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TRANSVERSE MOMENTUM CORRELATIONS IN PRODUCTION PROCESSES*

R. Blankenbecler and Thomas L. Neff
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

A simple method to analyze two-particle correlations of transverse momenta in multiparticle final states is described. A clear physical interpretation of such correlations is presented, and a preliminary analysis of several reactions involving three and four body final states is reported.

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Increasing machine energies have recently stimulated intense experimental and theoretical interest in multiparticle production processes. Much of this interest, however, has been focused on measuring multiplicities and studying similar properties of inclusive ($a + b \rightarrow c$ + anything) reactions. Theoretical models for production processes (parton, multiperipheral, multiregge, limiting fragmentation, etc.) have been used primarily to make either structural predictions such as scaling behavior or global and integrated predictions such as mean multiplicity as a function of energy, longitudinal distributions, and so forth.[1] Inclusive experiments do not generally allow direct detailed tests of basic model assumptions.

The surfeit of information in these processes is difficult to digest because of the lack of a clear choice among the large number of possible sets of variables with uncertain physical interpretations.[2] The purpose of this letter is to indicate how study of two particle corrections in multiparticle production processes can yield new insights into, and experimental tests of, the underlying production mechanisms. Although the method can be applied to quite general processes, in particular, the bi-inclusive type ($a + b \rightarrow d + d$ + anything), we limit ourselves to three and four particle (exclusive) final state events for purposes of illustration. The generalization of the analysis to higher multiplicities and inclusive reactions will be clear and the details will be the subject of a future communication.

A simple extension and combination of the multiperipheral and absorption models, in an impact parameter picture, will be used to suggest and parameterize correlations in the transverse momenta of two final state particles in the nonasymptotic region.[3] Our model is not meant to be taken very literally and it is probably not the only model that fits the data, but it does seem to work quite well. The impact parameter description has been used primarily for elastic and quasi-two-body processes.[4] In such reactions one represents the scattering amplitude at fixed energy as an integral over a two-dimensional impact parameter $\vec{b}$ with structure function $T(b^2)$. The natural extension to the inelastic three body final state is illustrated in Fig. 1a,b. At a fixed energy, the model leads to a differential cross section, in any frame colinear with the beam, of the form

$$\frac{d^4\sigma}{d^2k\, d^2n} = G(\vec{k},\vec{n}) \quad , \tag{1}$$

where $\vec{k}$ is the transverse momentum of the final (leading) particle corresponding to the incident laboratory projectile and $\vec{n}$ is the transverse momentum of the final particle corresponding to the target. Longitudinal momenta have been integrated out. Whether G depends on k^2 and n^2 only or depends strongly on $\vec{k}\cdot\vec{n}$ is our principal concern.

Since the transverse momenta are quite limited, it will be shown shortly that an experimentally and theoretically acceptable parameterization at large but fixed energy is

$$G(\vec{k},\vec{n}) = G_0\, e^{-a\vec{k}^2 - b\vec{n}^2 - c\vec{k}\cdot\vec{n}}\, \Phi(\vec{k},\vec{n},s) \quad , \tag{2}$$

where a and b measure the approximate "sizes" of projectile and target and $\Phi(\vec{k},\vec{n},s)$ is the residual phase space factor. This form is only suggested to provide a simple characterization of the data. A formula with a more acceptable theoretical pedigree will be discussed below. The exponential cutoffs on transverse momentum transfers will kinematically produce leading particles in the center-of-mass, as observed experimentally. The correlation parameter c is nonzero if rescattering or absorption between target and projectile in initial or final states is important. The simple multiperipheral model neglects such rescattering. This form is properly used only for fixed s but it will be assumed here that the size parameters have only a weak dependence on the energy. Note that, using momentum conservation, the distribution (2) can be rewritten in terms of any two final state momenta but the parameters a, b, c will always be defined as above. The parameter c is most accurately determined by performing a correlation between k and n.

A remaining important question is whether G_0 can have a dependence upon transverse momenta due to the spins of exchanged particles and quantum numbers of the produced particles. For example, in secondary pion production by rho and pion exchange along the ladder (see Fig. 1a), G_0 has only a weak dependence upon transverse momenta and can be taken to be a constant. This case will be referred to as transverse monopole radiation. However, for pion production by a rho and omega ladder, G_0 has a strong dependence on transverse momenta due to a spin coupling kinematic factor $[(\hat{z} \times \vec{k}) \cdot \vec{\pi}]^2$. This will produce transverse dipole radiation. These two noninterfering pion radiation patterns are shown in Fig. 1c as dashed lines, where the vector $\vec{k}$ defines the x-axis. The exponential factors in (2) distort the patterns to the forms shown by the solid lines.

The physical interpretation of the parameters a, b, c in the distribution (2) follow from the eikonal model depicted in Fig. 1b. The two circles indicate the domains of the impact parameters for the two particle elastic or quasi-elastic amplitudes. The full transition amplitude is

$$M(\vec{k},\vec{n}) = \quad d^2b_1 d^2b_2 \, e^{-i\vec{b}_1\cdot\vec{k}+i\vec{b}_2\cdot\vec{n}} \, T(\vec{b}_1)\, T(\vec{b}_2) \left\{A_{ii}(\vec{B})\, A_{ff}(\vec{B})\right\}^{\frac{1}{2}} \qquad (3)$$

where the A factors are an ancient way[5] of accounting for absorptive interactions between target and projectile in the initial and final states and $\vec{B} = \vec{b}_1 + \vec{b}_2$. In some approximations, A is the S-matrix; in others, $\sqrt{A}$ is $\frac{1}{2}(1+S)$. In any case, if this factor is equal to a constant for all contributing impact parameters, the correlation coefficient will be zero. The radius parameters in the two T-matrices and in the A-factor can be estimated from elastic and quasi-elastic reactions and used to predict $M(\vec{k},\vec{n})$. Since the A factors are small near B=0, the results may be relatively insensitive to the behavior of T near the origin. If not, the correlation analysis will lead to a test of the difference between the behavior of diffraction and Regge exchange near b=0 as has been discussed by H. Harari.[6] Diagrams with the particles interchanged in the final state have been neglected. This seems reasonable from the experimental longitudinal momentum distributions.

For comparison of these ideas with experiment, consider the reactions

$$\pi^- p \rightarrow \pi^- \pi^+ N \tag{4a}$$

$$K_L P \rightarrow K^{\mp} \pi^{\pm} P \ . \tag{4b}$$

The two reactions in (4b) give very similar distributions and are added together in the fits. To illustrate clearly that correlations are present in the data, let us define ϕ to be the angle between the transverse momenta of the scattered projectile and the nucleon. The experimental ϕ-distributions are shown in Fig. 2a (recall Fig. 1c). This angular correlation plot shows clearly that $c \neq 0$ since phase space with cutoffs on the momentum transfers is quite flat. Alternatively, a different view of the correlations can be obtained by using the transverse momentum of one of the particles to define the x-axis and then projecting the transverse momentum distribution of one of the remaining particles onto this and the orthogonal y-axis. This will be termed a cartesian correlation. We have arbitrarily chosen to display the correlation between the charged boson pair in each of the reactions (4a, b). Any pair could be selected, and the resulting distributions are equally distinctive and emphasize different aspects of the production process. Experimental results are plotted as cartesian correlations in Fig. 2b, c for the two reactions of interest. The secondary distribution in the $\pi^- P$ case is seen to be primarily transverse monopole radiation while that of $K_L P$ has a substantial dipole contribution.

The experimental results for the $\pi^- P$ reaction at 16 GeV/c are shown in Fig. 2b. The solid curve is our rough fit based on the parameterization of Eq. (2) with $a \cong 4$, $b \cong 18$, and $c \cong 7$. These parameters are not directly related to two particle scattering radii but the analytic form has the virtue of being very simple. The fit to the ϕ distribution is also satisfactory. The important point is that a value of $c \cong 0$ cannot fit the x and y widths and the ϕ distribution.

The parameterization of Eq. (3) has also been fit to the data assuming the T-matrices are of the form $\exp(-b^2/2R^2)$ and A_{ii} $(=A_{ff})$ is of the form $1 - \epsilon \exp(-b^2/2R_{12}^2)$. The values $\epsilon = 1.0$, $R_1^2 \cong 4$, $R_2^2 \cong 14$, and $R_{12}^2 \cong 8$ yield results virtually identical to the simple model discussed above. The radii R_2^2 and R_{12}^2 were chosen to roughly fit the pion-nucleon quasi-elastic and elastic scattering t-distributions.

The secondary pion distributions for the $K_L P$ reaction (with incident K_L beam energy between 6 and 10 GeV/c) are shown in Fig. 2c,[7] and clearly both monopole and dipole contributions must be included. The effective radius in the π-p subamplitude may, in general, be different for pion exchange (transverse monopole radiation) and omega exchange (dipole radiation). The simplest generalization of Eq. (2) is then

$$G(\vec{k},\vec{n}) = \left[g_0 \, e^{-b_0 n^2} + g_1 a b_1 \left((\hat{z} \times \vec{k}) \cdot \vec{\pi} \right)^2 e^{-b_1 n^2} \right] e^{-ak^2 - cn \cdot k} \qquad (5)$$

The data for events with invariant final state masses (subenergies) above the K* and Δ resonance regions is shown in Fig. 2a, c. The effect of the dipole term on the dominant monopole term is evident here. Despite a slight variation with invariant masses, a good average fit is obtained from Eq. (5) with $a \cong 6$, $b_0 \cong b_1 \cong 10$, $c \cong 6$, and $g_1/g_0 \cong 7.4$. The monopole and dipole contributions appear to have the same exponential behavior in this region, but the fits are not very sensitive to b_0. It appears that the parameters smoothly attain asymptotic values as the relative subenergies increase. The ratio of c to a, however, remains near one. It does not seem to become small as one might expect from simple multiperipheral or multiregge models. Thus one is led to the conclusion that absorption may be important even for large subenergies.

If the impact parameter distribution of T(b) varies rapidly, for example as one of the final pairs move through a resonance (presumably the formula is not valid if both pairs resonate) then this variation will be reflected in the values of the parameters a, b, and c. The values taken by these parameters will then depend on whether or not one is in the resonance region. In the K* resonance region, the data indicate that the dipole term strongly dominates but there is still a clear monopole term. The relative weights of these terms change as one moves through the resonance region. The azimuthal distribution provides a clear and intuitive way of separating these contributions. A detailed study of resonance behavior will be presented elsewhere.

Correlations very similar to the above survive in higher multiplicity events. In Fig. 3 we exhibit the π^- distributions relative to the leading proton in the reaction $\pi^+ p \rightarrow \pi^+ \pi^- \pi^+ p$. The x- and y-widths are strikingly different. An interesting further question is whether these strong correlations, which have similar forms in all the exclusive reactions discussed here, will survive the averaging implicit in the bi-inclusive reactions mentioned earlier. The above model, including absorptive corrections, suggests that they will.

In the sense that the Pomeron is connected with the restoration of many-particle s-channel unitarity (in a picture with t-channel exchanges) our analysis is indeed probing the nature of this "singularity", and the correlations it induces. One expects the description of the Pomeron in a t-channel description to be complicated (cuts, etc.). The absorption factor A in Eq. (3), for example, should in this picture be "Pomeron dominated" because of the large relative energy of the particles involved.

In conclusion, correlations of transverse momenta provide a simple way to analyze and parameterize certain aspects of multidimensional data. This will be particularly useful when extended to higher multiplicity events, as the method can be used to study correlations between any pair of final state particles. For example, the correlation between pion pairs in differing charge states should provide information on interactions and resonances in the final state. The presence of higher multipoles will be very interesting. This freedom should be of great utility in studying the properties intrinsic to, and connections between, the secondaries assigned to the fragmentation and pionization regions. The corresponding experimental analysis should provide detailed information about physical mechanisms and sensitive tests of models of production processes.

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7. We wish to thank SLAC experimental group B for providing this experimental data prior to publication.

FIGURE CAPTIONS

1. a) The general reaction where k_0 is incident boson and n_0 the incident baryon in reactions considered. b) Impact parameters in transverse configuration space. The discs indicate two-particle elastic or quasi-elastic amplitudes in this space. The secondary pion can be thought of as being produced from the overlap region. c) Transverse radiation patterns. The leading boson k defines the x-axis and the pure monopole and dipole patterns of the secondary are shown as dotted lines; these are distorted to the forms shown by the solid lines.

2. a) The angular correlations between the transverse momenta of the final state leading boson and baryon. b) Cartesian correlation plot for the $\pi^- p$ reaction as functions of π^+_x and π^+_y, where π^+ is the secondary pion and the x-axis is defined by the transverse momentum of the leading boson. c) Cartesian correlation plot for the $K_L P$ reaction as functions of π_x and π_y, where π is the secondary particle and the x-axis is defined by the transverse momentum of the leading boson (the kaon).

3. The cartesian correlation plot for the π^- relative to the final proton in the reaction $\pi^+ p \rightarrow \pi^+ \pi^- \pi^+ p$.

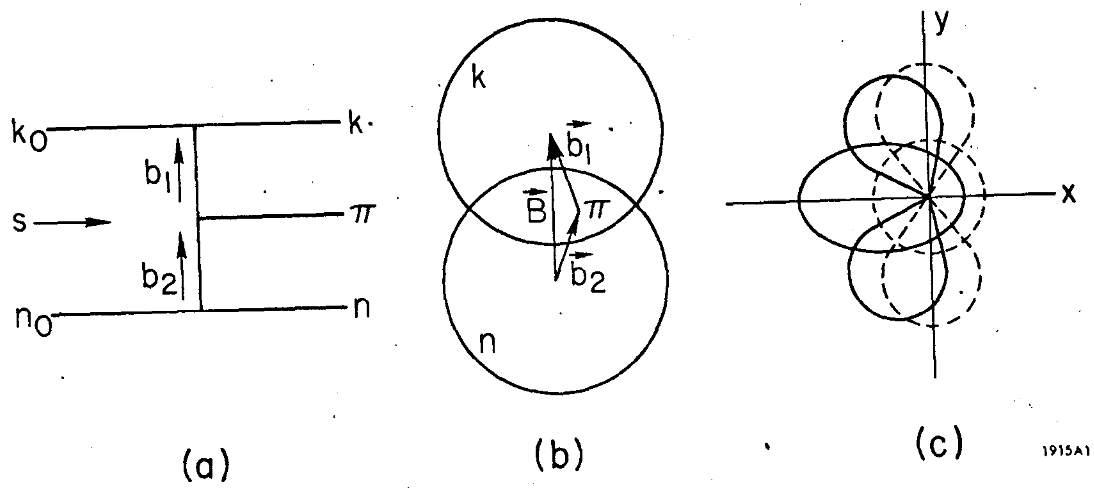


Fig. 1

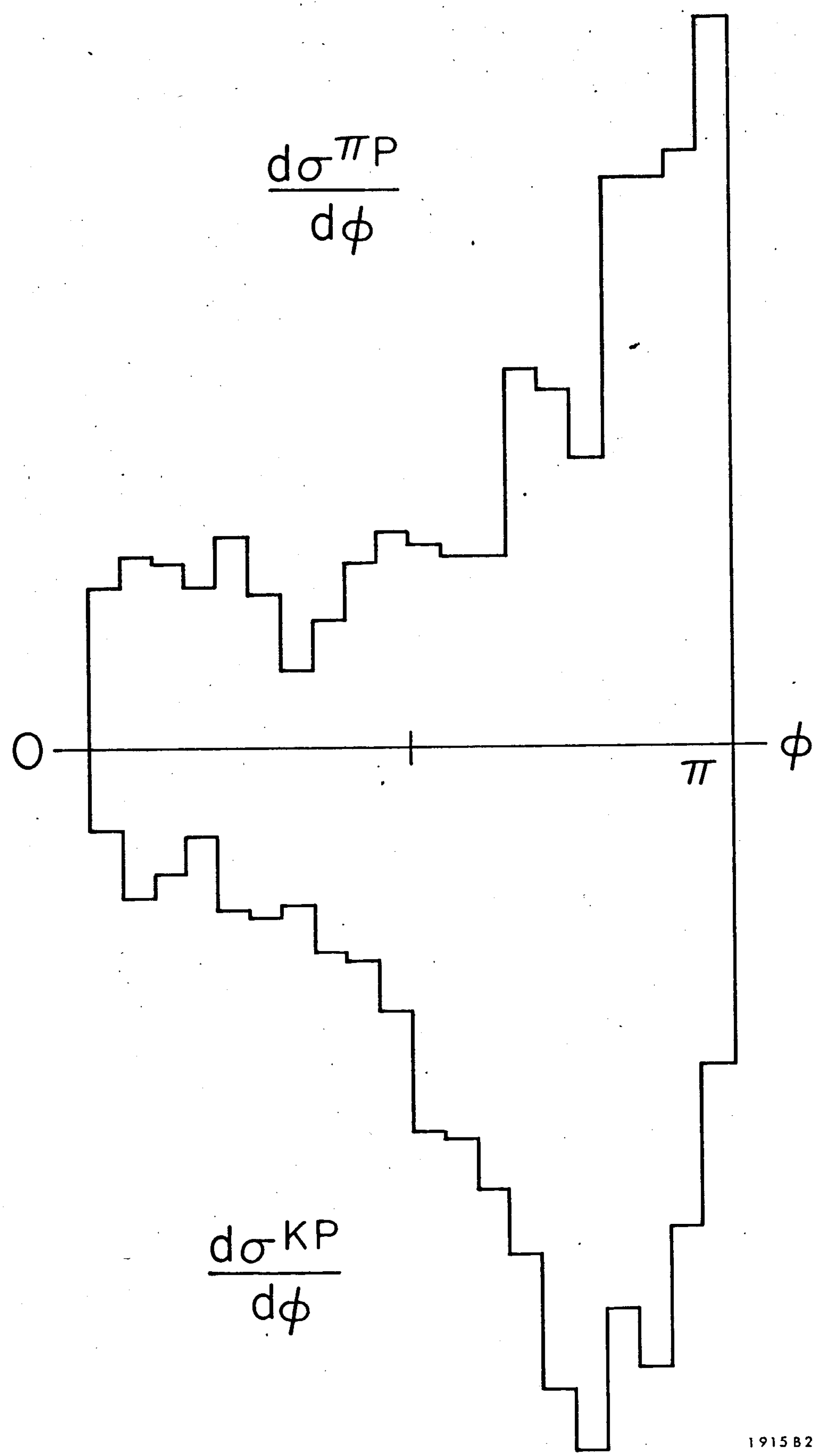
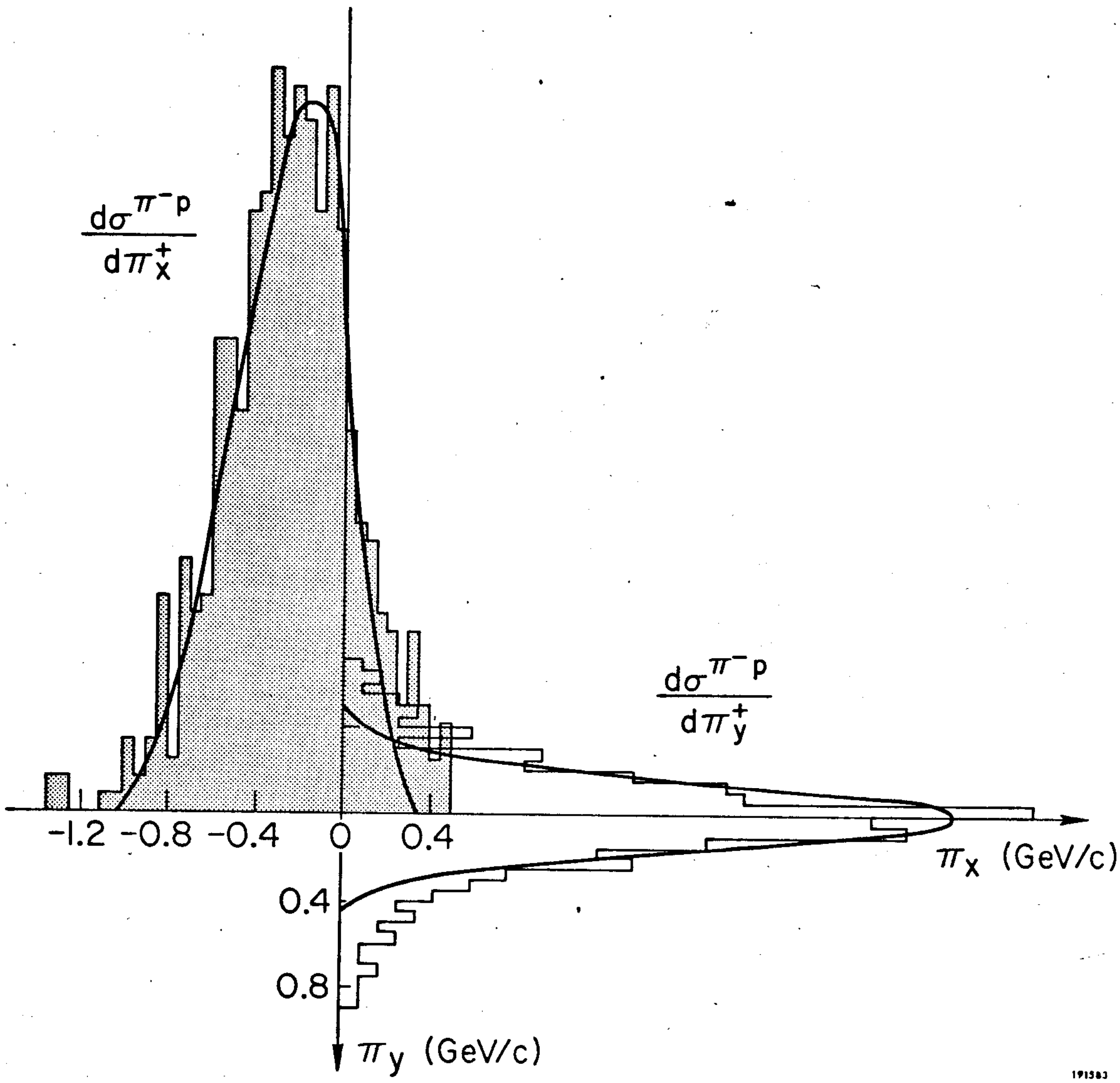
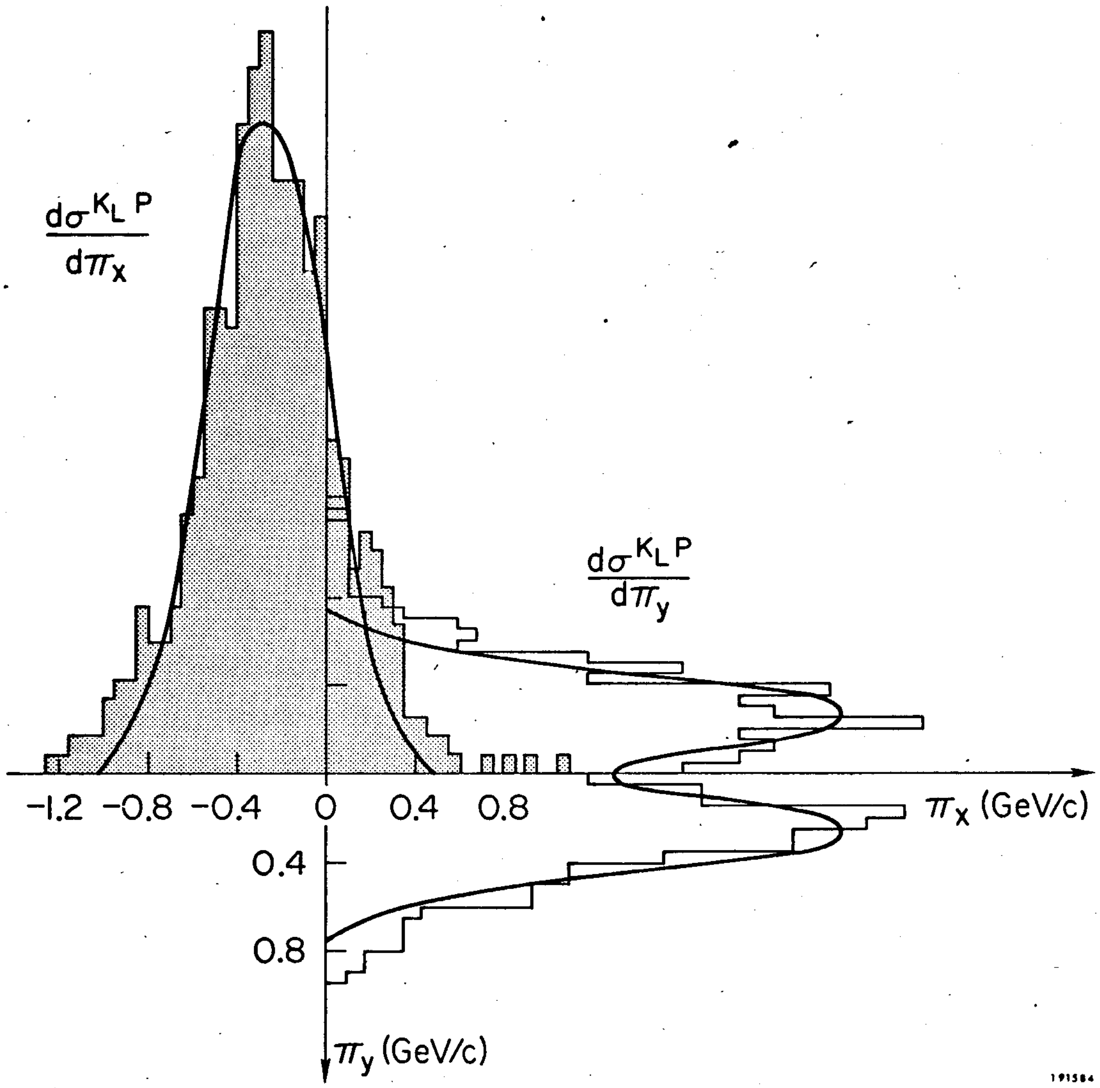


Fig. 2a



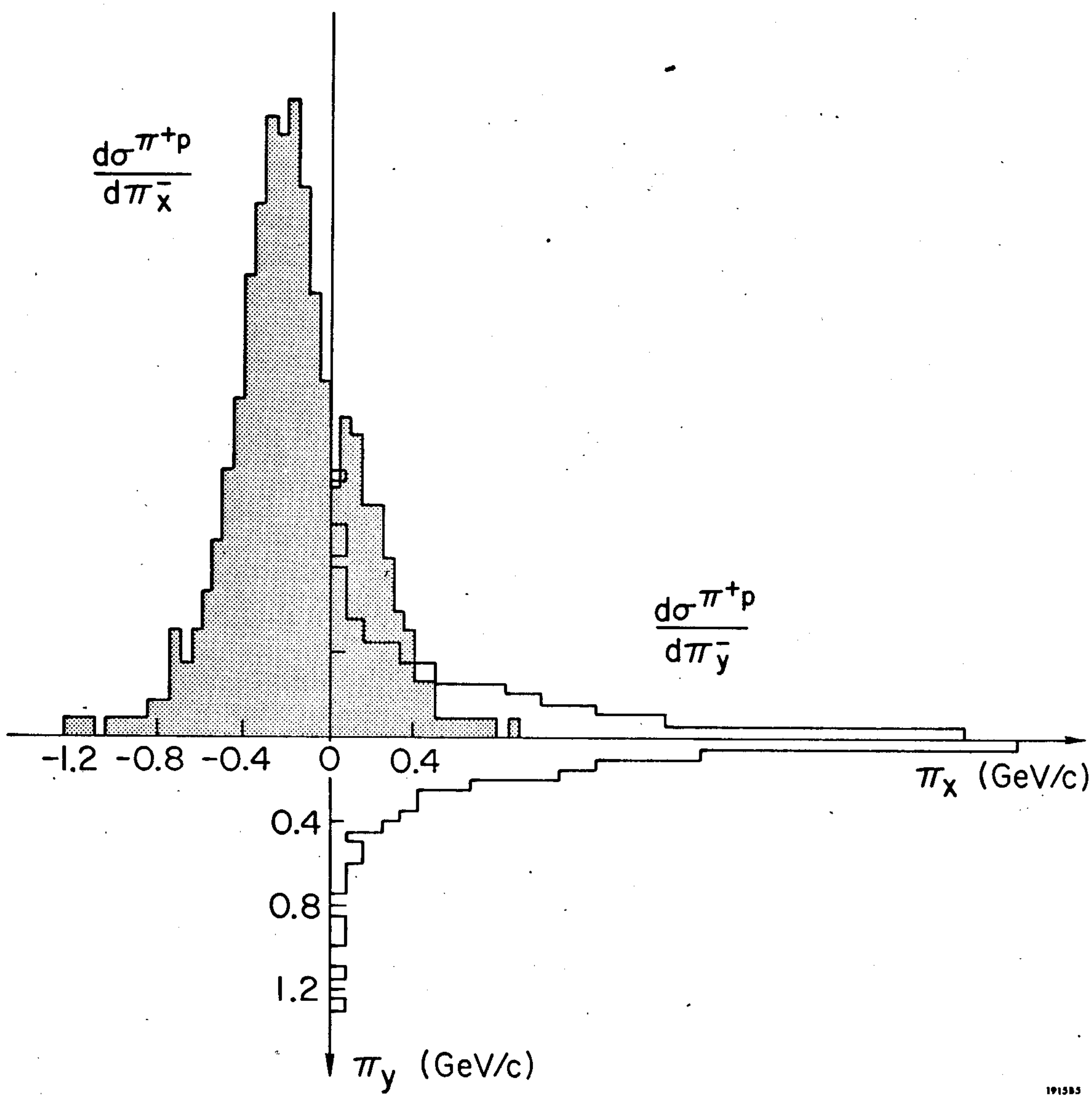
$\frac{d\sigma^{\pi^- p}}{d\pi_x^+}$
$\frac{d\sigma^{\pi^- p}}{d\pi_y^+}$
-1.2 -0.8 -0.4 0 0.4
0.4
0.8
π_x (GeV/c)
π_y (GeV/c)

Fig. 2b



$\frac{d\sigma^{K_L P}}{d\pi_x}$
$\frac{d\sigma^{K_L P}}{d\pi_y}$
-1.2
-0.8
-0.4
0
0.4
0.8
π_x (GeV/c)
0.4
0.8
π_y (GeV/c)

Fig. 2c



$\frac{d\sigma^{\pi^+p}}{d\pi_x^-}$
$\frac{d\sigma^{\pi^+p}}{d\pi_y^-}$
-1.2
-0.8
-0.4
0
0.4
π_x (GeV/c)
0.4
0.8
1.2
π_y (GeV/c)

Fig. 3