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SLAC-PUB-937
(TH) and (EXP)
July 1971

COORDINATE SPACE STRUCTURE IN THE PARTON MODEL*

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Abstract

We study the relation between the parton model and light cone analyses for highly inelastic leptonic processes. Results are displayed which follow from scaling laws in general, independently of specific parton model predictions. We conclude that the assumptions of the light cone analysis of inelastic electron scattering are supported by the parton model. However, the parton model matrix element for massive muon pair production is not light-cone dominated, nor does it have the same light cone singularity as inelastic electron scattering.

(Submitted to Physical Review Letters)

*Work supported in part by the U. S. Atomic Energy Commission.

†NSF Predoctoral Fellow.

Many theoretical explanations have been proposed to account for the experimental observation of scaling in highly inelastic electron scattering. Although each of these, by design, predicts scaling behavior for electron scattering in the Bjorken limit region, the relation of one approach to another is only poorly understood at present. The subject of this letter is the relation between two of the currently most popular of these approaches: the parton model of Feynman[1] and Bjorken[2] especially as developed by Drell, Levy and Yan[3]; and the study of light cone singularities proposed by Ioffe, Frishman, Brandt,[4] and others.

It is often noted that these two approaches achieve scaling predictions for inelastic electron scattering by assuming certain free field behavior: for the commutators near the light cone in the one instance, and for the scattering amplitude in particular regions of momentum space in the other. In order to understand better the significance of this similarity, we explore more quantitatively the relation between these two approaches, first for the well-studied subject of electron scattering and second for the production of massive lepton-antilepton pairs in high-energy hadronic collisions. Specifically, we display explicitly the coordinate space structure of the appropriate matrix elements which is implicit in the scaling laws derived in the parton model, and further display the role of singularities near the light cone in determining this scaling behavior. Our techniques may be applied to scaling laws derived independently of the parton model. For deep inelastic electron scattering we first derive our results from the scaling laws without discussing specific theoretical origins for the scaling. Afterwards we outline the derivation of this coordinate space behavior from the details of the parton model expressions for the matrix elements in question. This latter approach yields considerable insight into the physical origin of our results. Finally we analyze massive lepton pair production with similar techniques.

The results of our analysis may be summarized as follows. For highly inelastic lepton scattering, Bjorken scaling[5] implies that the matrix element has only free field singularities on the light cone and is smooth enough away from the light cone to support the usual arguments for light cone dominance. From the viewpoint of the parton model, the light cone singularity appears as the propagator of the quasi-elastically scattered parton.

In massive lepton pair creation, major differences between the light cone analysis and the parton model become evident. From the parton model scaling law of Drell and Yan[6] we find that the matrix element of interest has high-frequency oscillations far away from the light cone which invalidate the usual arguments for light cone dominance; and moreover, the matrix element on the light cone does not have the singularity that is present in deep inelastic electron scattering. In studying this process by light cone techniques,[7] one assumes not only that the light cone dominates but also that the leading singularity is that "measured" in inelastic electron scattering. The absence of this singularity in the parton model for this process is associated with the absence of an elastically scattered parton in the final state.

To begin, recall that inelastic electron scattering is characterized by the well-known structure functions W_1 and W_2 defined by:

$$W_{\mu\nu} = \int d^4y\, e^{iq\cdot y}\, \frac{4\pi^2 E_P}{M} \langle P \left| J_\mu(y) J_\nu(0) \right| P \rangle \tag{1A}$$

$$= -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1(q^2,\nu) + \frac{1}{M^2}\left(P_\mu - \frac{P\cdot q}{q^2}\, q_\mu\right)\left(P_\nu - \frac{P\cdot q}{q^2}\, q_\nu\right) W_2(q^2,\nu) \tag{1B}$$

where $P\cdot q \equiv M\nu$; $q^2 = -Q^2 \le 0$.

We restrict our attention to the contracted tensor W^μ_μ: the slightly more complicated general case together with the similarly straightforward extension to the current commutator will be handled elsewhere.[8]

Inelastic scattering experiments at SLAC[9] support the hypothesis that in the Bjorken limit (ν, $Q^2 \longrightarrow \infty$; $x = Q^2/2M\nu$ finite) W_1 and νW_2 become functions of x alone:

$$\mathrm{Lim}_{bj}\, M W_1\left(q^2,\nu\right) = F_1(x) \quad \text{and} \quad \mathrm{Lim}_{bj}\, \nu\, W_2\left(q^2,\nu\right) = F_2(x)$$

so that:

$$\mathrm{Lim}_{bj}\, W^{\mu}_{\mu}\left(q^2, \nu\right) = W^{\mu}_{\mu}(x) = \frac{1}{M}\left(-3\,F_1(x) + \frac{1}{2x}\, F_2(x)\right)$$

Since x is kinematically constrained to be between zero and one, we have:

$$W^{\mu}_{\mu}(x) = \int_0^1 d\eta\, \delta\,(\eta - x)\, W^{\mu}_{\mu}(\eta)$$

We now substitute into this the following identity:

$$2\pi\eta\,\delta\,(\eta - x) = \int d^4y\, e^{iq\cdot y} \Box \left\{\Delta_+\left(y,m^2\right) e^{iP_\eta\cdot y}\right\} \qquad \begin{array}{l} 0 < \eta < 1 \\ 0 < x < 1 \end{array}$$

where

$$\Delta_{\pm} \equiv (2\pi)^{-3} \int d^4k\, \theta\left(k_o\right) \delta\left(k^2 - m^2\right) e^{\mp ik\cdot y}$$

and P_μ is a four-vector satisfying: $P_\eta^2 = m^2$; $P_\eta \cdot q = \eta M\nu$. We obtain:

$$W^{\mu}_{\mu}(x) = \frac{1}{4\pi M} \int d^4y\, e^{iq\cdot y} \Box \left\{\Delta_+\left(y,m^2\right) \int_0^1 \frac{d\eta}{\eta^2} \left[F_2(\eta) - 6\eta F_1(\eta)\right] e^{iP_\eta\cdot y}\right\}$$

Up to terms whose Fourier transform vanishes in the scaling limit, the coordinate space structure of the matrix element is then:

$$\frac{4\pi^2 E_P}{M} \langle P|J_\mu(y)\, J^\mu(0)|P\rangle = \frac{1}{4\pi M} \Box \left\{\Delta_+\left(y,m^2\right) \int_0^1 \frac{d\eta}{\eta^2} \left[F_2(\eta) - 6\eta F_1(\eta)\right] e^{iP_\eta\cdot y}\right\} \qquad (2)$$

We quote the analogous result for the current commutator:

$$\frac{4\pi^2 E_P}{M}\langle P|\left[J_\mu(y),J^\mu(0)\right]|P\rangle = \frac{1}{2\pi Mi}\Box\left\{\Delta\left(y,m^2\right)\int_0^1 \frac{d\eta}{\eta^2}\left[F_2(\eta)-6\eta F_1(\eta)\right]\cos P\eta\cdot y\right\} \tag{3}$$

The invariant functions Δ and Δ_+ introduce free field singularities for the propagation of a particle of mass m on its mass shell. The apparent singularity in the η integral does not contribute in the scaling region as can be seen by putting in a cutoff, performing the Fourier transform indicated in Eq. (1A) and noting that the cutoff dependent terms vanish for large q^2 and ν.

The free field singularities in these expressions are multiplied by a smooth function of $P\cdot y$. Up to now, we have made no assumption about light cone dominance. However, since $P\cdot y$ remains fixed as Q^2 and ν become large, it follows that the rapidly varying exponential in Eq. (1A) damps all contributions relative to the singularity on the light cone. For the details of this argument, we refer to the work of Frishman.[4] Indeed, keeping only the leading light cone singularity of the Δ function in Eq. (3) we obtain the light cone representation of the matrix element of Jackiw, van Royen and West.[10]

This result may be obtained, together with some insight into the origin of the coordinate space behavior, from the parton model expression for the matrix element, which in an infinite momentum frame may be written[3]:

$$W^\mu_\mu = \frac{4\pi^2 E_P}{M}\sum_{n,i}|a_{n,i}|^2\int d^3P_i'\int d^4y e^{iq\cdot y}\langle P_i|j_\mu(y)|P_i'\rangle\langle P_i'|j^\mu(0)|P_i\rangle \tag{4}$$

$|a_{n,i}|^2$ is the probability for finding parton i in constituent state n of the target. W^μ_μ is the incoherent sum of the elastic scattering of partons by "bare" currents

$j^{\mu}(y)$. For spin zero partons:

$$\langle P_i|j_\mu(y)|P_i'\rangle\langle P_i'|j^\mu(0)|P_i\rangle = e^{i\left(P_i-P_i'\right)\cdot y}\,\frac{\left(P_i+P_i'\right)^2}{(2\pi)^6\,2E_i\,2E_i'}\,\left(\lambda_o^i\right)^2$$

where λ_o^i is the charge of the scattered spin-0 parton. Spin 1/2 partons yield the same contribution multiplied by -2. In the Bjorken limit $(P_i+P_i')^2 \simeq Q^2$; the remaining P_i' dependence in Eq. (4) combines to form the singular function $\Delta_+(y,m^2)$. The free field singularity found formally in the preceding paragraphs clearly originates in the free propagation of the elastically scattered parton. Equation (2) may be now obtained[8] from Eq. (4) by means of the parton model definitions of the structure functions $F_1(x)$ and $F_2(x)$.

We now turn to the process $P + P \rightarrow \mu^+ + \mu^- +$ "anything". In the limit of high energy and large dimuon mass, Drell and Yan[6] have derived a scaling law for this process from the parton model. The scaling law originates in the observation that parton-antiparton annihilations, shown in Fig. 1a, will predominate over parton bremsstrahlung, shown in Fig. 1b, in the above-mentioned limit. The differential cross section for producing a dimuon of mass Q^2 from incident particles of momenta P^μ and P'^μ and center-of-mass energy $\sqrt{s}$ is given by:

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{3Q^2}\left(\left[s-(M+M')^2\right]\left[s-(M-M')^2\right]\right)^{-1/2} W\left(Q^2,s\right) \tag{5}$$

ignoring the muon mass. $W(Q^2,s)$ is given by:

$$W\left(Q^2,s\right) = -4E_1E_2(2\pi)^5\int d^4y\;\Delta_+\left(y,Q^2\right)\langle PP'(\mathrm{in})|J_\mu(y)\,J^\mu(0)|PP'(\mathrm{in})\rangle \tag{6}$$

The scaling law of Drell and Yan asserts that in the limit $s\rightarrow\infty$ and $Q^2\rightarrow\infty$ with $\tau = Q^2/s$ fixed, $W(Q^2,s)$ becomes a function of τ alone:

$$\lim_{\substack{Q^2, s \to \infty \\ \tau \text{ fixed}}} W(Q^2, s) = W(\tau) = \int_0^1 \frac{d\eta_1}{\eta_1} \int_0^1 \frac{d\eta_2}{\eta_2} \delta(\eta_1 \eta_2 - \tau) \sum_a F_{2a}(\eta_1) F'_{2\bar{a}}(\eta_2) \frac{1}{\lambda_a^2} \tag{7}$$

$F_{2a}(\eta)$ is the contribution to the inelastic structure function, $F_2(\eta)$, of the proton from partons of type a; and similarly $F'_{2\bar{a}}(\eta)$ for the antipartons, and λ_a is the charge of parton a.

To obtain the coordinate space structure of Eq. (7), observe the following identity:

$$\int d^4y \, \Delta_+(y, Q^2) e^{i(P\eta_1 + P\eta_2) \cdot y} = \frac{2\pi}{s} \delta(\eta_1 \eta_2 - \tau)$$

The four vectors, $P\eta_1$ and $P'\eta_2$ are restricted to satisfy:

$$(P\eta_1 + P'\eta_2)^2 = \eta_1 \eta_2 \, s + \text{terms which vanish as } s, Q^2 \longrightarrow \infty .$$

In the center-of-mass, in which $P = \left(\sqrt{k^2 + M^2}, \vec{k}\right)$ and $P' = \left(\sqrt{k^2 + M'^2}, -\vec{k}\right)$ $P\eta_1$ and $P'\eta_2$ may be chosen to be: $P\eta_1 = \left(\sqrt{\eta_1^2 k^2 + M^2}, \eta_1 \vec{k}\right)$ and $P\eta_2 = \left(\sqrt{\eta_2^2 k^2 + M'^2}, -\eta_2 \vec{k}\right)$. Substituting this identity into Eq. (7), we obtain (up to terms whose integral against $\Delta_+(y, Q^2)$ vanishes in the limit):

$$\langle PP'(\text{in}) | J_\mu(y) J^\mu(0) | PP'(\text{in}) \rangle = -\frac{1}{(2\pi)^6} \sum_a \frac{1}{\lambda_a^2} \int_0^1 \frac{d\eta_1}{\eta_1} e^{iP\eta_1 \cdot y} F_{2a}(\eta_1)$$

$$\otimes \int_0^1 \frac{d\eta_2}{\eta_2} e^{iP\eta_2 \cdot y} F'_{2\bar{a}}(\eta_2) \tag{8}$$

The leading light cone singularity of the single particle matrix element of the current product (cf Eq. 2) does not enter this matrix element. In fact, Eq. (8) has no singularity at all on the light cone.[11] unless F_{2a} or $F'_{2\bar{a}}$ is far more badly behaved than the overall structure functions appear to be (again the apparent divergence at $\eta = 0$ does not contribute in the $s, Q^2 \to \infty$ limit). As Q^2 becomes large, it is not possible to hold the variables in this matrix element fixed as it was in inelastic electron scattering. The reason for this lies in the kinematic restriction that s must be larger than Q^2. Moreover, as $s \to \infty$, the product of exponentials of oppositely directed momentum vectors on the right of Eq. (8) develops high frequencies which compensate those of $\Delta_+(y, Q^2)$ and givesrise to contributions to $W(Q^2, s)$ from all parts of coordinate space.

We do not expect this result to be seriously altered if the colliding hadrons exchange wee partons (equivalent to Regge exchange) before or after the annihilation. In this case, the term $\sum_a \lambda_a^{-2} F_{2a}(\eta_1) F'_{2\bar{a}}(\eta_2)$ is replaced by an unknown function $\mathscr{F}(\eta_1, \eta_2, s)$.[6] Because wee exchanges can carry only asymptotically small amounts of momentum[1,6] and because exchange processes seem to become very smooth at large s, we do not expect $\mathscr{F}(\eta_1, \eta_2, s)$ to differ importantly from the factorized expression.

Lastly we note that the lack of a singularity in the matrix element state is linked, as we would expect, with the lack of a scattered parton in the final state. Note that the bremsstrahlung diagram, Fig. 1b, which in the parton model does not contribute in the scaling limit, does display the singularity which the annihilation diagram lacks. This completes the derivation of the results outlined earlier.

Clearly the techniques developed here can be applied to the many other processes for which scaling laws have been proposed: in the parton model these include[12] highly inelastic neutrino and antineutrino scattering, e^+e^- annihilation to a hadron plus anything, and high-energy photoproduction of massive muon pairs, among others.

We thank Professor Sidney Drell for his advice and encouragement.

Figure Caption

1. Parton model diagrams for $P + P \rightarrow \mu^+ + \mu^- +$ "anything": a. parton pair annihilation; b. parton bremsstrahlung.

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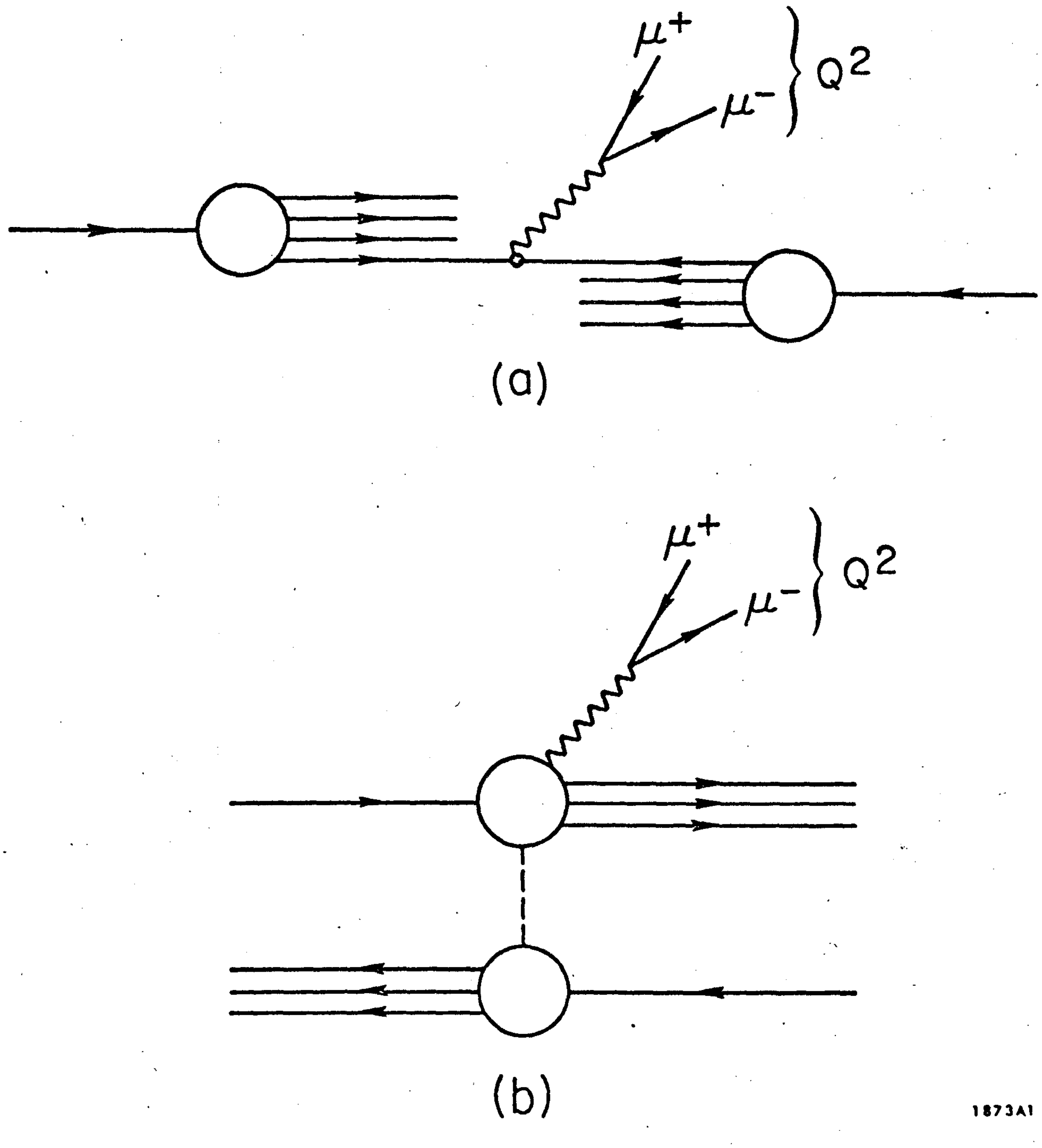

Fig. 1