2 G_2^{(1)} (Q^2) \qquad \text{(III.27)}$$

Because of its long range nature, we believe that this represents the "hadronic contribution" of the photon. Following Gribov (1), we visualize this term as due to the shadowing of the photon's vacuum polarization by the nucleon. Uncertainty principle arguments suggest that a vacuum polarization fluctuation of mass m exists for a distance of order $2\nu/(Q^2+m^2)$. This compares with the values of y_+ in (III.24) and (III.25). If this view is correct, there are many possible contributions to (III.27). The photon may make a virtual transition to charged pairs: pions, kaons, nucleons, etc., to vector mesons, or to more complicated virtual states. The simplest model for these terms is the ρ-dominance model (14); it also represents a reasonably well known and probably very important contribution to real photon absorption. In this model, the transverse and longitudinal cross sections are given by

$$\sigma_{(VT)} = \frac{\sigma_{\gamma T}}{\left(1 + Q^2/m_\rho^2\right)^2} J_T(x)$$

$$\sigma_{(VL)} = \frac{\sigma_{\gamma L}\left(Q^2/m_\rho^2\right)}{\left(1 + Q^2/m_\rho^2\right)^2} J_L(x) \qquad \text{(III.28)}$$

$$\xi = \sigma_{\gamma L}/\sigma_{\gamma T}$$

where ξ is the parameter introduced by Sakurai as the ratio of longitudinal to transverse ρ cross sections. The reason for the deviation of this ratio from unity is not clear, but we may attribute it to differences in the extrapolation from the ρ mass to space-like Q^2. The functions J_T and J_L are introduced to take into account the fact that the Q^2 dependence of (III.28) cannot be taken literally if $|x| \gtrsim 0.2$; they are unity for small x. Practically, the x-dependence of J_T is not too important since the transverse ρ contribution to νW_2 cuts off fast enough automatically as x increases. The longitudinal ρ contribution would give too large a value to νW_2 for $Q^2 \gg m_\rho^2$ and large x were it not for J_L. In our data analysis, we have used the function suggested by Sakurai. Such a function also emerges in a model calculation performed by Lee (14d). Clearly, the ρ contribution is of the form (III.27) for small x where $J_{T,L}$ are approximately 1. It is interesting that the leading ρ term in the Bj limit is provided by the longitudinal ρ part, while the transverse part goes to zero for any fixed x. As will be described later, when the data is fitted assuming these are the only $\underline{a}=1$ contributions, one finds $\xi \cong 0.6$. As seen from Fig. 5 these contributions are not insignificant compared to the total value of νW_2. The space-time behavior of the ρ contributions will be discussed in more detail later.

The $\underline{a}=2$ terms

When the $\underline{a}=1$ contributions are subtracted from νW_2, the result appears to be linear near x=0. Referring to (III.17), we infer that this is probably due to $\underline{a}=2$ contributions. From (III.18), such contributions also give $1/\nu$ terms in the real photon cross section. It is interesting to note that the slope $L_2^{(2)}$ of the νW_2 residue is quite comparable with the observed energy dependence of $\sigma_{\gamma p}$.

What is the physical interpretation of the $\underline{a}=2$ contribution? In our opinion, it represents an interference between the diffractive $\underline{a}=1$ term and the short range terms. This is suggested by its energy dependence, which is the geometric mean of the other two. We have no specific model for the $\underline{a}=2$ contribution; in particular, it is not clear that the slope at the origin ($G_2^{(2)}(0)$) should be the same as that at large Q^2 ($L_2^{(2)}$) although they would be expected to be of the same order of magnitude. Thus the agreement of the slope and the energy dependence of the real photon cross section may be slightly fortuitous.

Other values of $\underline{a}$

While the data do not require other values of a, Regge theory suggests an $\underline{a}=3/2$ contribution, corresponding to a $1/\nu^{1/2}$ energy behavior in the real photon cross section. Our data analysis indicates that $\underline{a}=3/2$ does not dominate $\underline{a}=2$, but a small contribution of this type cannot be ruled out.

IV. ANALYSIS OF THE EXPERIMENTAL DATA

Choice of Parameterization

We have discussed the relation between the various types of space-time behavior and the energy-momentum behavior of the inelastic structure functions. We now confront the published data (3) on inelastic electron scattering to try to identify these various types of contributions and deduce a self-consistent set forming the picture of the process. We wish to emphasize right from the start that the data is not yet sufficient to determine a unique picture. In fact, as we discussed earlier, a separation into a sum of such terms need not be unique, even in principle. The set of terms we select to parameterize the data seems to form a picture which is consistent with our space-time analysis and our knowledge of photon interactions in related processes. In addition it provides a good fit to the data over a wide range including low Q^2 points. Our picture attributes the nonscaling variation at small Q^2 and small x to the presence of long range terms when the photon interacts like an extended object via its "hadronic cloud". The presence of such nonscaling variations in the observed data has also been discussed by Nauenberg (16a) and Nachtman (16b).

The intuitive picture we have developed is a two component (17) one in which the short range contribution may be regarded as a "bare" interaction of the photon and the long range contribution is attributed to the "hadronic cloud" of the physical photon. Such a two component picture was first proposed by Gribov and Ioffe (1, 2), who, however, chose to disregard the short-ranged part giving as significant a contribution. On the other hand, we find that the data does have a significant amount of relatively short-ranged contributions and so it should not be ignored.

To test such a picture against the data we must parameterize these contributions in a reasonable manner. We choose as our scaling variable the quantity

$$x' \simeq x\left[1 - M/(2\nu) + 0(M/(2\nu))^2\right]$$

For $x \simeq 1$ we find $x' \simeq \bar{x}$ where $\bar{x}$ is the natural scaling variable introduced in Eq. (III. 4) when we transform to the light cone variables. When $\nu \gg M$ both x' and $\bar{x}$ approach the variable x. Phenomenologically and historically, when the lower energy data was plotted versus x' rather than x, it was found to fall on a more universal curve and a duality picture was better satisfied (18). Therefore x' was called a "better" scaling variable. The variable $\bar{x}$ was not tried, but our space-time analysis would indicate that $\bar{x}$ should be a better scaling variable since it occurs more naturally in the analysis. However, one can also offer possible theoretical reasons for x' to be a "better" scaling variable. One such reason, which applies to an analog of the short-ranged part, was offered in Ref. 19. In view of the ambiguities and multitude of such arguments we feel that the petty differences between various scaling variables cannot have great physical significance. The best choice of variables is, in our opinion, primarily an empirical question. So we use the variable x'. Then our results can be compared with other recent data analysis. Besides, the plots do look more universal.

A simple uncertainty principle argument leads us to picture the long-ranged terms as a shadowing by the nucleon of the vacuum polarization fluctuations of the photon. These contribute to the time-like normal threshold cuts in the complex q^2 plane for a fixed real s (19). Imbedded in these cuts are the vector meson poles which in some photon initiated reactions seem to dominate the cut contributions. Thus it seems reasonable to assume that one can parameterize this very long-ranged ($\sim 1/(y \cdot P)$) hadronic-like part of the photon interaction by diffracting vector mesons a la Sakurai (14). This is far more economical in parameters than a general spectral representation of the normal threshold cut contribution. Also, since the coupling of ρ^o to the photon is much stronger than that of ω^o and ϕ^o, we further simplify the analysis by using a single vector meson term whose mass we expect to be close to m_ρ. Though we believe that the diffracting vector meson states contribute significantly, we also believe that other low threshold multiparticle hadronic states should contribute to this hadronic part. The present data is insufficient to pin-point the location or the indicial power of the pole or to rule out the possibility of the contributions from the remaining thresholds and poles at $q^2 > 0$. Thus the present data analysis must be regarded as a very preliminary one which incorporates only the broad features of the underlying physics.

These arguments lead us to choose the following form for the contribution from the transverse and longitudinal parts

$$\begin{aligned} \sigma_{(VT)}(s, Q^2) &= \left\{ \sigma_{\gamma T} / \left(1 + Q^2/m_V^2\right)^2 \right\} J_T(s, Q^2) \\ \sigma_{(VL)j}(s, Q^2) &= \left\{ \sigma_{\gamma L} \left(Q^2/m_V^2\right) / \left(1 + Q^2/m_V^2\right)^2 \right\} J_{Lj}(s, Q^2) \end{aligned} \qquad \text{(IV.1)}$$

where $\sigma_{\gamma T}$, $\sigma_{\gamma L}$, m_V^2 are constant parameters, and the functions J_T and J_L reflect the arbitrariness (14c) in extrapolating the pure pole dominance hypothesis away from the pole. In the analysis we describe, we use the Sakurai's (14a) VMD choice for these functions, viz:

$$
\begin{aligned}
J_T(s, Q^2) &= 1 \\
J_{LS}(s, Q^2) &= \left\{1 + Q^2/(s-M^2)\right\}^{-2} = (1-x)^2
\end{aligned}
\tag{IV.2}
$$

With this choice, the parameters $\sigma_{\gamma T, L}$ are related to the vector meson total cross section by

$$
\begin{aligned}
\sigma_{\gamma T} &= (e/f_V)^2 \, \sigma_{Vp}^T \\
\sigma_{\gamma L} &= (e/f_V)^2 \, \sigma_{Vp}^L
\end{aligned}
\tag{IV.3}
$$

and their ratio is

$$
\xi = \frac{\sigma_{Vp}^L}{\sigma_{Vp}^T} = \frac{\sigma_{\gamma L}}{\sigma_{\gamma T}} \tag{IV.4}
$$

Such an identification need not be unique as was discussed by Wu-Ki Tung (14c). We also tried other possible choices for J_L. As we shall see, these do not work as well as J_{LS} in our analysis. These choices were

$$
J_{L1} = \left\{1 + Q^2/(s-M^2)\right\}^{-1} = (1-x) \tag{IV.5}
$$

$$
J_{L2} = \left\{4Q^2M^2 + (s + Q^2 - M^2)\right\}^{1/2}/(s-M^2) \tag{IV.6}
$$

We expect that terms in the range $1 < \underline{a} < 3$ may also be present, but we have no physical model for parameterizing them. It is also clear that corrections to our simplified parameterization of the $\underline{a}=1$ term, which dominates the small x region, would tend to mask the Q^2-dependence of the remaining terms. Therefore, as an expedient, we choose a Padé approximant in x' to parameterize the remaining contribution from terms with $\underline{a} > 1$ and short-range terms. Since experimentally $\sigma_L \ll \sigma_T$ and since the presently observed σ_L will be found to be adequately parameterized by VMD, we assume that these terms with $\underline{a} > 1$ are purely transverse. This, admittedly, may be an oversimplification. Thus we have chosen the functional form

$$X_{T(SC)}(s, x') \equiv (s-M^2)\sigma_{T(SC)}(s, x') = \frac{2MG(x')^{a-2}(1-x')^{\beta}}{1-2Bx' + Cx'^2} \qquad (IV.7)$$

for the contribution of the not so long-ranged (SC) terms to the transverse part of the experimenter's structure function X defined in Section II. In this form a and β are fixed parameters which are not varied by the fitting program. M is the target proton mass. G, B and C are the three variable parameters. In general, these parameters could be functions of s if one does fixed s fits to the x' behavior. Such a form is also suggested by computing (19) the contribution of single particle exchange graphs leading to two particle final states. Such graphs lead to functions with complex singularities (19) of the amplitude in the complex x' plane (at fixed s) which on Fourier transformation could lead to an exponential decay in $y \cdot P$ and thus act like a rapidly scaling short-range term. However, for our phenomenological analysis we just consider the above form to be a nice approximant.

With such a phenomenological model in mind, we have analyzed the published data (3) consisting of 198 points in the kinematic range

$$\left\{0.25 \leq Q^2 \leq 20.1 \text{ GeV}^2, \quad 3.6 \leq s \leq 26.8 \text{ GeV}^2, \quad 0.025 \leq x' \leq 0.825\right\}$$

Out of these there are 6 points in the range $\left\{0.25 \leq Q^2 \leq .5 \text{ GeV}^2,\ x' < 0.1\right\}$ and about 5 points in the range $\left\{0.5 < Q^2 < 1 \text{ GeV}^2,\ x' < 0.1\right\}$. Because of the low

Q^2 values these are usually not shown by others (3) on the "scaling" plots. On the other hand, to us there points are of special interest since they are expected to be most sensitive to the Q^2 dependence of the long range terms. All these points are plotted in Fig. 6 assuming $R = 0.18$.

Fits of the Real Photon Cross Sections

Our analysis consists of seeking the most rapidly scaling part of the data by subtracting out a reasonable long-ranged hadronic contribution. We believe that the relative proportion of these terms is already exhibited by the on-mass-shell photon absorption data which is consistent with the asymptotic form

$$\sigma_{\gamma p}(\nu) = \sigma_{\gamma p}(\infty) + G_p/\nu \tag{IV.8}$$

The experimental values of the parameters $\sigma_{\gamma p}(\infty)$ and G_p are strongly dependent on the low energy cut off ν_{min} on the data fitted. Our fits to the high energy data from UCSB (13) give

$$\begin{aligned} \sigma_{\gamma p}(\infty) &= 108.0 \pm 1.8\ \mu b \\ G_p &= 106.5 \pm 13.5\ \mu b\ \text{GeV} \\ \chi^2/DF = 33.3/34\ , &\qquad \nu_{min} = 3.7\ \text{GeV}^2 \end{aligned} \tag{IV.9}$$

However, the errors on these parameters could be misleading because of the cutoff dependence. Increasing the cutoff tends to decrease $\sigma_{\gamma p}(\infty)$ and increase G_p, and the range of these variations can be more than twice the size of the errors. Furthermore this fit is not unique. We can fit the data with equally good χ^2 by using the forms

$$\sigma_{\gamma p}(\nu) = \sigma_{\gamma p}(\infty) + G'_p/\nu + T'_p/\nu^2 \tag{IV.10a}$$

$$\sigma_{\gamma p}(\nu) = a_p + b_p/\nu^{1/2} \tag{IV.10b}$$

$$\sigma_{\gamma p}(\nu) = a'_p + b'_p/\nu^{1/2} + c'_p/\nu^{3/2} \tag{IV.10c}$$

In view of these experimental uncertainties we keep $\sigma_{\gamma p}(\infty)$ and G_p as variable parameters determined by out fits to the inelastic data. We then compare these with the real photon ($Q^2=0$) absorption data for consistency.

Our space-time analysis gives the leading terms which are permitted. For example, for $\underline{a}=1$, the cross section for large Q^2 can have a leading behavior $(1/Q^2)$ which gives a scaling contribution to νW_2 with finite intercept at $x'=0$. The asymptotic constant real photon cross section $\sigma_{\gamma p}(\infty)$ indicates the presence of such a term. From Eq. (IV.1), we note that the longitudinal vector cross section has this leading behavior while the transverse one has the nonleading behavior $(1/Q^4)$. Pure VMD is ruled out by the finite value of R for large ν and Q^2; but since we now admit the possibility of a short-range transverse contribution, the data do not contradict the presence of an important vector meson contribution. Our data analysis does not rule out the possibility of a small transverse $\underline{a}=1$, contribution which would have leading (i.e., $\propto 1/Q^2$) behavior and hence violate transverse VMD. For this reason, it would be interesting to study $\sigma_T(Q^2, \nu \to \infty)$ carefully as a function of Q^2. The longitudinal VMD term scales. Its contribution to νW_2 is $\left\{(4\pi^2 \alpha)^{-1} (m_V^2 \sigma_{\gamma L}) (1-x')^3\right\}$. It does not contribute to the real photon cross section which is purely transverse.

This transverse diffractive part is expected and found to be equal for proton and neutron targets. This is seen by a fit to the UCSB (13) neutron data which gives

$$\sigma_{\gamma n}(\infty) = 105.7 \times 3.2\ \mu b$$

$$G_n = 54.0 \pm 26\ \mu b\ \text{GeV} \tag{IV.11}$$

$$\chi^2/\text{DF} = 42/34, \qquad \nu_{min} = 3.7\ \text{GeV}$$

Clearly $\sigma_{\gamma p}(\infty) \simeq \sigma_{\gamma n}(\infty)$ within errors.

We believe that the remaining energy dependent part of the real photon absorption cross section reflects the contribution of $\underline{a}=2$ and shorter-range terms which lead to nonzero scaling functions when we go off shell ($Q^2 \neq 0$). We picture the G/ν term as the $\underline{a}=2$ term caused by interference of the long range part and a very short range term T/ν^2. If such interpretation as an interference term is meaningful then the Schwartz inequality requires

$$T_p \geq \left| \frac{G_p^2}{4\sigma_{\gamma p}(\infty)} \right| \tag{IV.12}$$

This leads to contributions which are hard to deduce from the present data since they are smaller than the errors on G_p and also somewhat ambiguous due to the correction terms discussed in Section III and seen in Eq. (VI.4).

A particular feature of a G/ν term in the cross section is that it leads by crossing symmetry to a real part in the forward Compton scattering amplitude of the form

$$\left\{ \mathrm{Re}\, f_1(\nu) \right\}_{\mathrm{sym}} = -\left\{ G/(2\pi^2) \right\} \ln(\nu/\nu_c) \tag{IV.13}$$

where ν_c is an arbitrary constant and leads to an arbitrary additive constant in the real part. Therefore, in the present non-Regge model one does not need the fixed pole at the wrong signature nonsense point $\alpha=1$ since one does not talk of a "Pomeron exchange" and one does not need the fixed pole at the right signature nonsense point $\alpha=0$ since the logarithmic real part is arbitrary up to an additive constant (20).

Since the shorter range terms with $\underline{a} > 1$ are expected to start probing the internal structure of the target nucleon, there is no reason to expect them to be the same for the proton and the neutron. This, of course, is evidenced experimentally by the observation that for the nondiffractive parts.

$$G_n/G_p \simeq 1/2 \tag{IV.14}$$

This has interesting implications for the data on inelastic electron scattering from neutron targets.

<u>Attempts to Obtain Scaling by Subtracting a Transverse VMD Term</u>

We first try to find a fit omitting the longitudinal VMD contribution. For this purpose, we use a constant R=0.18 and take $m_V^2 = m_\rho^2 = 0.585\ \mathrm{GeV}^2$ and try the following fit to the data as a minimum χ^2 fit.

$$X_p(s,x',\theta) = \left[\frac{(S-M^2)}{\left(1+Q^2/m_V^2\right)^2}\,\sigma_{\gamma T} + \frac{2MG(x')^{a-2}(1-x')^{\beta}}{1-2Bx'+Cx'^2}\right]\left[1+\epsilon(S,x',\theta)R\right] \tag{IV.15}$$

This fit in effect subtracts the transverse VMD contribution to obtain the most universal residue. We find that a best fit is obtained when we fix $\underline{a}=2$ and $\beta=4$ and $\sigma_{\gamma T}$ to be energy independent. The parameters of the best fit are

$$\begin{aligned} \sigma_{\gamma T} &= 81.7 \pm 5.8\ \mu b \\ G_p &= 154.1 \pm 7.0\ \mu b\ \mathrm{GeV} \\ B_p &= 0.31 \pm 0.08 \\ C_p &= 0.32 \pm 0.19 \\ \chi^2/DF &= 323.6/194 = 1.7 \end{aligned} \tag{IV.16}$$

The residual $\nu W_{2(SC)}$ and this fit are shown in Fig. 7a. From it we clearly see that the subtraction is leading to a more universal $\nu W_{2(SC)}$ than the total $\nu W_{2(TOT)}$. On the other hand the residual $\nu W_{2(SC)}$ is significantly less universal if we make an energy dependent subtraction by vector dominating the whole of $\sigma_{\gamma p}(\nu)$ rather than just $\sigma_{\gamma p}(\infty)$. This is seen in Fig. 7b. To us this indicates that it is only the asymptotic real photon cross section that should be vector dominated, corresponding to the photon acting purely as a hadronic state. It is interesting to note that after subtraction the residual νW_2 seems to vanish almost linearly with x' as $x' \to 0$. However, the slope G_p is somewhat higher and $\sigma_{\gamma T}$

somewhat lower than liked by the best fits to the real photon data. This is not too worrysome since one can find a compromise solution which is a reasonable fit to both sets of data and best fit to neither. Also as we explained $G = G_2^{(2)}(0)$ may not equal $L_2^{(2)}$. To us this simple subtraction indicates that at low Q^2 there is a very significant long range contribution in the data which can be reasonably parameterized by the transverse VMD. However, the rather poor χ^2 shown in (IV.16) indicates that some physical contribution is missing at this stage. We will see shortly how the inclusion of the longitudinal VMD term results in a dramatic improvement in the fit.

This model has interesting implications for the inelastic structure functions for the neutron. If we assume that R is equal for the neutron and proton targets, then experimentally

$$(\sigma_{\gamma T})_p \simeq (\sigma_{\gamma T})_n \qquad G_p \simeq 2\, G_n \tag{IV.17a}$$

So we may expect that the ratio

$$\frac{\nu W_{2n}(s,x')}{\nu W_{2p}(s,x')} \sim 1 \qquad \text{as} \quad x' \to 0 \tag{IV.17b}$$

since it is dominated by the diffractive $\underline{a}=1$ terms. The difference

$$\left\{\nu W_{2p}(s,x') - \nu W_{2n}(s,x')\right\} \sim \left\{M(G_p - G_n)/(2\pi^2\alpha)\right\} x' \qquad \text{as} \quad x' \to 0 \tag{IV.17c}$$

since it gets dominated by the $\underline{a}=2$ terms in our model. These predictions seem to be borne out by the present inelastic data (3). However, the errors are too large to provide a real test. Also as we will see ahead, we cannot rule out an additional small $\sqrt{x'}$ contribution coming from the $\underline{a}=3/2$ terms.

Combined Transverse and Longitudinal VMD Subtraction to Obtain Scaling

Since the subtraction of the transverse VMD alone did not leave a remainder that scaled well, we next study the consequences of assuming that the longitudinal cross section is purely a long range leading term parameterized by longitudinal VMD. We find that any one of the three choices for $\sigma_{(VL)j}$ fit the 23 experimentally separated data points (3) for $\sigma_L(s, Q^2)$. The parameters for these one parameter fits ($m_V^2 = m_\rho^2$) are as follows

(i) For $\sigma_{(VL)S}$ we get $\sigma_{\gamma L} = 43.4 \pm 8\ \mu b$ with $\chi^2/DF = 8/22 = 0.4$

(ii) For $\sigma_{(VL)1}$ we get $\sigma_{\gamma L} = 28.6 \pm 4.6\ \mu b$ with $\chi^2/DF = 4.8/22 = 0.2$

(iii) For $\sigma_{(VL)2}$ we get $\sigma_{\gamma L} = 2.9 \pm 0.6\ \mu b$ with $\chi^2/DF = 22.7/22 = 1.0$

(IV.18)

The smallness of the χ^2 indicates that the errors on the data are either not independent statistical errors or are very overestimated. Thus this separated data cannot distinguish between the various possibilities for σ_L.

We now try to fit the data under the assumption that the longitudinal cross section σ_L is completely dominated by ρ-meson a la Sakurai. The total transverse cross section σ_T is now assumed to be a sum of transverse VMD ($\sigma_{(VT)}$) and a shorter-range contribution $\sigma_{T(SC)}$ which leads to a rapidly scaling $\nu W_{2(SC)}$. So now the fitting formula is

$$X_p(x', s, \theta) = \left\{X_{(VT)} + X_{T(SC)}\right\} + \epsilon(x', s, \theta)\ X_{(VL)} \qquad \text{(IV.19a)}$$

where using Eqs. (IV.1), (IV.2), and (IV.7) we get the transverse parts to be

$$X_{(VT)}(x', s) \equiv (s - M^2)\ \sigma_{(VT)}(x', s)$$

$$\simeq \left\{x'^{-1}(1-x')^2\ m_V^2\, \sigma_{\gamma T}\right\} \left\{m_V^2/(x's)\right\} \to 0; \qquad s \gg \left(m_V^2/x', M^2\right) \qquad \text{(IV.19b)}$$

$$X_{T(SC)}(x', s) \equiv (s - M^2)\ \sigma_{T(SC)}(x', s)$$

$$= \left\{2MG\, x'^{\underline{a}-2}(1-x')^{\beta}\right\} \left\{1 - 2\,Bx' + Cx'\right\}^{-1} \qquad \text{(IV.19c)}$$

and the longitudinal part is

$$X_{(VL)}(x', s) \equiv (s-M^2)\ \sigma_{(VL)}(x', s)$$

$$\simeq \left\{x'^{-1}(1-x')^3 \left(m_V^2\, \sigma_{\gamma L}\right)\right\}; \qquad s \gg \left(m_V^2/x', M^2\right) \qquad \text{(IV.19d)}$$

In these expressions we should observe that scaling at a given x' occurs only when $s \gg \left\{m_V^2/x', M^2\right\}$. In the data analysis m_V^2, $\underline{a}$ and β are fixed parameters and the other 5 parameters are variable.

Using such an expression with $m_V^2 = m_\rho^2 = 0.585\ \mathrm{GeV}^2$ we find that the best fit is obtained when we fix $\underline{a}=2$, $\beta=4$. The parameters of the fit are

$$\begin{aligned} \sigma_{\gamma T} &= 97.5 \pm 6\ \mu b \qquad \xi = 0.58 \pm 0.09 \\ \sigma_{\gamma L} &= 56.3 \pm 5\ \mu b \\ G_p &= 117.5 \pm 7\ \mu b\ \mathrm{GeV} \\ B_p &= 0.63 \pm 0.07 \\ C_p &= 0.96 \pm 0.18 \\ \chi^2/DF &= 187/193 = 0.97 \end{aligned} \qquad \text{(IV.20)}$$

The residual $\nu W_{2(SC)}$ and this fit are shown in Fig. 8. From the χ^2 and Fig. 7 we clearly see that $\nu W_{2(SC)}$ in this case is significantly more universal than that obtained by assuming a constant R and subtracting just the transverse VMD contribution. This indicates that the Q^2 dependence of the longitudinal part is different from that of the transverse part. The energy dependence and the relative proportion of the above terms in νW_2 is shown in Fig. 5. From this figure it is clearly seen that for large x', the nonscaling caused by the transverse VMD term $\nu W_{2(VT)}$ is compensated by the longitudinal VMD term $\nu W_{2(VL)}$. The opposite happens for small x'. That is partly why the present parameterization works better than the one, in Eq. (IV. 15), without the longitudinal VMD. The energy dependence in $\nu W_{2(SC)}$ is due to the kinematic factors relating MW_1 and νW_2.

If we take this "ρ-dominance" model seriously enough to identify the parameters of our fits with the actual cross sections, then we would expect the real photon absorption cross section to be given approximately by

$$\sigma_{\gamma p}(\nu) = \sigma_{\gamma T} + G/\nu$$

$$\cong (98\pm6) + (118\pm7)/\nu \ \mu b \qquad \text{(IV.21a)}$$

and

$$\xi = \frac{\sigma_{\gamma L}}{\sigma_{\gamma T}} = \frac{\sigma^L_{Vp}}{\sigma^T_{Vp}} \cong 0.58 \pm .09 \qquad \text{(IV.21b)}$$

It is satisfying that this agrees surprisingly well with the measured real photon total absorption cross sections and the separated longitudinal cross section, although the latter is of course not a completely independent result.

This fit is rather sensitive to the choice of the factor J_{Lj} and the parameters m_V^2, $\underline{a}$ and β. If one uses the factor J_{L1} instead of J_{LS} one can again obtain a reasonable fit with a χ^2 of 1.1 per degree of freedom. However, J_{L2} leads to a poorer fit with a χ^2 of 1.6 per degree of freedom. This indicates that if a longitudinal VMD-type term is to be included it must fall at least as rapidly as $(1-x')^2$ for large x' and Q^2. This is not surprising. Within the framework of this model, J_{LS} is much more probable than either of the other two choices.

The sensitivity to β is also of a similar nature. If β was 3 instead of 4 one would again obtain a reasonable fit with a χ^2 of 1.1 per degree of freedom. This changes to 2.3 if β=2 and a very poor fit is obtained if β=5.

The sensitivity to a is rather strong. a=3 or 3/2 lead to poor fits. This indicates that the leading behavior at small x' corresponds to an a=2 term. However, this does not rule out small additional contributions with different values of a or even some energy dependent factors.

The fit is also sensitive to the value of the vector meson mass m_V. In fact a fit with $m_V^2 = 1.05\ \mathrm{GeV}^2$ gives the most minimum χ^2. Then the parameters are

$$
\begin{aligned}
\sigma_{\gamma T} &= 70.7 \pm 4.0\ \mu b \qquad \xi = 0.34 \pm 0.07 \\
\sigma_{\gamma L} &= 21.0 \pm 3.4\ \mu b \\
G_p &= 116.0 \pm 7.8\ \mu b\ \mathrm{GeV} \\
B_p &= 0.61 \pm 0.09 \\
C_p &= 1.23 \pm 0.25 \\
\chi^2/\mathrm{DF} &= 173/192 = 0.9
\end{aligned}
\tag{IV.22}
$$

This fit is shown in Fig. 9. Comparing Fig. 8 and Fig. 9 we observe that by changing m_V^2 from 0.585 to 1.05 we have removed the systematic deviation of the data from the fit in the region $0.09 \leq x' \leq 0.3$. What is the meaning of this result? The fact that χ^2 was reduced by 14 by changing the vector meson mass indicates, at least, that there was some systematic correlation remaining in the data when fitted at the ρ mass. This may imply that some small, but significant, physics may have been omitted from our fitting procedure. The most obvious possibility is that the use of a single vector meson was too great a simplification. There is also a mass continuum, and the larger mass fit may indicate that the "center of gravity" of this continuum is well above the ρ mass. This seems implausible to us because of the poor agreement of (IV.22) with the real photon cross section. If we were to constrain the parameters so that they simultaneously give a reasonable fit to the real photon cross section, the smaller mass is clearly preferred. We are also influenced by the likelihood that the low mass non-resonant 2π virtual states (usually associated with the "Drell processes") give an appreciable (10-20%) contribution to the real photon cross section. A more likely possibility is that this small systematic correlation may be due to a neglected $\underline{a}$=3/2 contribution or to the oversimplification of incorporating the $\underline{a}$=2 term only in the scaling contribution. However, lacking a detailed model of the Q^2 dependence of these terms, we were unable to do more. Because of systematic errors in the data due to effects such as radiative corrections, it is probably premature to attach too much significance to this rather small effect.

The presence of these systematic errors in the data was indicated to us by the small values ($\ll 1$) of χ^2 per degree of freedom obtained when we tried to make fixed s fits to the data, with our parameterization. In the absence of independent statistical errors, the χ^2 alone need not be a very reliable criterion for the goodness of the fit, and so one must exercise some caution in fitting this data. In particular, one should also calculate the mean deviation to check that there are no systematic deviations. For example using such tests we found that if we kept $m_V^2 = m_\rho^2 = 0.585$ GeV^2 and relaxed the χ^2 to be not quite a minimum one could reduce the systematic deviation in $0.09 \leq x' \leq 0.3$ by increasing $\sigma_{\gamma T}$ and decreasing G. This just illustrates some of the practical ambiguities in the determination and the interpretation of the parameters of our fits.

In the present two component picture, R does not diverge but instead scales when $Q^2 \gg m_V^2$. In Fig. 10 we plot the value of R obtained from our parameterization and at present energies it seems to increase with Q^2 at a fixed x'. Ultimately it will scale. This again appears consistent with the present separated data as seen in Fig. 6b.

Consequences for Sum Rules

From our best fits we can extract the scaling functions and evaluate the various sum rules. The results are as follows

(i) The Bjorken and Paschos sum rule (21b)

$$\int_0^1 dx' \, \nu W_{2(SC)}(x') \simeq 0.09$$

$$\int_0^1 dx' \, \nu W_{2(TOT)}(x') \simeq 0.15 \pm 0.02 < 2/9$$

where $\nu W_{2(TOT)} = \nu W_{2(SC)} + \nu W_{2(VL)}$ since $\nu W_{2(VT)} \to 0$.

(ii) The Gottfried sum rule (22)

$$\int_0^1 \frac{dx'}{x'} \, \nu W_{2(SC)}(x') \simeq 0.46 < 1$$

$$\int_0^1 \frac{dx'}{x'} \, \nu W_{2(TOT)}(x') \simeq \infty$$

The Bjorken and Paschos sum rule leads to a value too small to be compatible with a quark parton model without gluons (23). The Gottfried sum rule diverges. However, if we believed that only the short range contribution should be associated with the nature of the nucleon and thus the partons then we might only integrate $\nu W_{2(SC)}/x'$. This would then lead to a value much less than 1 in contradiction with the field theory result of Drell, Levy, and Yan (21c). However, as they emphasize (24), the parton picture is not covariant and only applies in the infinite momentum frame. In their model the multiplicative powers of x' in $\nu W_2(x')$ which are different from 2 (indicating long ranges) are obtained when they exponentiate a series of terms. So then we may really need to integrate $\nu W_{2(TOT)}(x')/x'$. Then their inequality for the Gottfried sum rule will be satisfied (as $\infty > 1$) by the present data if and only if they allow the presence of the longitudinal component. This, of course, will give $R \neq 0$.

V. DETAILED DERIVATION OF THE FINITE ENERGY REPRESENTATION

Assumptions

We now give the details of the derivation of the finite energy representation that we have used in Section III for the inelastic structure functions and cross sections. This was based on the following assumptions on the space time structure of the forward matrix element of the current commutator:

(i) On the surface ($y^2=0$) of the light cone, the invariant functions $C(y^2, y\cdot P)$ have the leading singularity which is weak enough to lead to asymptotically "scaling" structure functions. This restriction leads to the following canonical light cone structure

$$
\begin{aligned}
iC_1(y^2, y\cdot P) &= \epsilon(y\cdot P)\ \delta(y^2)\ f_\delta(y^2, y\cdot P) + \epsilon(y\cdot P)\ \theta(y^2)\ f_\theta(y^2, y\cdot P)\\
iC_2(y^2, y\cdot P) &= \epsilon(y\cdot P)\ \theta(y^2)\ f_2(y^2, y\cdot P)\\
iC_T(y^2, y\cdot P) &= \epsilon(y\cdot P)\ \theta(y^2)\ f_T(y^2, y\cdot P)\\
iC_L(y^2, y\cdot P) &= \epsilon(y\cdot P)\ \theta(y^2)\ y^2 f_L(y^2, y\cdot P)
\end{aligned}
\qquad (V.1)
$$

where $f_j(y^2, y\cdot P)$ are real and even functions of $y\cdot P$ which are integrable near the surface of the light cone $y^2=0$. The mass spectrum condition requires that the one dimensional fourier sine transforms like

$$
\mathscr{F}_2(x) = 8\pi \int_0^\infty dt\ \cos\,(xt)\ \frac{\partial}{\partial t}\left\{t\, f_2(0,t)\right\} \qquad (V.2)
$$

have finite support $-1 \leq x \leq 1$. This is not obtained in our analysis but most easily seen (4a) using either the DGS (11) or the JLD (11) representation. Examples of such light cone behavior are easily obtained using free field theory (4). That is why we call these singularities in Eqs. (V.1) as the "canonical singularities".

(ii) At large longitudinal distances $f_j(y^2 \simeq 0,\ y\cdot P)$ either fall off exponentially or have a pure power law behavior of the form

$$f_j(y^2, y\cdot P) \sim \frac{h_j^{(a)}(y^2)}{|y\cdot P|^a} \qquad \text{for} \quad y\cdot P \gg 1 \tag{V.3}$$

where a is a real number. This is a reasonable (6, 7) physical assumption since it leads to structure functions W_i which are compatible with the experimental data. It is not the most general one, but that it is consistent with the mass spectrum condition is seen by inverting Eq. (V.2). Theoretical models of such behavior will be investigated elsewhere.

Fourier transforming this assumed space time structure we can represent the inelastic structure functions as

$$\nu W_2(Q^2, \nu) = 2\ M^2 Q^2 \nu \int_{-\infty}^{\infty} d^4y\ \sin(y\cdot P)\,\theta(y\cdot P)\,\theta(y^2) f_2(y^2, y\cdot P) \tag{V.4a}$$

$$MW_1(Q^2, \nu) = 2\ \frac{M\nu}{Q^2}\ M^2 Q^2 \nu \int_{-\infty}^{\infty} d^4y\ \sin(y\cdot P)\,\theta(y\cdot P)\,\theta(y^2) f_T(y^2, y\cdot P) \tag{V.4b}$$

Here we have used the functions C_2 and C_T for the sake of symmetry and to clearly see the approach to scaling. However, we must remember that the pairs of functions that occur in a kinematic singularity free decomposition of $C_{\mu\nu}$ are either the pair $\{C_1, C_2\}$ or the pair $\{C_T, C_L\}$. These are related by

$$C_1 = (P\cdot\partial)^2\, C_L \tag{V.5a}$$

$$C_2 = C_T + \Box\, C_L \tag{V.5b}$$

using Eqs. (V.1) the two sets are seen to have consistent leading light cone singularities since as a distribution

$$y^2 \delta(y^2) \equiv 0 \tag{V.6}$$

These related pairs with different light cone singularities merely reflect the fact that the nature of this singularity is intimately connected with the tensor chosen to multiply the invariant function. There is no unique "natural" choice. Our choice involves only θ-function singularities on the surface of the light cone. As we will see at small Q^2, this can lead to nontrivial Q^2 dependence even in parts of the structure functions which ultimately "scale" for large Q^2. This is because of behavior inside the light cone. A term involving leading δ-function singularity masks this Q^2 dependence by restricting the function to the surface of the light cone right from the start. The nontrivial Q^2 dependence is then incorporated in the nonleading terms. This rearrangement corresponds to writing a function as

$$\hat{F}(Q^2, x) = F(\infty, x) + K(Q^2, x)$$

where (V.7)

$$K(Q^2, x) \to 0 \qquad \text{as} \quad Q^2 \to \infty$$

Since we are interested in the approach to scaling, it is convenient for us to choose the functions C_2 and C_T. All our conclusions can of course be reproduced using any other choice provided one is careful about the nonleading terms. They can also be derived using other methods. In fact Brown (12) has checked some of these results using the JLD representation. Pedagogically, however, our method is somewhat more direct.

Since from Eqs. (V.4) we find that MW_1 is of the same form as νW_2 except for the additional factor $(1/2x)$ we only discuss νW_2. The results for MW_1 can then be easily written down. So we start with Eq. (III.7) and first discuss the conditions for its convergence and the nature of our approximation before obtaining our representation for νW_2.

Convergence of the Integrals

In our analysis the assumption of scaling restricts the nature of the light cone singularities of C_i. Similarly the allowed values of a that can lead to a Regge-like behavior are restricted by the condition that the asymptotic power law $1/(y \cdot P)^{a}$ be continuable to small $y \cdot P$ without making the integrals diverge. Since these restrictions on the assumed behavior of f_j depend upon the conditions that the integral in Eqs. (III.7), (III.8) and (V.4) converge, it is of interest to study, briefly, these conditions. Here we study the convergence of these integrals. Since $W_i(\nu, x)$ can be measured at any given point (ν, x) the overall integrals must exist as ordinary functions rather than distributions (which only make sense when integrated with a test function). To study this convergence it is convenient to write Eq. (V.4) in terms of ordinary $(t, z, \vec{y}_\perp)$ variables rather than the light cone variables introduced in Eq. (III.5).

As an example we consider

$$\nu W_2(\nu, Q^2) = 2\pi Q^2 \; \nu I(\nu, q_3) \qquad \text{(V.8a)}$$

where from Eq. (V.4a)

$$I(\nu, q_3) \equiv 2\int_0^\infty dt \sin \nu t \int_0^t dz \cos (q_3 z) \int_0^{(t^2-z^2)} d\lambda f_2(\lambda, t) \qquad \text{(V.8b)}$$

The assumptions on f_2 guarantee convergence in the finite t region, so we need to study the convergence of this integral at the limit $t=\infty$.

We first reduce the integral to a two-dimensional one by integrating by parts with respect to z

$$I(\nu, q_3) = + \frac{4}{q_3} \int_0^\infty dt \sin \nu t \int_0^t dz \sin(q_3 z)\ z\ f_2(t^2 - z^2, t) \tag{V.9}$$

Consider the z-subintegration, which is

$$K(q_3, t) = \int_0^t dz\ z \sin(q_3 z)\ f_2(t^2 - z^2, t) \tag{V.10a}$$

$$= \frac{1}{2} \int_0^{t^2} d\lambda \sin\left(q_3 \sqrt{t^2 - \lambda}\right) f_2(\lambda, t) \tag{V.10b}$$

Since we expect f_2 to decrease like t^{-1}, the final z-integration will be convergent if the other explicit t-dependence in (V.10b) does not lead to a result which cancels out this decrease. A reasonable way to assure convergence seems to be to require

$$\int_0^\infty d\lambda\ f_2(\lambda, t) = \psi(t) < 0(t^{-\epsilon}) \qquad (\epsilon > 0) \tag{V.11}$$

(In fact, we expect $\epsilon \geq 1$.) The t-dependence in the sin factor of (V.10b) then seems harmless, but we have not tried to give a rigorous proof that it causes no trouble. In fact, it is not difficult to construct examples where K decreases less rapidly than $\psi(t)$. However, we assume that for reasonable physical functions (V.11) implies convergence of (V.8).

The Approximations

To obtain our representation we need an asymptotic expansion of the fourier integrals (V.4). This can be done by using (8, 9) the Reiman Lebesgue lemma or the method of stationary phase depending on the nature of the integrand. These have already been discussed in Section III. The asymptotic expansion is obtained by repeated partial integrations (8) as follows

$$\int_a^b dy \begin{matrix}\sin\\ \cos\end{matrix}(\nu y)\, G(y) = -\sum_{m=1}^{r}\left[\frac{1}{\nu^m}\left\{\frac{\partial^{m-1}G}{\partial y^{m-1}}\right\}\begin{matrix}\sin\\ \cos\end{matrix}\left(\nu y + m\frac{\pi}{2}\right)\right]_a^b$$

$$+\frac{1}{\nu^r}\int_a^b dy \left\{\frac{\partial^r G}{\partial y^r}\right\} \begin{matrix}\sin\\ \cos\end{matrix}\left(\nu y + r\frac{\pi}{2}\right) \qquad \text{(V.12)}$$

where r is any nonnegative integer such that all terms in the expansion converge.

In our approximation to the structure functions we will always ignore terms of order Q^2/ν^2 $(=Mx/\nu)$. Because of this we could make the approximation of ignoring y_- in the second argument of f_2 in Eq. (III.7) and freely throwing away the contribution at infinity on integration by parts. We now justify this and show that the corrections to this approximation are of order Mx/ν compared to the terms which are kept.

To see this we can perform one integration by parts with respect to y_- in Eq. (III.7) and get

$$\nu W_2(x,\nu) = 4\pi x\nu \int_0^\infty dy_+ \int_0^\infty dy_- \cos(\bar{\nu}y_- - \bar{x}y_+)\left\{2y_+\, f_2\left(2y_+y_-, y_+ + \frac{1}{2}y_-\right)\right.$$

$$\left. + \int_0^{2y_+y_-} d\lambda\, \frac{1}{2}\frac{\partial}{\partial y_+} f_2\left(\lambda, y_+ + \frac{1}{2}y_-\right)\right\} \qquad \text{(V.13a)}$$

Our approximation ignores the second term and replaces $\left(y_+ + \frac{1}{2}y_-\right)$ by just y_+ in the first term. It allows ignoring the surface terms at infinity in further integrations by parts with respect to y_- since $f_2(2y_+y_-, y_+)$ vanishes as $y_- \to \infty$ due to its first argument. The correction to our approximation can be written as

$$\nu W_{2C}(x,\nu) = 4\pi x\nu \int_0 dy_+ \int_0 dy_- \cos(\bar{\nu} y_- - \bar{x} y_+) \Big\{ 2y_+ \Big[f_2\Big(2y_+y_-, y_+ + \frac{1}{2}y_-\Big)$$

$$- f_2(2y_+y_-, y_+) \Big] + \int_0^{2y_+y_-} d\lambda \frac{1}{2} \frac{\partial}{\partial y_+} f_2\Big(\lambda, y_+ + \frac{1}{2}y_-\Big) \Big\} \qquad \text{(V.14a)}$$

and the total νW_2 as

$$\nu W_2(x,\nu) = 4\pi x\nu \int_0^\infty dy_+ \int_0^\infty dy_- \cos(\bar{\nu} y_- - \bar{x} y_+) \frac{\partial}{\partial y_-} \int_0^{2y_+y_-} d\lambda\, f_2(\lambda, y_+) + \nu W_{2C}$$

$$\text{(V.14b)}$$

We can perform one more partial integration with respect to y_- obtain

$$\nu W_{2C}(x,\nu) = -4\pi x \int_0^\infty dy_+ \int_0^\infty dy_- \sin(\bar{\nu} y_- - \bar{x} y_+) \Big\{ 2y_+ \frac{\partial}{\partial y_-} \Big[f_2\Big(2y_+y_-, y_+ + \frac{1}{2}y_-\Big)$$

$$- f_2(2y_+y_-, y_+) \Big] + y_+ \Big[\frac{\partial}{\partial y_+} f_2\Big(\lambda, y_+ + \frac{1}{2}y_-\Big) \Big]_{\lambda = 2y_+y_-}$$

$$+ \int_0^{2y_+y_-} d\lambda \frac{1}{4} \frac{\partial^2}{\partial y_+^2} f_2\Big(\lambda, y_+ + \frac{1}{2}y_-\Big) \Big\} .$$

$$= 0\left[x/(\nu R_p)\right] \qquad \text{(V.15)}$$

Since the integrand in parenthesis $\{ \}$ in Eq. (V.15) is an integrable function of y_- and y_+ we can use the Reiman Lebesgue lemma (or perform one more partial integration) to see that the integral in Eq. (V.15) decreases at least as fast as $1/\nu$. Therefore this correction term has the leading behavior $(x/\nu)\, G_{C2}(x)$ which is similar to the nonleading short range terms in Eq. (III.9). These vanish in the Bjorken limit but give a contribution of order $1/\nu^2$ to the total cross section at any given Q^2, like the short range terms in Eq. (III.9). Therefore they affect the coefficient T of the $1/\nu^2$ term in Eqs. (IV.10) and (IV.12) which makes the interpretation of Eq. (IV.12) somewhat ambiguous. This can also be seen from Eq. (V.45).

Long Range Contributions

Let us now consider the remaining contribution to νW_2.

$$\nu W_2(x,\nu) - \nu W_{2C}(x,\nu) = 4\pi x\nu^2 \int_0^\infty dy_+ \int_0^\infty dy_- \sin(\bar{\nu}y_- - \bar{x}y_+) \int_0^{2y_+y_-} d\lambda\, f_2(\lambda, y_+) \tag{V.16}$$

Here it is understood that we must ignore the contribution at infinity on integrations by part with respect to y_- in agreement with Eq. (V.14b).

We first determine the contribution of a term like Eq. (III.12) in f_2 which has pure power behavior in y_+ for large y_+ and is moderated by a smooth cutoff as $y_+ \to 0$. For example

$$f_2^{(a)} \sim \frac{h_2^{(a)}(\lambda)}{y_+^a}\, g_2^{(a)}(y_+/R_p)$$

where (V.17)

$$g_2^{(a)} \to \begin{cases} 1 & \text{when } y_+/R_p \gg 1 \\ 0(y_+/R_p)^a & \text{when } y_+/R_p \ll 1 \end{cases}$$

such a term is illustrated in Fig. 4.

If we change to the variables $\beta = \bar{\nu}y_-$ and $\eta = y_+/\bar{\nu}$ then the contribution of such terms to νW_2 is

$$\nu W_2^{(a)}(x,\nu) \simeq (2\pi Q^2/\bar{\nu}^{a-1}) \int_0^\infty \frac{d\eta}{\eta^a} \int_0^\infty d\beta \sin\left(\beta - \frac{1}{2}Q^2\eta\right) \int_0^{2\eta\beta} d\lambda\, h_2^{(a)}(\lambda)\, g_2^{(a)}\left(\frac{\eta\bar{\nu}}{R_p}\right)$$

$$+ 0\left[x/(\nu R_p)\right] \tag{V.18}$$

The variable $\bar{\nu}$ has disappeared from the integrand except for the argument of $g_2^{(a)}(\eta\bar{\nu}/R_p)$. If the power $\underline{a}$ is such that the integral remains convergent (near $\eta \simeq 0$) even when $g_2^{(a)}$ is replaced by 1 everywhere, then we can write

$$\nu W_2^{(a)}(x,\nu) = (1/\bar{\nu}^{\underline{a}-1})\, Q^2\, G_2^{(a)}(Q^2) + \nu \mathcal{W}_2^{(a)} + 0\left[x/(\nu R_p)\right] \tag{V.19a}$$

where

$$G_2^{(a)}(Q^2) = 2\pi \int_0^\infty \frac{d\eta}{\eta^{\underline{a}}} \int_0^\infty d\beta \sin\left(\beta - \frac{1}{2} Q^2 \eta\right) \int_0^{2\eta\beta} d\lambda\, h_2^{(a)}(\lambda) \tag{V.19b}$$

$$\nu \mathcal{W}_2^{(a)} = (2\pi Q^2/\bar{\nu}^{\underline{a}-1}) \int_0^\infty \frac{d\eta}{\eta^{\underline{a}}} \int_0^\infty d\beta \sin\left(\beta - \frac{1}{2} Q^2 \eta\right) \int_0^{2\eta\beta} d\lambda\, h_2^{(a)}(\lambda) \left[g_2^{(a)}\left(\frac{\eta\bar{\nu}}{R_p}\right) - 1\right] \tag{V.19c}$$

This rearrangement of the integral has been done by adding and subtracting the dashed pieces to $f_2^{(a)}$ as shown in Fig. 4. The first term reflects the effect of extrapolating the large longitudinal distance behavior all the way to very small distances by adding a short range piece which is subtracted by the part $\nu \mathcal{W}_2^{(a)}$.

To determine the range of $\underline{a}$ for which this decomposition is possible we have to study the convergence of the integral in Eq. (V.19b) for small η. So we integrate with respect to β to obtain a result of the form

$$G_2^{(a)}(Q^2) = \int_0^\infty \frac{d\eta}{\eta^{\underline{a}}} \left[\cos\left(\frac{1}{2} Q^2 \eta\right) S^{(a)}(\eta) - \sin\left(\frac{1}{2} Q^2 \eta\right) C^{(a)}(\eta)\right] \tag{V.20a}$$

where

$$S^{(a)}(\eta) = 2\pi \int_0^\infty d\beta \sin\beta \int_0^{2\eta\beta} d\lambda\, h_2^{(a)}(\lambda) = -8\pi\eta^2 \int_0^\infty d\beta \sin\beta\, h_2^{(a)\prime}(2\eta\beta)$$

$$\simeq -8\pi\eta^2\, h_2^{(a)\prime}(0) + 0(\eta^3) \tag{V.20b}$$

and

$$C^{(\underline{a})}(\eta) = 2\pi \int_0^\infty d\beta \cos\beta \int_0^{2\eta\beta} d\lambda\, h_2^{(\underline{a})}(\lambda) = -4\pi\eta \int_0^\infty d\beta \sin\beta\, h_2^{(\underline{a})}(2\eta\beta)$$

$$\simeq -4\pi\eta\, h_2^{(\underline{a})}(0) + 0(\eta^2) \qquad \text{(V.20c)}$$

The integrals in (V.19) clearly converge if $\underline{a} < 3$ due to the small η behavior in (V.20). For large Q^2 we can partially integrate in Eq. (V.20a) and use (V.20b, c) to obtain the asymptotic behavior

$$G_2^{(\underline{a})}(Q^2) \simeq \frac{1}{2}(Q^2/2)^{\underline{a}-2} L_2^{(\underline{a})} \qquad \text{(V.21)}$$

which leads to a "Regge-like" scaling form when Q^2 and ν are large

$$\nu W_2^{(\underline{a})}(x, \nu) \simeq |x|^{\underline{a}-1} L_2^{(\underline{a})} \qquad \text{(V.22)}$$

The question of how large ν and Q^2 have to be before the scaling behavior of Eq. (V.22) sets in was discussed in Section III. There we saw that we need at least $\nu R_p \gg 1$ for the leading term of the asymptotic expansion to be a good approximation. We also saw that we need $Q^2 \gg m^2$ (when $\mathscr{R}$ is below $\mathscr{L}$ in Fig. 3) to have the integral obtain contribution from very close to the surface of the light cone and thus scale. We also saw in Eq. (III. 6) that such Regge-like behavior is only meaningful for region of small $|x| \lesssim 1/R_p \sim 0.2$.

The behavior of the total virtual photon absorption cross section at fixed Q^2 is then found to be

$$\sigma^{(\underline{a})}(Q^2, \nu) = G^{(\underline{a})}(Q^2)/\nu^{\underline{a}-1} \qquad \text{(V.23)}$$

This is Regge behavior corresponding to a Regge trajectory with the intercept

$$\alpha(0) = 2-\underline{a}\,, \qquad \underline{a} < 3 \qquad \text{(V.24)}$$

A remarkable feature of these results is the intimate relation between the Regge power of ν dependence and the power of Q^2 dependence. They are both determined by the longitudinal distance behavior and are such as to lead to a scaling νW_2 (10). It should, however, be observed that the relation (V.24) depends on the nature of the light cone singularity also. In particular if we did not have the scaling assumption to restrict the nature of this singularity to be "canonical", we would not get a similar unique relation. Such a situation would occur if one performed similar analysis for hadronic reactions (7) like the forward $\pi\pi$, πN, NN, KN scattering, etc. There for each different type of light cone singularity one could obtain a different analog of the relation (V.24). In case the light cone singularity structure in these hadronic reactions was "canonical" like the forward Compton scattering, then we could expect the long range terms to be rather important since the mass Q^2 of the probe is relatively small.

Examples of Long Range Terms

To gain familiarity with the representation in Eq. (V.20) we consider a model for the functions $S^{(a)}$ and $C^{(a)}$ which involve the nonleading terms in η. Such nonleading terms are also interesting since they are sensitive to the derivatives of the commutator at the surface of the light cone. Naively, this is seen by using the distribution theory result for any positive integer n

$$\frac{1}{n!}\int_0^\infty d\beta\, \beta^n\, e^{i\beta} = e^{i(n+1)\pi/2} \tag{V.25a}$$

This is obtained from

$$\lim_{\epsilon \to 0+}\int_0^\infty d\beta\, \beta^n\, e^{i(x+i\epsilon)\beta} = \frac{e^{i(n+1)\pi/2}}{x^{n+1}}\int_0^\infty d\gamma\, \gamma^{(m+1)-1}\, e^{-\gamma} = \frac{e^{i(n+1)\pi/2}}{x^{n+1}}\, \Gamma(n+1)$$

$$\text{for } \mathrm{Re}\, x > 0 \text{ and } \mathrm{Re}\, n > -1 \tag{V.25b}$$

If we use the Taylor expansion near the light cone

$$\int_0^{2\eta\beta} d\lambda\, h_2^{(a)}(\lambda) = \sum_{n=1}^{\infty} \frac{(2\eta\beta)^n}{n!} \left[\frac{d^{n-1}}{d\lambda^{n-1}} h_2^{(a)}(\lambda)\right]_{\lambda=0} \tag{V.26}$$

we obtain for small η

$$S^{(a)}(\eta) = 2\pi \sum_{n=1}^{\infty} (2\eta)^n \left[\frac{d^{n-1}}{d\lambda^{n-1}} h_2^{(a)}(\lambda)\right]_{\lambda=0} \sin\{(n+1)\pi/2\} \tag{V.27a}$$

$$C^{(a)}(\eta) = 2\pi \sum_{n=1}^{\infty} (2\eta)^n \left[\frac{d^{n-1}}{d\lambda^{n-1}} h_2^{(a)}(\lambda)\right]_{\lambda=0} \cos\{(n+1)\pi/2\} \tag{V.27b}$$

Let us first consider the free field model of Ref. 4b consisting of the Born and seagull diagrams. In this model one gets the canonical light cone singularity with

$$f_2(y^2, y\cdot P) = \frac{\sin (y\cdot P)}{y\cdot P} \tag{V.29a}$$

The lack of y^2 dependence indicates the absence of form factors. We also note that (V.29a) is an example of the behavior obtained from s-channel resonance terms and that it is not within the class of functions we have assumed for f_2. This may indicate that our class is too narrow. For the present, however, our class is broad enough to cope with the data. It must be kept in mind, though, that future experimental results may demand a still broader class. Barring unforeseen difficulties, we believe that the present arguments could easily be extended to a more general set of functions. In any case, (V.29a) seems unphysical since it leads to

$$\nu W_2 \cong 4\pi^2 x \left[\delta(x+1) - \delta(x-1)\right] \tag{V.29b}$$

which is not a physically measurable quantity.

Another, more interesting, example is the diffractive cross section associated with the ρ-meson. We expect $h_T^{(1)}$ to have oscillations associated with the ρ-propagator; for example, it might be a Bessel function of argument $m_\rho\sqrt{2\beta\eta}$. In (V.19b), we are interested in the large η region where we expect the contribution to veer inside the light cone. Using a typical asymptotic form for such an oscillating function, we expect the β-integration to be like

$$\int_0^\infty d\beta \cos\left(\beta - \frac{1}{2}Q^2\eta\right) 2\eta h_T^{(1)}(2\beta\eta)$$

$$\cong 2\eta \int_0^\infty d\beta \cos\left(\beta - \frac{1}{2}Q^2\eta\right) \sin\left(m_\rho\sqrt{2\beta\eta} + b\right) \mu(\beta\eta) \qquad \text{(V.30)}$$

The factor μ is a slowly varying modulating factor, and b is some phase shift in this asymptotic function. We will use the stationary phase approximation to discuss this integral. First the addition formula is used to get sine functions of arguments

$$\beta - \frac{1}{2}Q^2\eta - m_\rho\sqrt{2\beta\eta} - b$$

$$\beta - \frac{1}{2}Q^2\eta + m_\rho\sqrt{2\beta\eta} + b$$

The first of these has a stationary phase at

$$\beta = \frac{1}{2}m_\rho^2\eta \qquad \left(\text{i.e.}\quad y_- = \frac{1}{2}\frac{m_\rho^2}{\nu^2}y_+\right)$$

while the second leads to a rapidly oscillating integrand, and hence negligible integral. The first phase may be rewritten

$$\left(\sqrt{\beta} - m_\rho\sqrt{\frac{\eta}{2}}\right)^2 - \frac{1}{2}\left(Q^2 + m_\rho^2\right)\eta - b \qquad \text{(V.31)}$$

and the resulting integral is easily seen to be of the form

$$\text{Const} \times m_\rho \; \eta^{3/2} \; \mu\left(\frac{1}{2} m_\rho^2 \eta^2\right) \; \cos\left[\frac{1}{2}\left(Q^2 + m_\rho^2\right)\eta + b'\right] \qquad (V.32)$$

where b' differs from b by $\pi/4$. Strictly speaking, (V.32) has been derived as a good approximation only for large η. However, we will try to match it to the correct form for small η in (V.20b,c) in order to reproduce the VMD expressions. Therefore we require that

$$\eta^{3/2} \; \mu\left(\frac{1}{2} m_\rho^2 \eta^2\right) \; \cos\left(\frac{1}{2} m_\rho^2 \eta + b'\right) \propto \eta^2 \qquad (V.33a)$$

$$\eta^{3/2} \; \mu\left(\frac{1}{2} m_\rho^2 \eta^2\right) \; \sin\left(\frac{1}{2} m_\rho^2 \eta + b'\right) \propto \eta \qquad (V.33b)$$

If $b' \neq \frac{\pi}{2}$, we need

$$\eta^{3/2} \; \mu \propto \eta^2 \qquad (V.33c)$$

If $b' = \frac{\pi}{2}$, we need

$$\eta^{3/2} \; \mu \propto \eta \qquad (V.33d)$$

We may as well consider the two independent cases $b' = 0, \frac{\pi}{2}$. For $b' = 0$, comparing (V.33c) and (V.20c), we note that $h_T^{(1)}(0) = 0$, and we expect a non-leading Q^2 dependence. In this case, if we call the modulating factor ϕ,

$$G_T^{(1)} = -\sigma_\infty m_\rho^4 \int_0^\infty d\eta \; \eta \cos\left[\frac{1}{2}\left(Q^2 + m_\rho^2\right)\eta\right] \phi\left(\frac{1}{2} m_\rho \Gamma_\rho \eta\right), \quad (b'=0) \qquad (V.34)$$

We have anticipated the physical interpretation and normalization of the result by supplying the subscript T for transverse VMD and σ_∞ for our simplifying assumption that all the asymptotic cross section is in this term. The correct form of ϕ is not known, but a simple exponential leads to a very nice result:

$$\phi(\xi) = e^{-\xi} \Rightarrow G_T^{(1)} = \left\{\frac{\sigma_\infty m_\rho^4}{\left(Q^2 + m_\rho^2\right)^2 + m_\rho^2 \Gamma_\rho^2}\right\} \left\{\frac{\left(Q^2 + m_\rho^2\right)^2 - m_\rho^2 \Gamma_\rho^2}{\left(Q^2 + m_\rho^2\right)^2 + m_\rho^2 \Gamma_\rho^2}\right\}$$

This has a resonant behavior when $Q^2 \cong -m_\rho^2$ (but not with quite the right structure), and for spacelike Q^2, the approximation $\Gamma_\rho \to 0$ leads to the transverse VMD expression we have used in our data analysis.

We shall not discuss the other case $b' = \frac{\pi}{2}$ in detail. Suffice it to say that it leads to a result

$$\int_0^\infty d\eta \, \sin\left[\frac{1}{2}\left(Q^2 + m_\rho^2\right)\eta\right] \phi\left(\frac{1}{2} m_\rho \Gamma_\rho \eta\right) = \frac{2\left(Q^2 + m_\rho^2\right)}{\left(Q^2 + m_\rho^2\right)^2 + m_\rho^2 \Gamma_\rho^2} , \quad \left(b' = \frac{\pi}{2}\right) \tag{V.36}$$

This has the leading Q^2 behavior (as it must, since $h(0) \neq 0$ in this case). It is not directly of longitudinal VMD form, but a linear combination of (V.35) and (V.36) can obviously give that form.

<u>Short Range Contribution</u>

The next step is to study terms with $\underline{a} \geq 3$ for which the large longitudinal distance behavior cannot be extrapolated to short distances. We can also consider the shorter range terms which decay exponentially as a function of longitudinal distance and the correction terms in $\nu W_2^{(\underline{a})}$ in Eq. (V.19c) along with the $\underline{a} \geq 3$ terms and call these collectively the "short range terms". In this case we can integrate Eq. (III.7) by parts twice with respect to y_- to get Eq. (III.8). The behavior of the terms in Eq. (III.8) even in the presence of power law terms with $\underline{a} \geq 3$ is the same as discussed in Section III. The special case $\underline{a}=3$ is interesting in that it leads to functions of the form $\frac{x}{\nu} \ell n \, x$ in the second term. When $\underline{a} < 3$ the correction term in Eq. (III.8) is no longer small at large ν unless Q^2 is also large.

Thus the short range contribution is of the form

$$\nu W_2^{(SR)}(x, \nu) = x^2 \, F_2^{(SR)}(x) + (x/\nu) \, G_2^{(SR)}(x) + 0(x/\nu^2) \tag{V.37}$$

This leads to a universal function of x once ν is large irrespective of Q^2. Such short range terms are expected to be important in the very deep inelastic region $|x| \gtrsim 1/(MR_p)$.

With this representation it is easy to study the correction term $\nu W_2^{(a)}$ due to replacing $g_2^{(a)}$ by 1 for $\underline{a} < 3$. By definition

$$\left\{g_2^{(a)}(y_+/R_p) - 1\right\} \to 0 \quad \text{as} \quad y_+/R_p \to \infty \tag{V.38}$$

So either it is another power in the range $\underline{a} < 3$ and leads to "Regge-like" scaling behavior or it is a short range term and leads to a universal behavior. Therefore, if the longitudinal behavior of the current commutator with canonical light cone singularities consists of a power fall off at large $y \cdot P$ which gets moderated at small distances inside the target proton, then the asymptotic structure functions can be expressed as a sum of short range terms like $x^2 F^{(SR)}(x)$ and Regge terms like $L^{(a)}|x|^{\underline{a}-1}$. Of course in general this decomposition need not be unique since we are not expanding in terms of linearly independent functions. It is also not physically meaningful for $|x| > 1/(MR_p)$ as was discussed in Section III. However, it is covariant since y^2 and $y \cdot P$ are invariants.

The reason that the short range terms lead to a universal behavior requiring only large ν is that they always restrict the integral to obtain contribution from regions very close to the surface of the light cone even when Q^2 and x are small. To probe the inside of the light cone and thus obtain nontrivial Q^2 dependence we need small Q^2 and long range terms.

Delta Function Light Cone Singularities

At this point it is interesting to note that had we used the function C_1 instead of C_T in MW_1 we would have obtained

$$MW_1 = (2x)^{-1}\, \nu W_2 - MW_{1\delta} - MW_{1\theta} \tag{V.39a}$$

where (if $M \equiv 1$)

$$MW_{1\delta} = -4\pi x \left(1 + \frac{1}{2}(\bar{x}/\bar{\nu})\right)^{-1} (\nu/\bar{\nu}) \int_0^\infty d\beta \int_0^\infty dt \ \sin(\beta - \bar{x}\, t)\ f_\delta(0, t) \tag{V.39b}$$

where $t \equiv y_+ + \frac{1}{2} y_-$ and $\beta = \bar{\nu} y_- (1 + \bar{x}/(2\bar{\nu}))$ and the integral is interpreted using the distribution theory as in Eqs. (III.11) and (V.25). This is a "universal function" of x once $\bar{x}/\bar{\nu} = Q^2/(2\nu^2) \ll 1$ for any Q^2. Thus it shows no nontrivial Q^2 dependence and leads to Regge-like behavior if

$$f_\delta(0, t) = 4t^2 f_L(0, t) \sim t^{2-\underline{a}} \tag{V.40}$$

The nontrivial Q^2 dependence is now contained in the nonleading term

$$MW_{1\theta} = \left(\frac{1}{\nu}\right) (2\pi Q^2 \nu) \int_0^\infty dy_- \int_0^\infty dy_+ \ \sin(\bar{\nu} y_- - \bar{x} y_+) \int_0^{2y_+ y_-} d\lambda\, f_\theta\left(\lambda, y_+ + \frac{1}{2} y_-\right) \tag{V.41}$$

This leads to terms similar to νW_2 except for the additional factor $(1/\nu)$. Therefore $MW_{1\theta}$ gives vanishing contribution in the Bjorken limit. However at small Q^2 it gives a nontrivial Q^2 dependent contribution to the scaling terms and acts like the correction $K(Q^2, x)$ in the example of Eq. (V.7).

VI. SUMMARY AND CONCLUSION

Let us summarize our results. First we note that our representation is independent of whether Q^2 is space like or time like. However, it only represents the possible leading behavior under the assumption of scaling. What terms actually exist in any given kinematic region has to be determined by experiment or a theoretical model which satisfy our assumptions.

We find that for any given $|Q^2|$ when we take "the Regge limit $\nu R_p \gg 1$ we can represent

$$\nu W_2(Q^2, \nu) = \sum_{\underline{a} < 3} \left(\frac{M}{\nu}\right)^{\underline{a}-1} \left(\frac{Q^2}{M^2}\right) G_2^{(\underline{a})}(Q^2/M^2) + x^2 F_2^{(SR)}(x) + 0\left[x/(\nu R_p)\right] \tag{VI. 1a}$$

and

$$MW_1(Q^2,\nu) = \sum_{\underline{a}<3} \left(\frac{M}{\nu}\right)^{\underline{a}-2} G_T^{(\underline{a})}(Q^2/M^2) + \frac{1}{2}\, x\, F_T^{(SR)}(x) + 0\left[1/(\nu R_p)\right] \qquad \text{(VI. 1b)}$$

In these expressions the "universal" terms and the orders of magnitude of corrections are exhibited explicitly. When we let $|Q^2| \gg m^2$ we get the asymptotic expansion

$$G_j^{(\underline{a})}(Q^2/m^2) \simeq \frac{1}{2}\left(Q^2/(2M^2)\right)^{\underline{a}-2} L_j^{(\underline{a})} + 0\left((Q^2/m^2)^{\underline{a}-3}\right) \text{ for } \underline{a}<3 \qquad \text{(VI. 2)}$$

The mass m^2 is the "characteristic mass" of the dominating hadronic component of the photon.

Then in the "finite Bjorken limit $\nu R_p \gg 1$ and $|Q^2| \gg m^2$" we obtain the representations for the "scaling functions"

$$\mathscr{F}_2(x) \equiv \nu W_2(x) = \sum_{\underline{a}<3} |x|^{\underline{a}-1} L_2^{(\underline{a})} + x^2 F_2^{(SR)}(x) \qquad \text{(VI. 3a)}$$

$$\mathscr{F}_1(x) \equiv MW_1(x) = \frac{1}{2}\sum_{\underline{a}>3} |x|^{\underline{a}-2} L_T^{(\underline{a})} + \frac{1}{2}\, x\, F_T^{(SR)}(x) \qquad \text{(VI. 3b)}$$

$$\mathscr{F}_L(x) \equiv \nu W_2(x) - 2x\, MW_1 = \sum_{\underline{a}>3} |x|^{\underline{a}-1}\left(L_2^{(\underline{a})} - L_T^{(\underline{a})}\right) + x^2\left(F_2^{(SR)}(x) - F_T^{(SR)}(x)\right) \qquad \text{(VI. 3c)}$$

As we discussed in Section III and Eq. (III. 26), such representation of the scaling functions is meaningful only in the region of small x (i.e., $|x| \lesssim 1/R_p \sim 0.2$). For larger values of x we start probing inside the target and the power law gets moderated to lead to a behavior like the short range terms. Furthermore kinematics require these scaling functions to vanish for $|x| > 1$. In our data analysis we mocked up these effects by introducing the factors $J_{L,T}(x)$ which are nearly unity for $|x| \lesssim 1/R_p$ but may fall off at larger x. We should emphasize here that such factors are not derivable from the integrals in a general way, but they may result from particular models. As we have already discussed, a unique separation into such component terms may not be possible. However, a dominant power $\underline{a}$ may be discernible from the data at small x, as in our data analysis.

It is also convenient to write the representation for the transverse and longitudinal total virtual photon absorption cross sections.

$$\sigma_T(Q^2,\nu) = \frac{4\pi^2\alpha}{M^2(1-x)} \left\{ \sum_{1 \le \underline{a} \le 3} (M/\nu)^{\underline{a}-1} G_T^{(\underline{a})}(Q^2/M^2) \right.$$

$$\left. + \frac{1}{2}(M/\nu)^2 \left[\left(\frac{Q^2}{2M^2}\right) F_T^{(SR)}(x) + G_T^{(SR)}(x) + G_{C2}(x) \right] + 0\left[M^2/(\nu^3 R_p)\right] \right\} \tag{VI. 4a}$$

$$\sigma_L(Q^2,\nu) = \frac{4\pi^2\alpha}{M^2(1-x)} \left\{ \sum_{1 \le \underline{a} \le 3} (M/\nu)^{\underline{a}-1} \left[G_2^{(\underline{a})}(Q^2/M^2) - G_T^{(\underline{a})}(Q^2/M^2) \right] \right.$$

$$+ \left(\frac{Q^2}{M^2}\right) \sum_{1 \le \underline{a} \le 3} (M/\nu)^{\underline{a}+1} G_2^{(\underline{a})}(Q^2/M^2)$$

$$+ \frac{1}{2}(M/\nu)^2 \left[\left(\frac{Q^2}{2M^2}\right)\left(F_2^{(SR)}(x) - F_T^{(SR)}(x)\right) + \left(G_2^{(SR)}(x) + G_{C2}(x) - G_T^{(SR)}(x) - G_{CT}(x)\right) \right]$$

$$\left. + 0\left[M^2/(\nu^3 R_p)\right] \right\} \tag{VI. 4b}$$

where $G_{C2,T}(x)$ is the contribution of the correction mentioned in Eq. (V.15). The bound $\underline{a}=1$ is due to unitarity or the Froissart bound. Gauge invariance requires that

$$\lim_{Q^2 \to 0} \sigma_L(Q^2,\nu) = 0 \tag{VI. 5}$$

This imposes the conditions

$$G_2^{(a)}(0) = G_T^{(a)}(0) \tag{VI.6a}$$

$$G_2^{(SR)}(0) + G_{C2}(0) = G_T^{(SR)} + G_{CT}(0) \tag{VI.6b}$$

Therefore the total real photon absorption cross section becomes

$$\sigma_\gamma(\nu) \equiv \sigma_T(0,\nu) = \frac{4\pi^2\alpha}{M^2}\left\{\sum_{1\leq a\leq 3} (M/\nu)^{a-1}\, G_2^{(a)}(0) + \frac{1}{2}(M/\nu)^2\left[G_2^{(SR)}(0) + G_{C2}(0)\right] + 0\left[M^2/(\nu^3 R_p)\right]\right\} \tag{VI.7}$$

On the other hand there is no a priori reason for $L_2^{(a)}$ and $L_T^{(a)}$ to be equal. Thus in the presence of the a=1 terms the longitudinal scaling function $\mathscr{F}_L(x)$ could have a finite intercept as $x \to 0$ (with $\nu, Q^2 \to \infty$). The longitudinal VMD term is an example of such behavior.

The Total Scaling Functions

Before we conclude it is useful to obtain a representation for the total scaling functions in the presence of the long range terms, and also to discuss some implications of the mass spectrum condition (4a).

If we integrate by parts with respect to y_+ and y_- in Eq. (V.14) we obtain

$$\nu W_2(x,\nu) \equiv \mathscr{F}_2(x,\nu) = \mathscr{F}_2(x) + \mathscr{F}_{2C}(x,\nu) \tag{VI.8a}$$

where the scaling function is represented as

$$\mathscr{F}_2(x) \equiv 8\pi \int_0^\infty dt\, \cos(xt)\, \frac{\partial}{\partial t}\left\{t f_2(0,t)\right\} \tag{VI.8b}$$

And $\mathscr{F}_{2C}(x,\nu)$ represents the correction to scaling at finite energies

$$\mathscr{F}_{2C}(x,\nu) \equiv (x/\nu)\, G_{C2}(x) + 8\pi \int_0^\infty dy_+ \int_0^\infty dy_-\, \cos(\bar{\nu}y_- - \bar{x}y_+)\, \frac{\partial^2}{\partial y_+ \partial y_-}\left\{y_+ f_2(2y_+y_-, y_+)\right\} \tag{VI.8c}$$

As already discussed in Section V, at large Q^2 the magnitude of these corrections can be of order $\left\{x/(\nu R_p),\ m^2/Q^2\right\}$. The integral in Eq. (VI.8b) is well defined for both the short and the long range terms. To extract the a=1 term explicitly we subtract the limit

$$\lim_{t\to\infty} 8\pi\, t f_2(0,t) \equiv L_2^{(1)} \tag{VI.9}$$

in Eq. (VI.8b) and integrate by parts with respect to t to get

$$\mathscr{F}_2(x) = L_2^{(1)} + x\int_0^\infty dt\, \sin(xt)\left\{8\pi\, t f_2(0,t) - L_2^{(1)}\right\} \tag{VI.10}$$

This is similar to the result obtained by Jackiw, Van Royen and West (4b) and others (4), which can now be used even in the presence of a=1 term without resort to distribution theory. However our representation for $\mathscr{F}_L(x)$ is different from theirs due to the presence of the long range terms.

$$\mathscr{F}_L(x) = \left\{L_2^{(1)} - L_T^{(1)}\right\} + x\int_0^\infty dt\, \sin(xt)\left\{8\pi t\left[f_2(0,t) - f_T(0,t)\right] - \left[L_2^{(1)} - L_T^{(1)}\right]\right\} \tag{VI.11}$$

These long range terms in $\mathscr{F}_L(x)$ would also clearly destroy the Schwinger term sum rule derived by Jackiw, Van Royen and West (4b).

The mass spectrum condition requires the functions $\mathscr{F}_2$ in Eqs. (VI.8, VI.10) to vanish for $|x| > 1$. So we can Fourier transform Eq. (VI.8b) to obtain

$$f_2(0,t) = \frac{1}{4\pi^2}\int_0^1 dx\left(\frac{\sin xt}{xt}\right)\mathscr{F}_2(x) \tag{VI.12}$$

This equation, which is based on scaling and finite support of $\mathscr{F}_2(x)$ implies that the total $f_2(0, y\cdot P)$ is an entire function of $y\cdot P$. This has already been emphasized by Brandt and Preperata and others (25). Similarly $f_T(0, y\cdot P)$ is an entire function of $y\cdot P$. However, it should be clear that the total f_2 may be considered as a sum of terms, as in Fig. 4, which are individually singular and lead to functions that are nonvanishing for $|x| > 1$ as long as their sum has the proper analyticity and support. Such is the situation for our separation in the case of the long range terms. This separation was useful to study the small $|x|$ behavior which is controlled by the behavior of f_2 at large $y \cdot P$. But it is useless when discussing $\mathscr{F}_2(x)$ for large x (near x=1) or the integrals of $x^2 \mathscr{F}_2(x)$ that occur in the Cornwall-Norton sum rules (25).

We can use Eq. (VI.12) to see that realistic scaling functions $\mathscr{F}_2(x)$ lead to $f_2(0,t)$ whose asymptotic behavior can be adequately described by the class of functions considered by us. To be specific, we consider two typical examples:

(i) Suppose $\mathscr{F}_2(x) = (1-|x|)^2\, \theta(1-|x|)$. Then using Eq. (VI.12) we get

$$4\pi^2 f_2(0,t) = t^{-1}\int_0^t d\alpha \left(\frac{\sin\alpha}{\alpha}\right) - 2t^{-2} + t^{-2}\left[\cos t + t^{-1} \sin t\right]$$

$$\longrightarrow \begin{cases} 1/3 & \text{as } t \to 0 \\ (\pi/2)/t & \text{as } t \to \infty \end{cases} \tag{VI.13}$$

This is like a leading $\underline{a}=1$ term. However we should also note that in this example f_2 contains nonleading terms $t^{-2} \cos t$ and $t^{-1} \sin t$ which are like (V.29a) and which do not belong to the class of function we have choosen since they have long range ($\underline{a} < 3$) but oscillate.

(ii) If $\mathscr{F}_2(x) = |x|(1-|x|)^3\, \theta(1-|x|)$, then we get an $\underline{a}=2$ term

$$4\pi^2 f_2(0,t) = t^{-2} - 6t^{-5}(t - \sin t)$$

$$\longrightarrow \begin{cases} 1/20 & \text{as } t \to 0 \\ 1/t^2 & \text{as } t \to \infty \end{cases} \tag{VI.14}$$

From these examples we see that the oscillations of $f_2(0,t)$ occur due to finite support of $\mathscr{F}_2(x)$ and do not affect the leading smooth power fall off at large distances. Therefore they are unimportant for the behavior near $x=0$. However, these oscillations will strongly affect the threshold behavior near $|x|=1$. To the extent that s-channel resonances lead to oscillations, like in Eq. (V.29a), they may be expected to affect this threshold behavior. The implication of these observations for threshold behavior and duality (18) will be investigated elsewhere.

Final Remarks

What we have done is to supplement Bjorken's (5) original proof of asymptotic scaling to finite energies. Once we accept his proof of the existance of the asymptotic scaling functions, we can restrict (4) the singularity structure on the surface of the light cone to be "canonical". Then we can perform our analysis which shows that these scaling functions should be attained rapidly at finite ν and Q^2. In fact the conditions for scaling to set in are

$$\nu R_p \gg 1 \quad \text{and} \quad Q^2/m^2 \gg 1 \tag{VI. 15}$$

where R_p is a size characteristic of the target proton (say the proton radius) and m^2 is a mass characteristic of the hadronic constituents of the incident photon. We find that the rapidity of approach to scaling and the form of the scaling functions obtained (specially at small x) are strongly dependent on the form of the longitudinal distance dependence of the current commutator. A very slow fall off at large longitudinal distances can lead to a slower approach to scaling as we increase Q^2. Interestingly, a slow power fall off at large longitudinal distance leads naturally to a Regge behavior in ν for the total cross sections and a Regge like term in x for the scaling functions at small x. Furthermore there exists a very simple relation (V.24) between the powers of ν, Q^2 and x and the power of the fall off in $y \cdot P$. As we discussed earlier, this relation will be sensitive to the nature of the light cone singularity. In particular if we did not have the scaling assumption to restrict the nature of this singularity to be canonical we would not get a similar unique relation. Therefore, care must be exercised in extending our analysis to on-mass-shell hadronic reactions (7).

Since we were dealing with a causal commutator, we could determine the order of magnitude of the transverse sizes involved in the interaction. We discussed these and their consequences for momentum transfer distributions in Section III. In particular the momentum transfer distribution would be expected to broaden at large Q^2. (15b). Such a simple discussion would not have been possible had we been dealing with a noncausal object.

Our space-time analysis and the data analysis leads us to interpret photon interactions in terms of a two component picture. These components are the short-ranged "bare" interactions and the long-ranged "hadronic" interactions of the physical photon. Interestingly, the effect of both these contributions seems to be present even at relatively low Q^2. We have no means, at present, to decide whether these represent two distinct dynamical processes or just the two extreme features of a single dynamical process. The possibility of extending such space-time analysis to nonforward amplitudes was already anticipated by Gribov, Ioffe, and Pomeranchuk (1). The discussion of such extensions and construction of models to understand the pure power law (7) fall off in $y \cdot P$ will be left for later publications.

The picture of photon interaction that we have developed is applicable to any target. In particular the target could be the nucleus. As a matter of fact the present work was motivated, in part, by trying to understand the physics of photon absorption in nuclei, which has some of the features of strong interactions (1, 26). It is expected that if the photon interacts very locally with a nucleon, the presence of other nucleons should not influence the probability of interaction with any particular nucleon (aside from the very small absorption); and the total cross section should be proportional to the number A of nucleons.

The observation of strong shadowing effects, like slower A dependence, is then circumstantial evidence of the long range part of photon absorption. Presumably this strongly shadowed part is the $\underline{a}=1$ "hadronic cloud". On the other hand, we do not expect the short range part to be shadowed at all. Then the interesting question that arises is the extent to which the other Regge-like terms, with $1 < \underline{a} < 3$, get shadowed. We expect them to be partially shadowed. Attempts to determine how partial is this shadowing will be made elsewhere. As an example if the $\underline{a}=2$ term was really an interference between the strongly shadowed $\underline{a}=1$ term and the unshadowed short-ranged term then it is not unreasonable to expect it to be partially shadowed. To the extent that the relative proportion of these terms, in our two component model, changes as Q^2 increases we would expect the amount of shadowing to decrease at large Q^2.

This interference model would also have interesting implications for the photo- and electroproduction of vector mesons off nucleons. In particular we expect that the vector mesons will come primarily from the diffractive ($\underline{a}=1$) and the interference ($\underline{a}=2$) parts of the amplitude, and not from the direct short-ranged part.

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FIGURE CAPTIONS

1. Kinematics of inelastic electron scattering.

2. The "hadronic" contribution of a physical photon in the rest frame of the target. The left half of the diagram represents the virtual photon transforming to hadrons some distance before reaching the nucleon and creating some final state at the vertical dashed line. The complete diagram represents the imaginary part of the forward Compton scattering amplitude, which by unitarity is related to the square of the matrix element. If the photon disappears at the origin of space-time, its point of re-emission is represented by y.

3. Schematic space-time picture of forward Compton scattering. The four vector y represents the space-time gap in the propagation of a virtual photon due to its interaction with a nucleon. The shaded area of dimension R_p represents absorption and re-emission inside the proton (short range contribution). The strip of thickness $1/\nu$ along the light cone represents the region of principle contribution in y_- if y_+ is not too large. For larger y_+, this region veers inside the light cone as indicated by the curve $\mathscr{C}$. The place where this happens is indicated by $\mathscr{L}(\sim \nu/m^2)$ and $\mathscr{R}(\sim \nu/Q^2)$ indicates the characteristic distance which is important in the long range contribution.

4. Decomposition of a cutoff power law term into a pure power law plus short range term.

5. The results of fitting a model consisting of VMD terms plus a scaling contribution to deep inelastic electron scattering data. The bottom curve is the transverse VMD contribution, the second is the non-VMD contribution, the third is the longitudinal VMD contribution, and the top one is the total theoretical fit to the data.

6. The deep inelastic electron proton scattering data.

   a. νW_2 calculated assuming constant $R = 0.18$.

   b. The observed values of R.

7. The results of subtracting a transverse VMD contribution to obtain the most universal residual $\nu W_{2(SC)}$. In (a), this VMD is applied to a constant real photon cross section, and in (b), to an energy dependent one.

8. The results of subtracting both transverse and longitudinal VMD contributions using $m_V^2 = m_\rho^2$ are shown in the top curve. The bottom one shows νW_2 from the experiment using the longitudinal cross section which results from the fit.

9. The same as Fig. 8 with m_V^2 adjusted to give the most universal residual.

10. The ratio R obtained from our fits shown in Figs. 8 and 9.

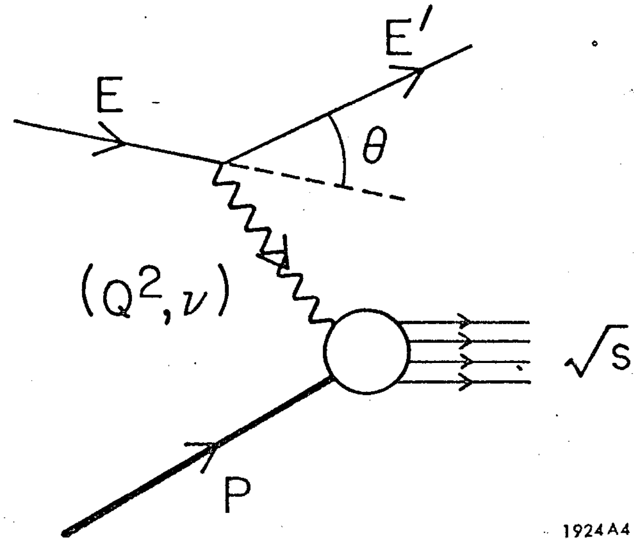


Fig. 1

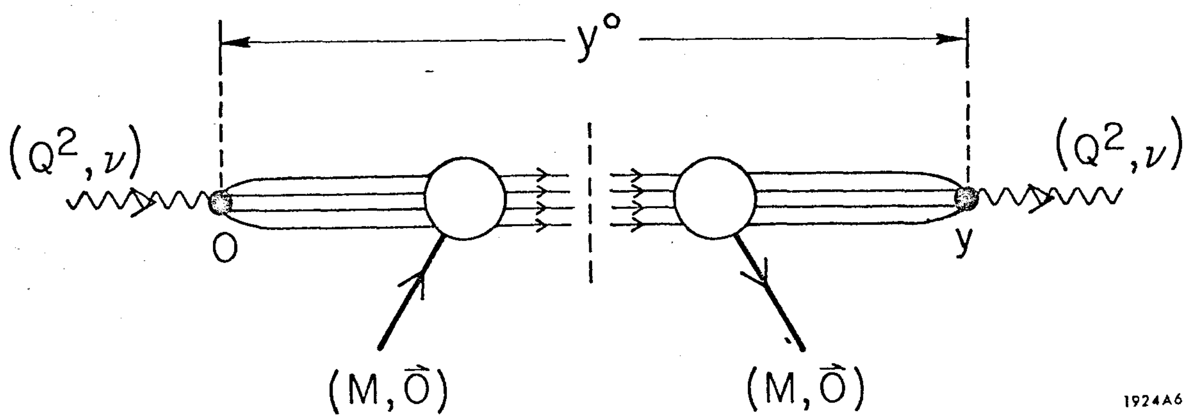
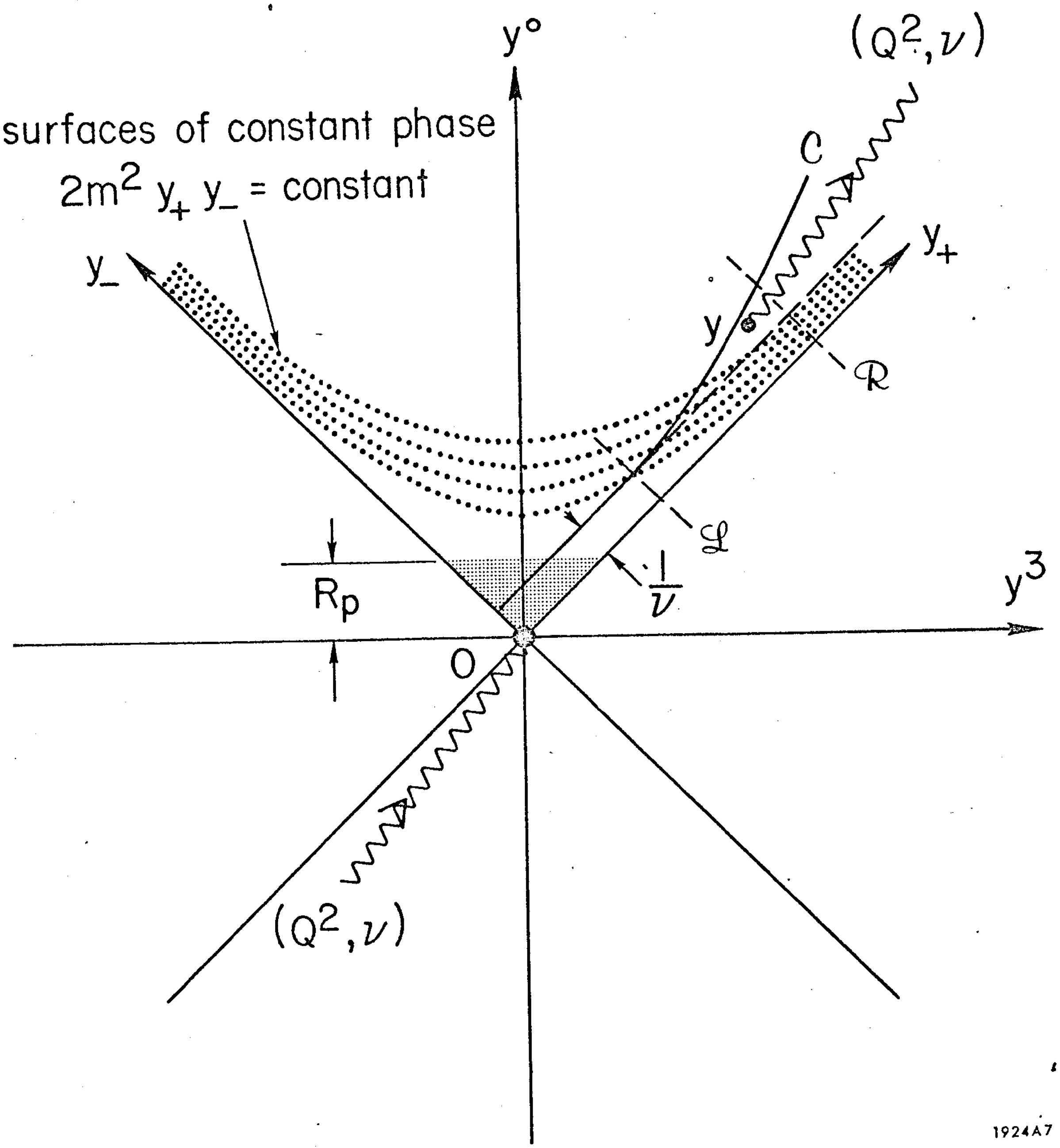


Fig. 2



y^0
(Q^2, ν)
surfaces of constant phase
$2m^2 y_+ y_- =$ constant
C
y_-
y_+
y
$\mathcal{R}$
$\mathcal{L}$
R_p
$\frac{1}{\nu}$
y^3
O
(Q^2, ν)
1924A7

Fig. 3

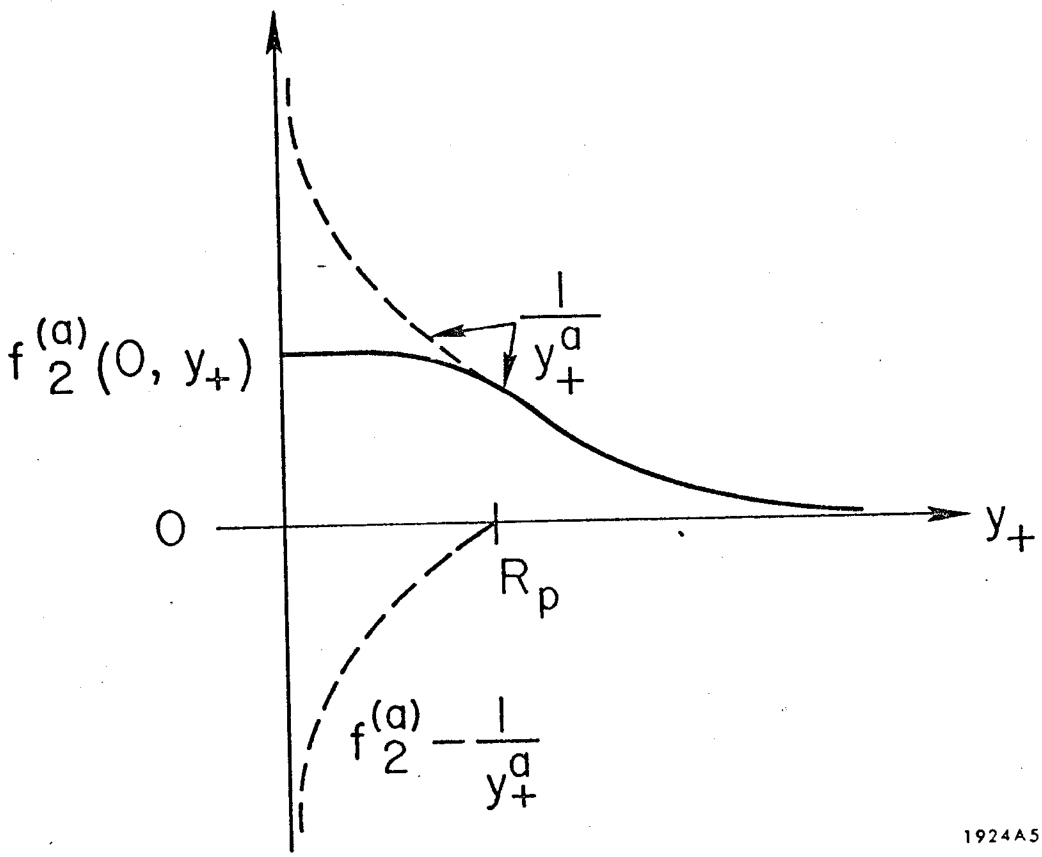
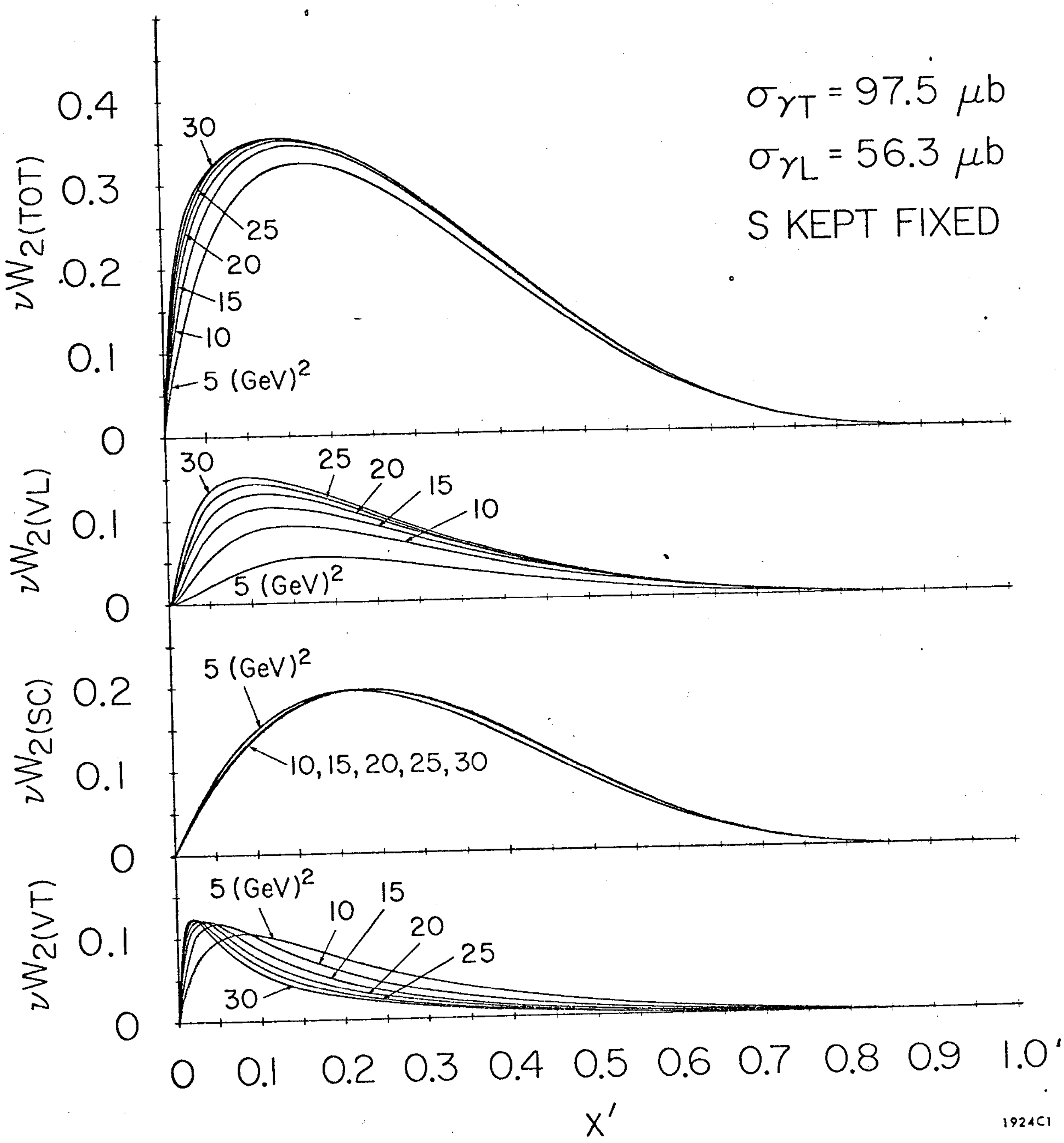


Fig. 4



$\sigma_{\gamma T}$ = 97.5 μb
$\sigma_{\gamma L}$ = 56.3 μb
S KEPT FIXED
νW_2(TOT)
νW_2(VL)
νW_2(SC)
νW_2(VT)
5 (GeV)^2
10, 15, 20, 25, 30
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
X′
1924C1

Fig. 5

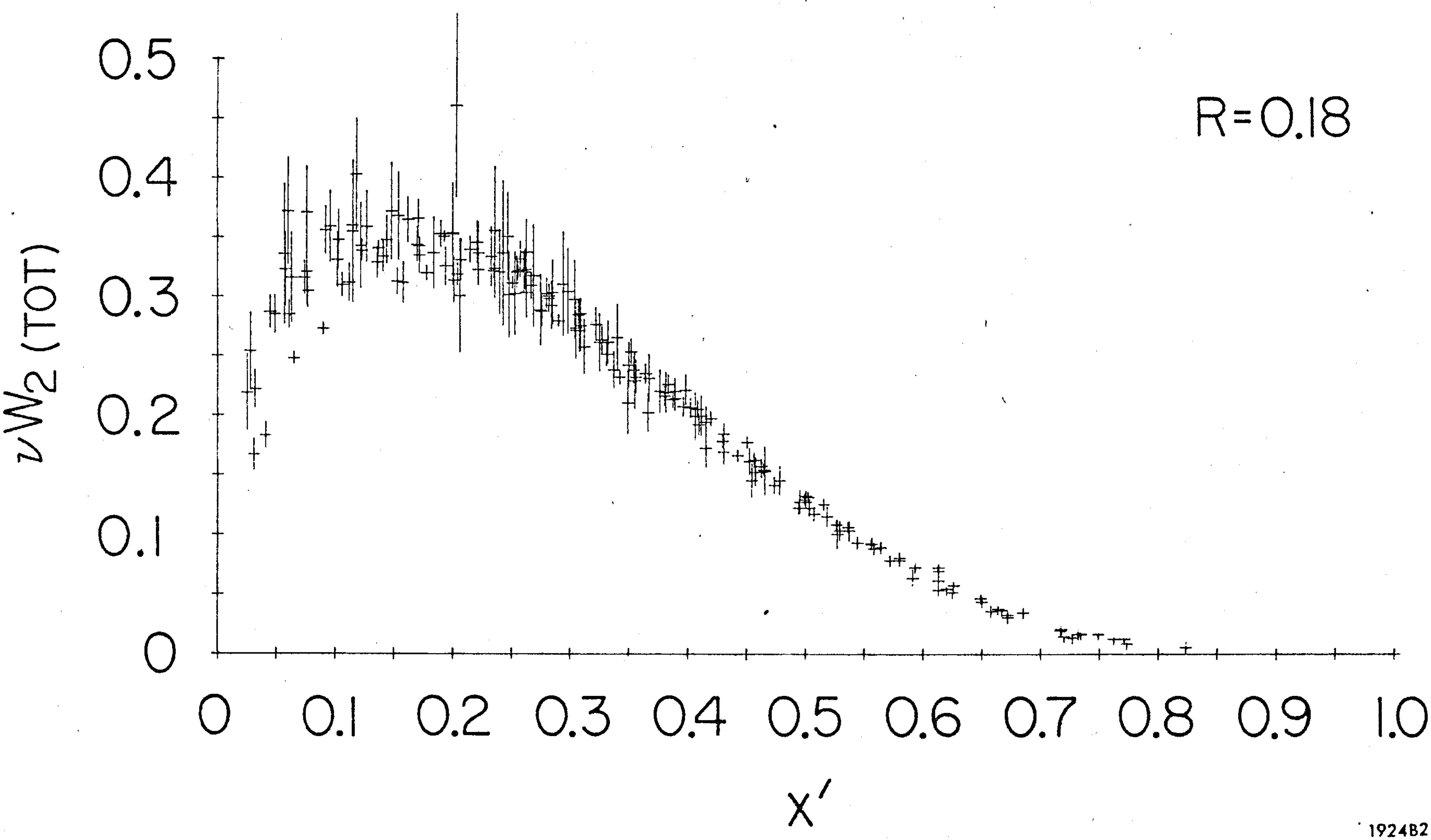
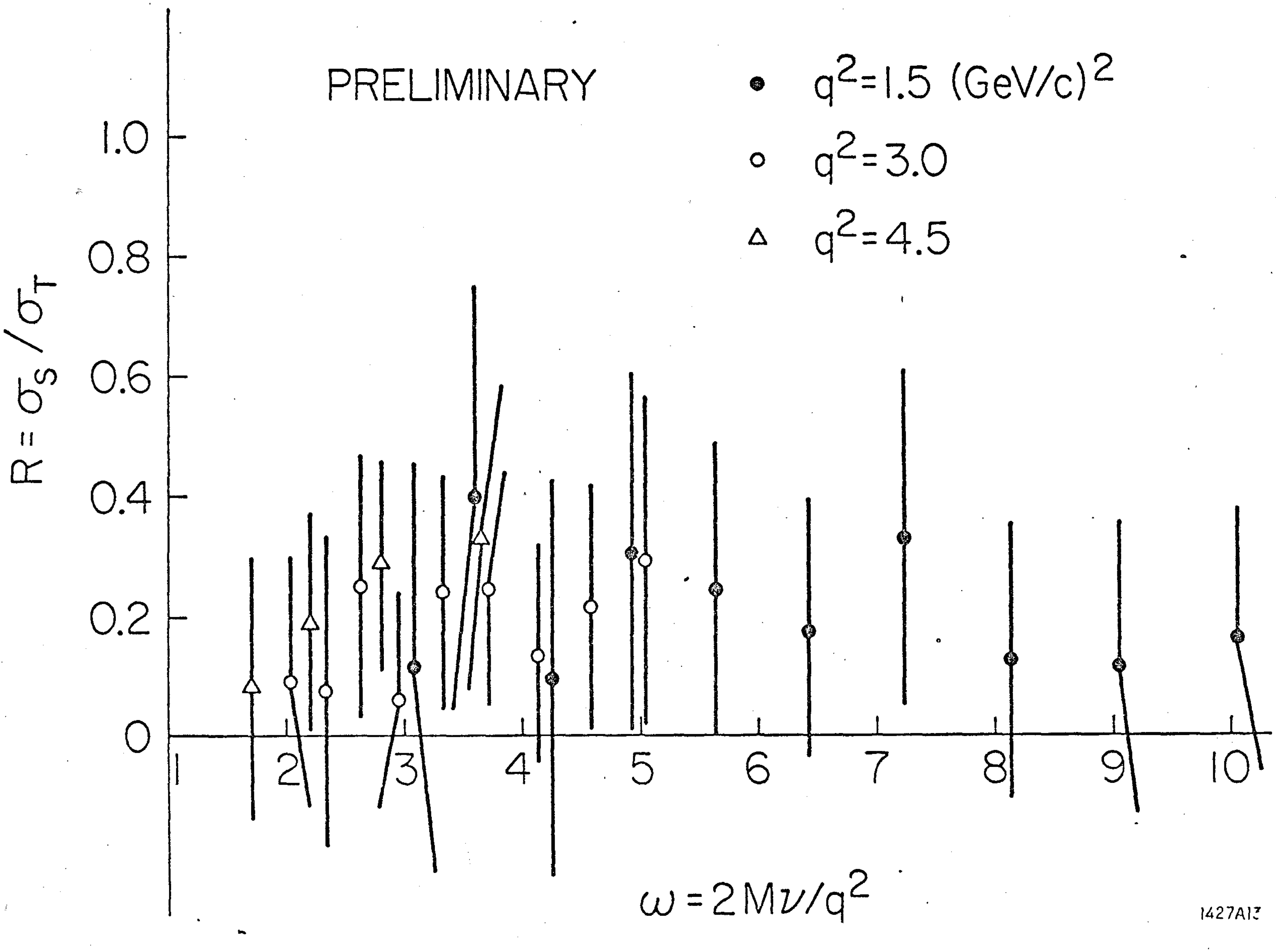
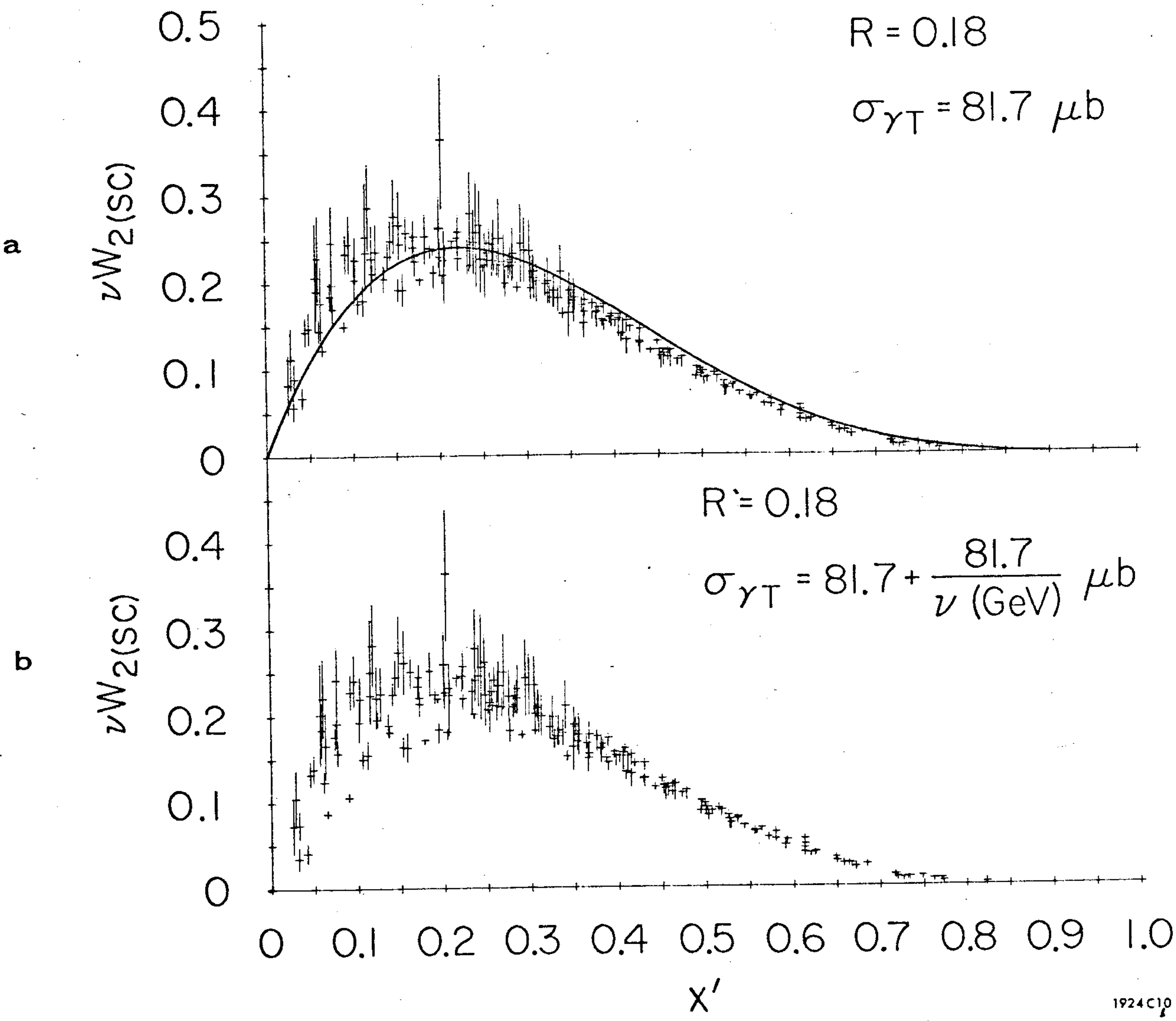


Fig. 6a



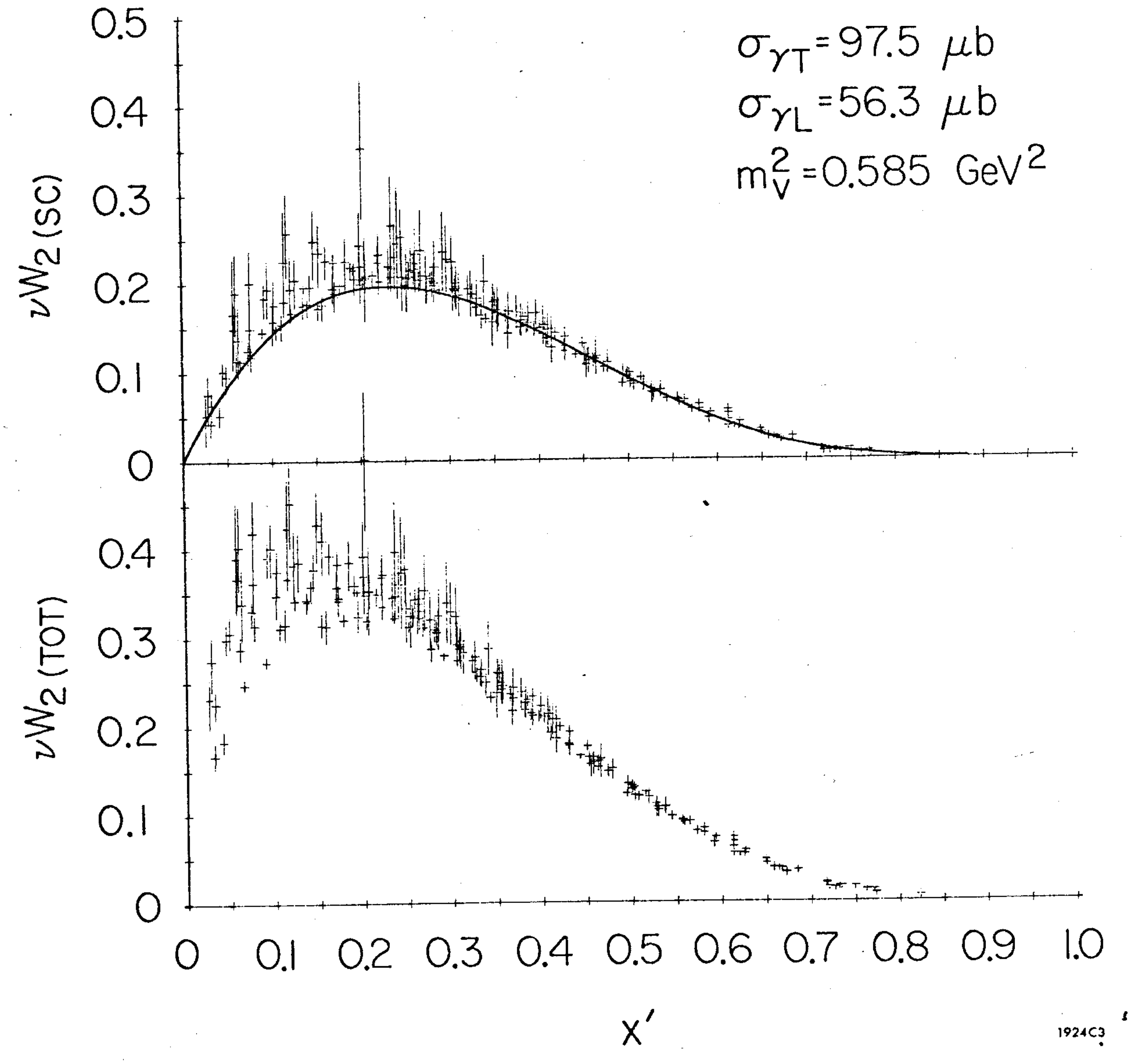
PRELIMINARY
q²=1.5 (GeV/c)²
q²=3.0
q²=4.5
R = σS / σT
1.0
0.8
0.6
0.4
0.2
0
1
2
3
4
5
6
7
8
9
10
ω = 2Mν/q²
1427A13

Fig. 6b



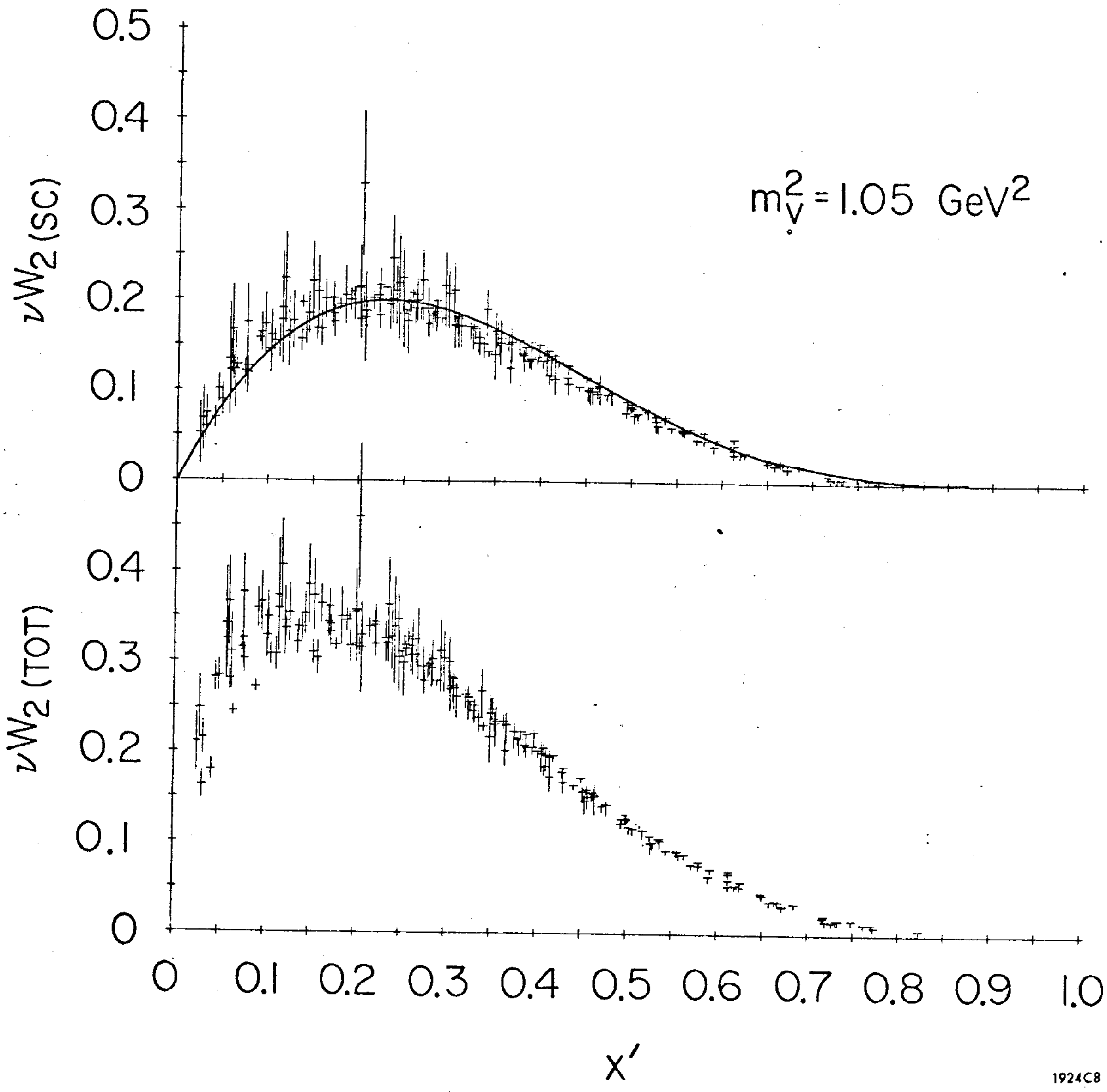
a
b
R = 0.18
$\sigma_{\gamma T}$ = 81.7 μb
R = 0.18
$\sigma_{\gamma T} = 81.7 + \frac{81.7}{\nu\ (\mathrm{GeV})}\ \mu$b
$\nu W_{2(SC)}$
$\nu W_{2(SC)}$
0.5
0.4
0.3
0.2
0.1
0
0.4
0.3
0.2
0.1
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
X′
1924C10

Fig. 7



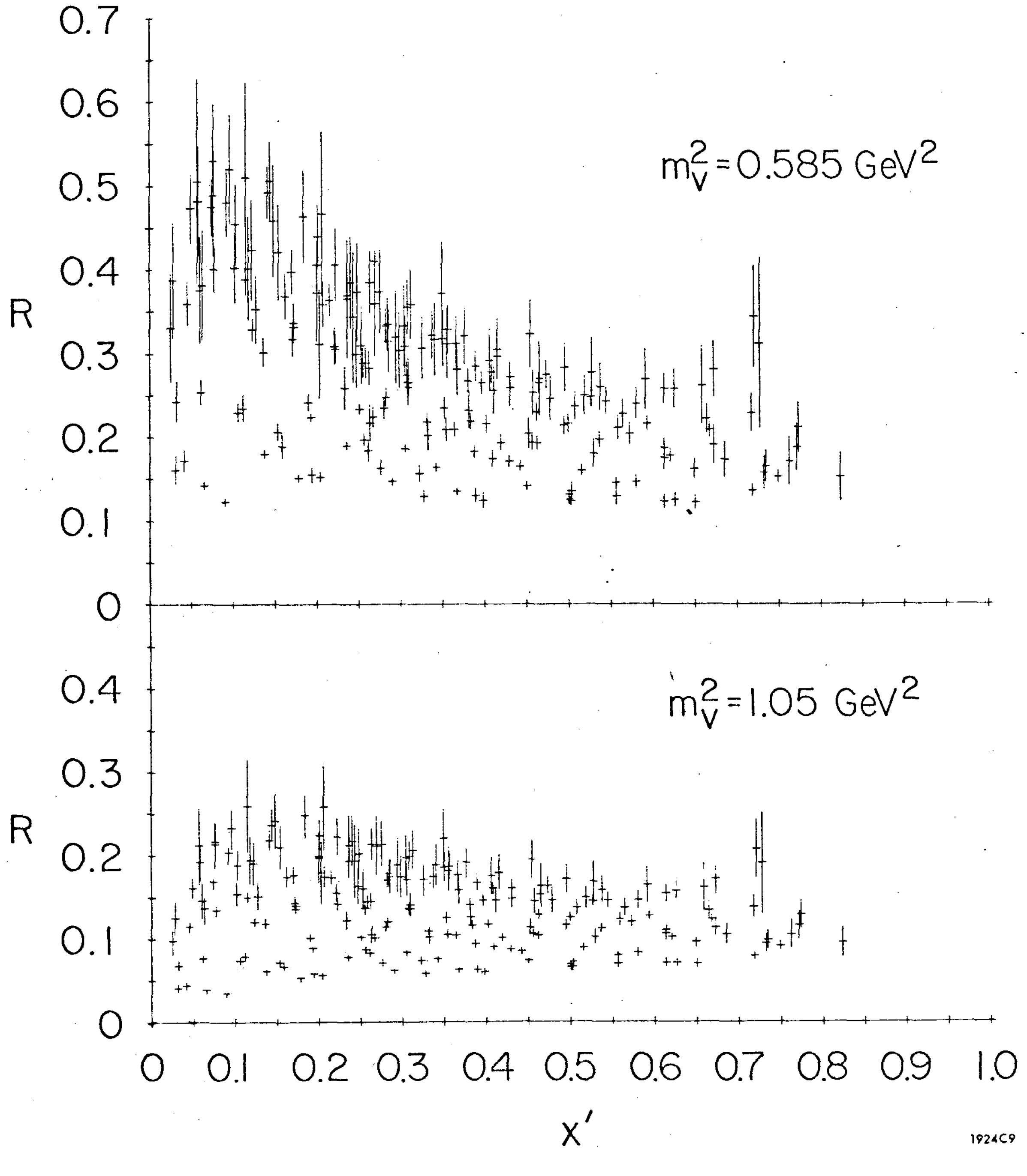
$\sigma_{\gamma T}$ = 97.5 μb
$\sigma_{\gamma L}$ = 56.3 μb
m_V^2 = 0.585 GeV^2
νW_2 (SC)
νW_2 (TOT)
0.5
0.4
0.3
0.2
0.1
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
X′
1924C3

Fig. 8



0.5
0.4
0.3
0.2
0.1
0
νW_2 (SC)
$m_V^2 = 1.05\ GeV^2$
0.4
0.3
0.2
0.1
0
νW_2 (TOT)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
X′
1924C8

Fig. 9



0.7
0.6
0.5
0.4
R
0.3
0.2
0.1
0
$m_V^2 = 0.585$ GeV^2
0.4
0.3
R
0.2
0.1
0
$m_V^2 = 1.05$ GeV^2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
X′
1924C9

Fig. 10