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MULTIPLICITY GROWTH AND LEADING PARTICLE ENERGY LOSS*

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ABSTRACT

The relation between the multiplicity growth and the energy loss spectrum of an incident particle, as implied by Poisson emission with a classical spectrum, appears to be obeyed experimentally in PP collisions.

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The Brookhaven experiments[1] on the behavior of the final proton in highly inelastic PP collisions at GeV revealed a spectrum of puzzling simplicity: to a first approximation the emerging proton can have any value of longitudinal momentum with equal probability, as long as that momentum is large. That is, the cross section $d\sigma/dP_L$ in $P+P \rightarrow P+$ (anything) for fast final protons is, aside from the narrow elastic and quasi-elastic structures at the maximum momentum, approximately independent of P_L. (It is worth noting that this statement, or that of any power law behavior $d\sigma/dP_L \cdot P^a$, holds in all frames reached by Lorentz transformations leaving the proton moving relativistically in the same direction.)

Subsequent experiments[2] have confirmed these results. In Fig. 1 we show the CERN results at 19 GeV/c. Since that time, some preliminary understanding of the situation at very high energy, beyond the region where resonances and quasi-two body reactions dominate the scattering channels, has been achieved.

The average multiplicity[3] appears to increase logarithmically at very large energy so that, ignoring a constant term,

$$\overline{N} = C \ln s \quad . \tag{1}$$

The constant C is approximately one if we multiply the experimental charged multiplicity of (0.7 ± 0.1) by 3/2 to attempt to account for undetected π^0's. The multiparticle production spectra may be dominated by "soft pions,"[4] whose probability distribution peaks for very low energy. These features are reminiscent of a simple "bremsstrahlung picture," an idea entertained, with just these points in mind, at least as far back as 1942,[5] and elaborated recently in the context of "scaling" by Feynman. In "bremsstrahlung" the abrupt acceleration of the charge[7] leads to a spectrum of radiated photons containing on the average equal amounts of energy per unit frequency interval:

$$\frac{d\overline{E}(\omega)}{d\omega} = C \quad , \tag{2}$$

which in view of $\overline{N}(\omega) = \overline{E}(\omega)/\omega$ gives

$$\frac{d\overline{N}(\omega)}{d\omega} = \frac{C}{\omega} \tag{3}$$

which is the low energy peaking of the radiated particles.

If we imagine giving the radiated "photons" a small mass, λ, we anticipate that the spectrum of the energy radiated resembles Fig. 2. Then, since the upper end point of the spectrum must be determined by the energy available to be radiated, integration of Eq. (4) gives the logarithmic increase of multiplicity. From this standpoint the emerging fast proton in P + P→ P + (anything) and presumably leading particles in general may be thought of as initial particle slowed down by radiative energy loss much as the electron loses energy to photons in electron scattering.

This "radiative tail" or "straggling" may be calculated if we can find the spectrum of energy fluctuations of the radiation field, since the energy in the radiation must be that lost by the incident particle. The energy in the radiation field, on the other hand, may be calculated from two simple assumptions very much in this general spirit:

(A) The probability of N emission into a given frequency ω_i of the radiation field is given by a Poisson distribution

$$P(N_i) = \frac{(\overline{N}_i)^{N_i}}{N_i!} \; e^{-\overline{N}_i} \quad .$$

This is equivalent to assuming that emissions into different ω_i are statistically independent, which seems possible for soft radiation from a very energetic particle, as long as a large fraction of the initial energy is not lost.

(B) The parameter $\overline{N}_i$ characterizing the Poisson distribution is given as a function of frequency ω by the "classical" relation Eq. (3); $d\overline{N}(\omega)/d\omega = C/\omega$. This may be taken as a reflection of the equal distribution of energy in the various momentum states of the radiation field, as Feynman[6] puts it.

Now the probability $P(\epsilon)$ of finding an energy ϵ in the radiation field is the probability of the various configurations N_1, N_2, subject to the constraint $\epsilon = N_1\omega_1 + N_2\omega_2 + \ldots$ or

$$P(\epsilon) = \sum_{N's} \delta(\epsilon - N_1\omega_1 - N_2\omega_2 \ldots)\, P(N_1)P(N_2)\ldots \tag{4}$$

Introducing the fourier transform of the δ function, using

$$\sum_{N_1} e^{-i\omega_1 N_1 t} \frac{(\overline{N}_1)^{N_1}}{N_1!} e^{-\overline{N}_1} = \sum_{N_1} \frac{(e^{-i\omega_1 t}\,\overline{N}_1)^{N_1}}{N_1!} e^{-\overline{N}_1} = e^{\overline{N}_1 (e^{i\omega_1 t} - 1)} ,$$

and passing from the sum to an integral in the exponents leads to

$$P(\epsilon) = \int_{-\infty}^{+\infty} dt\, e^{i\epsilon t}\, e^{C\int_0^{\epsilon} \frac{d\omega}{\omega}(e^{-i\omega t}-1)}\, e^{-C\int_{\epsilon}^{E} \frac{d\omega}{\omega}} .$$

The effective mass of the radiated particles has been taken to zero. The last term represents the probability of there being no radiation of quanta with energy greater than ϵ, up to the maximum energy available, E. Introduction of the variables ω/ϵ and ϵt leads immediately to

$$P(\epsilon) \sim \frac{d\sigma}{d\epsilon} \sim \frac{1}{\epsilon}\,(\epsilon/E)^C \tag{5}$$

(This is to be compared with the radiative correction formula for electron energy loss[7] $\frac{d\sigma}{d\epsilon} = \frac{\alpha A \sigma}{\epsilon} (\epsilon/E)^{\alpha A}$. That the parameter C is in fact essentially the C of Eq. (1) may be seen by considering each of the incident protons separately in the center-of-mass system, for example. Each proton has a maximum energy $\sqrt{S}$ and radiates half the produced particles. Thus for proton 1, $\overline{N}_1 = C \ell n E_1$. It is significant that with the logarithmic multiplicity law this argument is not actually tied to the center-of-mass or any other Lorentz frame, as long as both protons are relativistic. In some other Lorentz frame proton 1 will have an incident energy E_1' and an average multiplicity attributed to it due to its "bremsstrahlung endpoint" at E_1'. $N_1' = C \ell n E_1'$, while the other proton has $\overline{N}_2' = C \ell n E_2'$. But since $\overline{N}_1' + \overline{N}_2' = C \ell n E_1' E_2' \approx C \ell ns$ is an invariant, this language makes sense. This idea that produced particles are uniformly distributed with respect to the logarithmic energy variable, or rapidity,[6] is thus part of the consistency of the approach. Indeed, the idea that we may divide up the produced particles between the incident particles differently but consistently in various Lorentz frames, could be viewed as the reason "why" the multiplicity is logarithmic.

A precedent exists in the bremsstrahlung of an electron scattered in the coulomb field of a nucleus: In the rest frame of nucleus the electron is said to radiate because it is accelerated by the coulomb field. On the other hand, the process may also be treated[8] in the rest frame of the electron by the Weizacker-Williams method, according to which the field of the fast moving nucleus looks like a pulse of photons which scatter off the electron. In one frame we say the radiation comes from the electron, in the other from the nucleus, but it is the same radiation.

In any event, with $C \approx 1$, Eq. (5) does give a flat energy loss distribution. It is amusing, as P. Tsai has pointed out to us, that in the radiation problem language we can say this is like the passage through an absorber of about one (actually .75) radiation length.

In Eq. (5) E really only serves as a convenient scale factor, small energy losses do not depend on the behavior of the radiated energy near the endpoint of the spectrum. The data, nevertheless, seems to be sensibly flat up to surprisingly large fractional energy losses. If we boldly assume that Eq. (5) can be approximately taken all the way up to 100% energy loss, then we can get the average fractional energy loss for the leading particle

$$\frac{\overline{\epsilon}}{E} = \frac{C}{1 + C} \quad , \tag{7}$$

giving $\overline{\epsilon}/E = 1/2$ with $C \approx 1$, which is not inconsistent with suggestions from cosmic ray evidence.[9]

So far we have assumed that the pions actually observed are the primary emitted units. If the basic Poisson-emitted objects were always pion pairs, say, then we should take C in Eq. (1) at half its experimental value since we would want the multiplicity of pairs. Knowing the precise value of C is, of course, tied up with knowing the missing neutrals, but even in the extreme case that it were as small as 1/2, Eq. (5) still gives a rather flat spectrum. What it is, if anything, that is Poisson emitted may eventually be determined from the multiparticle spectra themselves.

A simple but nonetheless necessary experimental test of this general approach derives from our implicit assumption that the proton slows down due to radiation and not by transferring any energy to the other proton. This means that all the energy of, say, the forward proton must be found in the forward-going particles. Observation of frequent assymetric energy partitions between the forward and backward particles in the center-of-mass would be a strong mark against any picture of the type discussed here.

The agreement with experiment here must be regarded as very preliminary since the flat energy distribution for the first final protons has only been observed at relatively low total energies, where the logarithmic multiplicity law can hardly be regarded as fully in play. Furthermore, precise measurements of $\overline{N}(s)$, including missing neutrals, are still to come. It will be interesting to see if the flat energy loss spectrum persists at high energy, and to see results like $P + P \rightarrow N +$ (anything) at high energy. It will also be interesting to see if similar results are obtained with other beams. K mesons would be an interesting analog to protons in this regard, multiproduction of strangeness being small, the leading particle should be readily identifiable in the final state.

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6. R. P. Feynman, Phys. Rev. Letters 23, 1415 (1969), and in High Energy Collisions (Stonybrook Conference, 1969), C. N. Yang et al., (Gordon and Breach Science Publishers, New York). On page 255 Feynman alludes to the calculation we give below. Unfortunately an ambiguity of wording makes it unclear that we arrive at the same answer. However, Professor Feynman informs us that his result is meant to refer to the integral energy loss up to $(1-x) = \bar{\epsilon}/E$, which then agrees with our Eq. (5).
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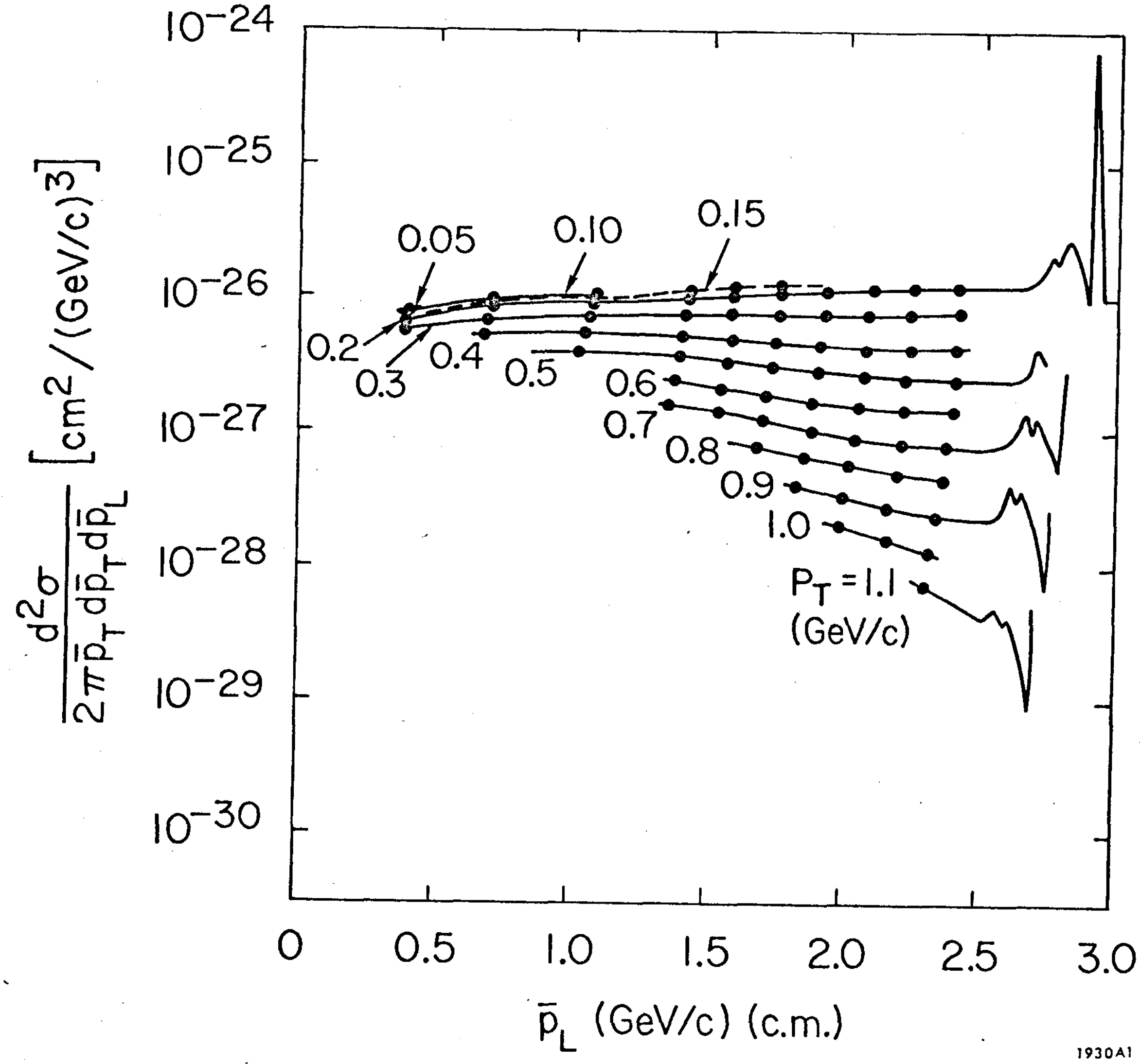

10^{-24}
10^{-25}
10^{-26}
10^{-27}
10^{-28}
10^{-29}
10^{-30}
$\frac{d^2\sigma}{2\pi \bar{p}_T d\bar{p}_T d\bar{p}_L}$ $[cm^2/(GeV/c)^3]$
0.05
0.10
0.15
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
P_T = 1.1 (GeV/c)
0
0.5
1.0
1.5
2.0
2.5
3.0
$\bar{p}_L$ (GeV/c) (c.m.)
1930A1

Fig. 1

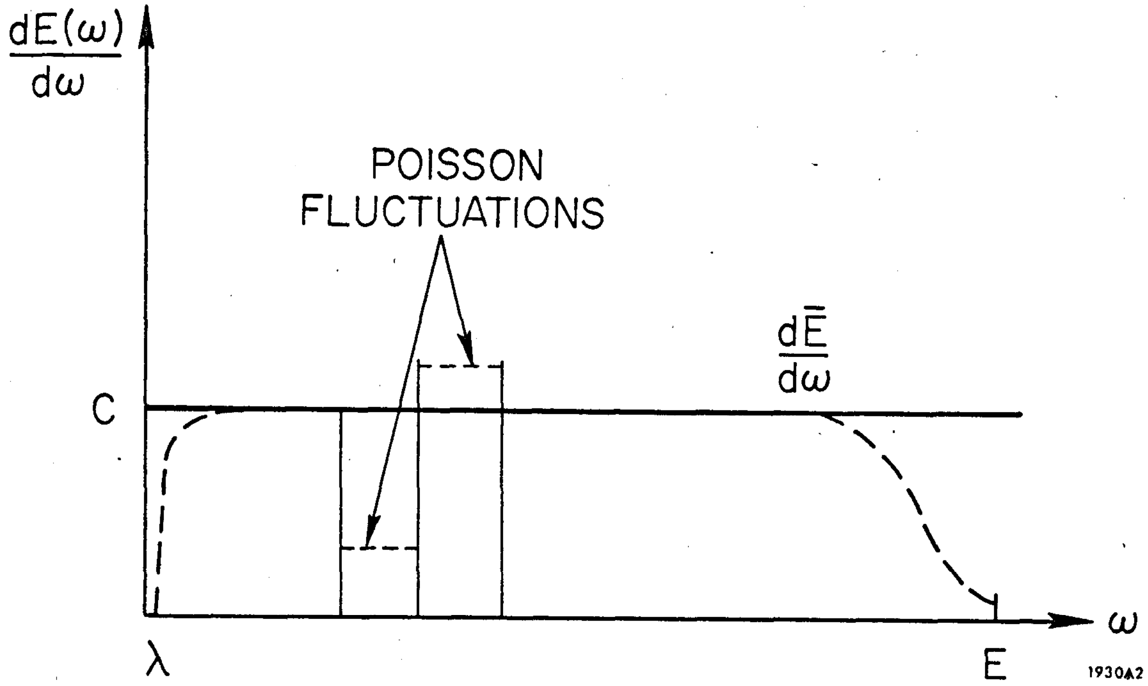

Fig. 2