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VIRTUAL PHOTON-PHOTON ANNIHILATION AND PION MASS DIFFERENCE*

Tung-Mow Yan

Laboratory of Nuclear Studies**
Cornell University, Ithaca, New York 14850

and

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

On the basis of current algebra and PCAC a sum rule is derived which relates the electromagnetic pion mass difference to the differential cross section for a charged pion pair production near the threshold through virtual photon-photon annihilation in electron-electron (positron) collision.

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**Permanent address.

An important implication of the recent work[1] on two-photon annihilation mechanism in electron-electron and electron-positron scatterings (Fig. 1a) at high energies is that the fundamental process of photon-photon interactions can be studied through these colliding beam facilities. This appears to offer a practical means for investigating this theoretically fundamental yet experimentally very difficult process. Most of the theoretical studies[2] so far made the so-called "equivalent photon" approximation which treats both photons as real and restricts the final electrons to very small scattering angles. Recently, Brodsky, Kinoshita and Terazawa,[3] and Walsh[4] have considered the interesting possibility of studying the deep inelastic electron-photon scattering by allowing one of the electrons to suffer a wide angle scattering. Even though the cross section is expected to decrease rather rapidly with increasing momentum transfers, it is clearly of great theoretical interest to explore what one can learn from the additional information gained by allowing both electrons to have finite momentum transfers. In this letter such an example is considered. It is the production process of a $\pi^+\pi^-$ pair near the threshold[5] in the colliding beam experiments via the annihilation of two virtual photons of equal but arbitrary space-like masses. Using algebra of currents, hypothesis of partially conserved axial current (PCAC), and soft pion technique, we derive a sum rule analogous to the Cottingham formula[6] which relates the electromagnetic pion mass difference to the amplitude for a $\pi^+\pi^-$ pair production near the threshold.

The process of interest is[7]

$$e(p_1) + e(p_2) \to e(p_1') + e(p_2') + \gamma(q_1) + \gamma(q_2) \tag{1a}$$

$$\gamma(q_1) + \gamma(q_2) \to \pi(k_1) + \pi(k_2) \tag{1b}$$

and described in Fig. 1a. We will always work in the colliding beam cm system. Momenta of various particles are designated in parentheses following the particle symbols in (1). By momentum conservation, we have

$$q_1 = p_1 - p_1' , \qquad q_2 = p_2 - p_2' \tag{2}$$

Algebra of currents and hypothesis of PCAC determine the amplitude for (1b) at the unphysical point

$$q_1 + q_2 = 0 , \qquad k_1 = k_2 = 0 \tag{3}$$

From this unphysical point one must extrapolate to the nearest physical point, namely, the production threshold $k_{10} = k_{20} = m_\pi$, $\underset{\sim}{k}_1 = \underset{\sim}{k}_2 = 0$. In addition to q_1^2 and q_2^2, there are two invariant variables available in the process (1b), namely, $s \equiv (q_1 + q_2)^2$, and $q_1 \cdot k_1$. These two quantities vary from zero to $4m_\pi^2$, and m_π^2, respectively, in the minimum extrapolation. As a consequence of the space-like nature of the two virtual photons, the range of extrapolation for s and $q_1 \cdot k_1$ from the unphysical point (3) to the threshold is only of order m_π^2 and it is <u>independent</u> of the virtual photon squared masses $q_1^2 = q_2^2$. Thus, the usual smoothness assumption of PCAC suggests that the amplitude at the unphysical point (3) should be a good approximation to the amplitude at the physical threshold. The correction introduced by the extrapolation will be discussed in more detail at the end of this paper.

The hadron dynamics for (1) is contained in the tensor

$$M_{\mu\nu} = i \int (dx)\, e^{-iq_1 \cdot x} \langle k_1 k_2 | (J_\mu(x)\ J_\nu(0))_+ | 0 \rangle \tag{4}$$

where $J_\mu(x)$ is the hadronic electromagnetic current. In the soft pion limit, $k_1, k_2 \rightarrow 0$, application of PCAC and algebra of currents reduces $M_{\mu\nu}$ to the following form[8]

$$M_{\mu\nu} \xrightarrow[k_1, k_2 \rightarrow 0]{} \frac{1}{(2\pi)^3 \sqrt{2\omega_1 2\omega_2}} \frac{8}{F_\pi^2} \int (dx)\, e^{-iq_1 \cdot x} \langle 0 | (V_\mu(x) V_\nu(0))_+ - (A_\mu(x) A_\nu(0))_+ | 0 \rangle \tag{5}$$

where F_π is the usual pion decay constant and $V_\mu(x)$, $A_\mu(x)$ are the third component of the strangeness conserving vector and axial vector current, respectively. In terms of the spectral functions $\rho_V(m^2)$ and $\rho_A(m^2)$ for the vector and axial vector current as introduced by Weinberg,[9] and assuming the validity of Weinberg's first spectral sum rule[9]:

$$\int dm^2 \, \frac{\rho_V(m^2) - \rho_A(m^2)}{m^2} = F_\pi^2 \tag{6}$$

we obtain

$$M_{\mu\nu} \xrightarrow[k_1, k_2 \to 0]{} \frac{2}{(2\pi)^3 \sqrt{2\omega_1 2\omega_2}} \left(g_{\mu\nu} - \frac{q_{1\mu} q_{1\nu}}{q_1^2} \right) f(Q_1^2) \tag{7}$$

where

$$f(Q_1^2) = \frac{1}{F_\pi^2} \int dm^2 \, \frac{\rho_V(m^2) - \rho_A(m^2)}{Q_1^2 + m^2} \tag{8}$$

with $Q_1^2 = -q_1^2 > 0$. The validity of Weinberg's first sum rule (6) is crucial for gauge invariance and absence of additional seagull terms. Thomson limit is ensured by $f(0) = 1$, which again is a consequence of (6).

When extrapolated to the physical threshold $M_{\mu\nu}(q_1, q_2)$ must satisfy the crossing symmetry

$$M_{\mu\nu}(q_1, q_2) = M_{\nu}(q_2, q_1) \tag{9}$$

and the gauge invariance

$$q_1^\mu M_{\mu\nu} = M_{\mu\nu} q_2^\nu = 0 \tag{10}$$

A gauge invariant generalization of (7) to $q_1 + q_2 \neq 0$ is unique if terms quadratic in the pion momenta are neglected. Finally

$$M_{\mu\nu} = \frac{2}{(2\pi)^3 \sqrt{2\omega_1 2\omega_2}} \left(g_{\mu\nu} - \frac{q_{2\mu} q_{1\nu}}{q_2 \cdot q_1} \right) f(Q^2) \tag{11}$$

where $Q^2 = -q_1^2 = -q_2^2$. The differential cross section for (1) with the special kinematics

$$\underset{\sim}{p}_1 + \underset{\sim}{p}_2 = \underset{\sim}{q}_1 + \underset{\sim}{q}_2 = \underset{\sim}{p}'_1 + \underset{\sim}{p}'_2 = 0\,, \qquad q_1^2 = q_2^2 \tag{12}$$

can now be given in terms of $f(Q^2)$:

$$\frac{d\sigma}{dE'_1 d\Omega_1 dE'_2 d\Omega_2} = \frac{1}{8\pi}\left(\frac{\alpha^2}{\pi}\right)^2\left(\frac{s_0}{Q^2}\right)^2\left(\frac{x}{Q^2+\frac{1}{2}s}\right)^2\sqrt{1-\frac{4m_\pi^2}{s}}$$
$$\times\left[\frac{1}{4}\,(1+x^2)^2 + x^2\cos^4\frac{\theta}{2}\right]\left[f(Q^2)\right]^2 \tag{13}$$

where $s_0 = (p_1+p_2)^2$, and $E'_1 = E'_2$ are the energies of the two final electrons, respectively; Ω_1 and Ω_2 are the solid angles of the two final electrons $(\underset{\sim}{p}'_1 + \underset{\sim}{p}'_2 = 0)$, θ is the angle between $\underset{\sim}{p}_1$ and $\underset{\sim}{p}'_1$; and $x = E'/E$ (E is the energy of the initial electrons). In (13) we have neglected the electron mass in the numerator and allowed the invariant mass squared of the two final pions to be slightly above the threshold, i.e., $s \equiv (q_1+q_2)^2 = (k_1+k_2)^2 \gtrsim 4m_\pi^2$.

Now, Das et al.[10] have derived an expression for the electromagnetic mass difference of the pions based on the same assumptions that lead to (11). It is given by

$$m_{\pi^\pm}^2 - m_{\pi^0}^2 = \frac{3\alpha}{4\pi}\int_0^\infty dQ^2\, f(Q^2) \tag{14}$$

Combining (13) and (14) together, we arrive at a sum rule:

$$m_{\pi^\pm}^2 - m_{\pi^0}^2 = \frac{3\alpha}{4\pi}\int_0^\infty dQ^2 \lim_{s\to 4m_\pi^2}\left\{\frac{\frac{d\sigma}{dE'_1 d\Omega_1 dE'_2 d\Omega_2}\,(ee\to ee\pi^+\pi^-)\big|_{\underset{\sim}{p}_1+\underset{\sim}{p}_2=\underset{\sim}{p}'_1+\underset{\sim}{p}'_2=0}}{\frac{1}{8\pi}\left(\frac{\alpha^2}{\pi}\right)^2\left(\frac{s_0}{Q^2}\right)^2\left(\frac{x}{Q^2+\frac{1}{2}s}\right)^2\sqrt{1-\frac{4m_\pi^2}{s}}\left[\frac{1}{4}\,(1+x^2)^2+x^2\cos^4\frac{\theta}{2}\right]}\right\}^{1/2} \tag{15}$$

This sum rule is the central result of this Letter. It is reminiscent of Cottingham's formula except that the pion mass difference is expressed in terms of the virtual

Compton scattering amplitude in the crossed channel; its validity also depends on the reliability of PCAC and Weinberg's first spectral sum rule. It should be noted that the algebraic sign of the pion mass difference is not determined theoretically by the sum rule. The choice in (15) corresponds to the empirical situation.

We make several comments on the sum rule (15).

1. In the original calculation of Das et al.,[10] it was assumed that ρ_V and ρ_A are dominated by ρ and A_1, respectively. Weinberg's second sum rule[9] and the KSRF[11] relation were further assumed. A value of 5 MeV is then obtained which is in reasonable agreement with experiment. Sum rule (15) does not require the existence of A_1 whose experimental status is still not totally settled, nor the validity of KSRF relation. However, Weinberg's second sum rule must be assumed in order that the pion mass difference be finite.

2. It is known that in a nonlinear chiral Lagrangian approach, pion mass difference is logarithmically divergent if terms proportional to m_π^2/m_ρ^2 are kept.[12] To eliminate this divergence Schwinger[13] and Lee[14] proposed to introduce a form factor in the ρ-photon vertex. In such a model the pion mass difference is finite, but its value can no longer be predicted. Once rendered finite, terms proportional to m_π^2/m_ρ^2 are small ($\sim 10\%$) for a wide range of choices of the form factor[15] and therefore can again be neglected. Under this approximation sum rule (15) still holds since (13) and (14) involve the same unknown form factor.

3. In view of the above remarks our sum rule offers a somewhat better test of PCAC. So long as the neglect of pion mass is a reasonable approximation, any additional assumption made will affect both (13) and (14) in such a way that the sum rule remains valid.

4. Although the reaction rate for (1) decreases as Q^2 increases at fixed incident beam energy, it should also be noted that the differential cross section (13),

increases with beam energy at fixed value of Q^2. At sufficiently high energy and high luminosity, in principle, it should be possible to determine the function $f(Q^2)$ from (13). To give a rough idea about the order of magnitude involved here, we present the following numbers. If $s = (2.5\ m_\pi)^2$, $Q^2 = 0.5\ (\text{GeV})^2$, and $s_0 = 36\ (\text{GeV})^2$, then (13) gives $d\sigma/dE_1' d\Omega_1 dE_2' d\Omega_2 = 0.7\times 10^{-34}\left[f(Q^2)\right]^2\ \text{cm}^2/\text{GeV}^2$, $f(Q^2) = 0.4$ assuming ρ and A_1 dominance, Weinberg's second spectral sum rule, and KSFR relation. Suppose $dE_1' = dE_2' = 50$ MeV, $d\Omega_1 = d\Omega_2 = 0.1$, then $d\sigma = 0.3\times 10^{-38}$ cm^2. This is too small to be measurable at presently available colliding beam facilities.

5. To same order in the fine structure constant, there are other mechanisms which could produce a charged pion pair in the final state. A charged pion pair can be produced by the decay of a virtual photon which is radiated by one of the electron lines (Fig. 1b). For electron-electron colliding beam these are the only additional diagrams. It is, however, not the case for electron-positron colliding beam. For the latter, there are also the diagrams as shown in Fig. 2. In the soft pion limit the amplitude associated with Fig. 2b can be again described by the same structure function (8) with time-like q^2, and with obviously much less reliable extrapolation to the threshold. Estimates based on pole dominance and Weinberg's spectral sum rules give a negligible contribution to (13) since both photon propagators are of order $\frac{1}{s_0}$ and the structure function provides further damping when s_0 is beyond the resonance region ($\sqrt{s_0} \gtrsim 1.5$ GeV). We note that all the other diagrams (Fig. 1b and Fig. 2a) are described by the pion electromagnetic form factor and produce a charged pion pair with opposite charge conjugation from that produced in Fig. 1a. Thus, if the charge of the individual pions is not observed, there is no interference in the cross section between the two types of amplitudes. Furthermore, since the pion pair

produced by Bremsstrahlung is in p-wave state, these diagrams give no contribution[16] to (15) in the limit $s \rightarrow 4m_\pi^2$.

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FOOTNOTES AND REFERENCES

1. S. J. Brodsky, T. Kinoshita, and H. Terazawa, Phys. Rev. Letters 25, 972 (1970), and Cornell preprint (CLNS-152). For other papers on this subject see the references cited in the second paper.
2. S. J. Brodsky et al. of Ref. 1 have carried out exact calculations in special cases for purpose of comparison with equivalent photon method.
3. S. J. Brodsky, T. Kinoshita, and H. Terazawa, to be published.
4. T. Walsh, to be published.
5. The processes of two real photon annihilation into soft n charged pion pairs have been considered by H. Terazawa, Phys. Rev. Letters 26, 1207 (1971). Aviv, Hari Das, and Sawyer, Phys. Rev. Letters 26, 591 (1971), and Abers and Fels, Phys. Rev. Letters 26, 1512 (1971) have, on the other hand, considered the annihilation of two real photons into soft $n\pi^0$.
6. W. N. Cottingham, Ann. Phys. 25, 424 (1963).
7. Since it is irrelevant for the two-photon annihilation whether the incident particles are electrons or positrons, we shall call them both electrons in this paper unless specified otherwise.
8. This result was also obtained by H. Terazawa in Ref. 5.
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16. If s is not close to $4m_\pi^2$, a standard calculation shows that the p-wave contribution in our special kinematics is proportional to the electron mass squared and therefore negligible for the e^-e^- case [cf. F. Calogero and C. Zemach, Phys. Rev. 120, 1860 (1960). These authors claim that this contribution vanishes not merely in the relativistic limit. Their arguments seem to have neglected the spin dependence which exists when the electron mass is not zero]; it can be a significant background to (13) when $Q^2 \gtrsim s$ for the e^-e^+ case, however.

FIGURE CAPTIONS

1. a. A charged pion pair production by two-photon annihilation in e^-e^- or e^-e^+ colliding beam experiments.

   b. A charged pion pair production by Bremsstrahlung in e^-e^- or e^-e^+ scattering diagrams. The radiated virtual photon can be from any of the four charged lines.

2. a. A charged pion pair production by Bremsstrahlung in e^+e^- annihilation diagrams. The radiated virtual photon can be from any of the four charged lines.

   b. A charged pion pair production involving Compton amplitude with time-like photons in e^+e^- annihilation channel.

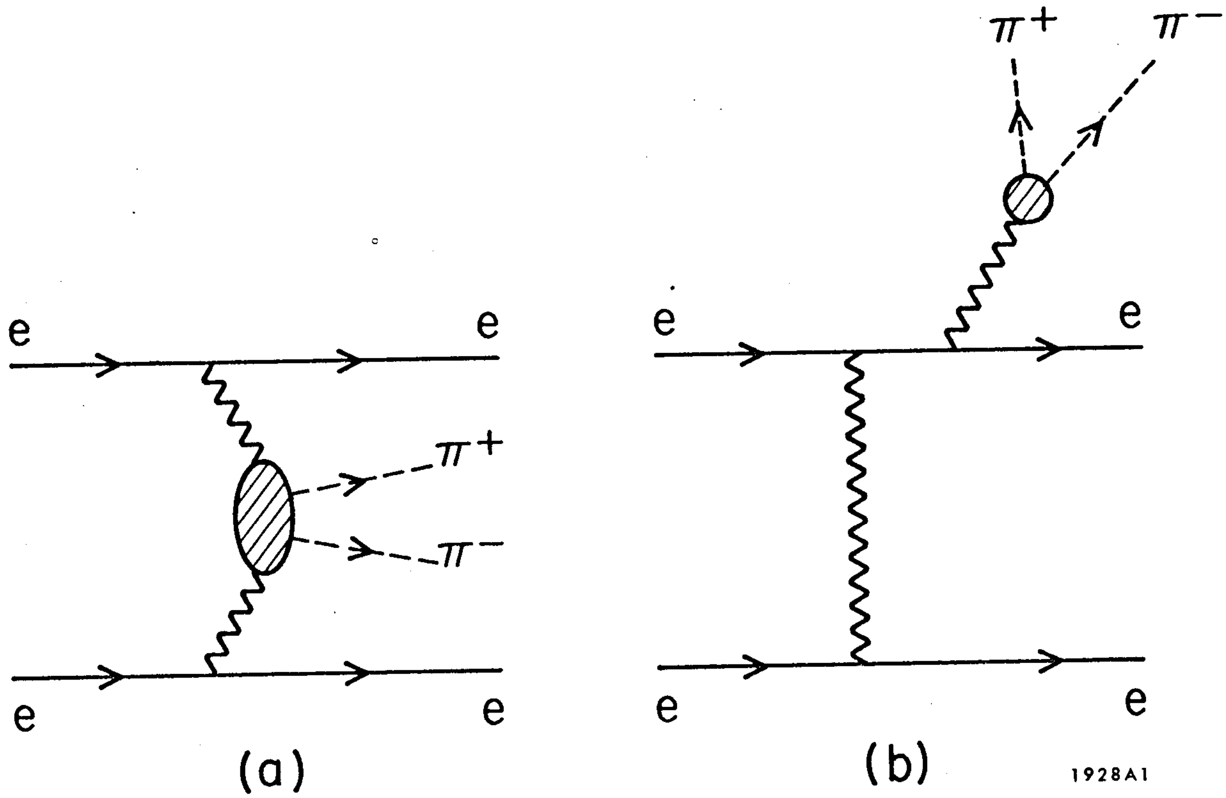

Fig. 1

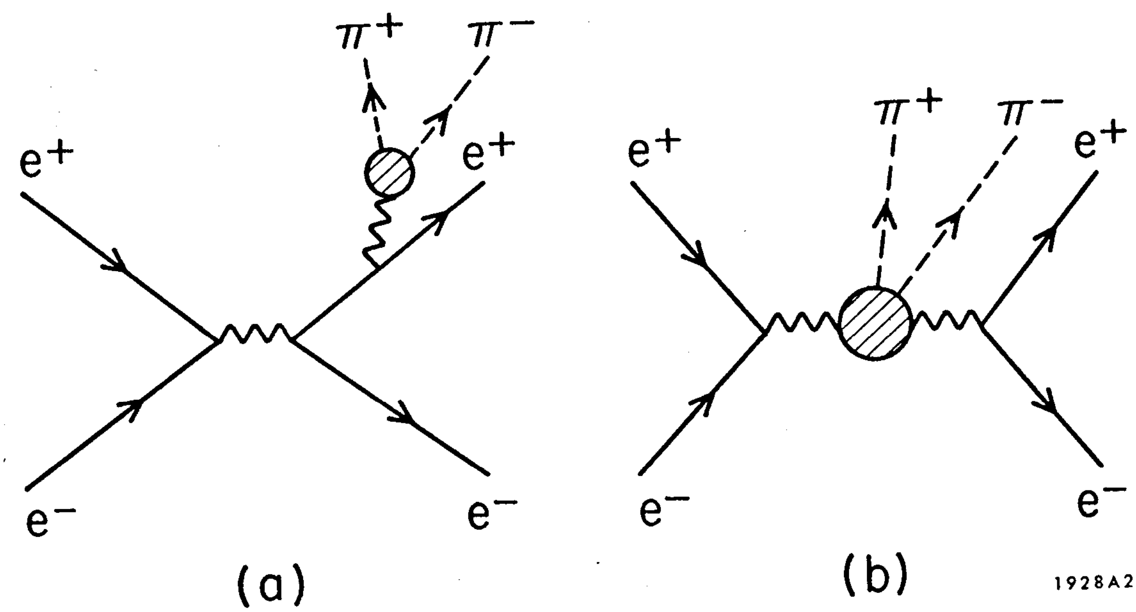

Fig. 2