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INELASTIC LEPTON SCATTERING IN GLUON MODELS*

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ABSTRACT

A formal study of quark models with interactions due to scalar, pseudoscalar or vector fields is presented. It is shown that all the results which have been derived in quark parton models in which the details of the nucleon's constitution are not specified can be obtained formally using naive canonical manipulations of operators. In the case that there is no vector field some new results are obtained which would provide an experimental measurement of the proportion of scalar or pseudoscalar gluons in the nucleon.

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Introduction

Some time ago we studied generalized quark parton models and abstracted those results which might be true more generally[1]. In fact, we showed[1] that the most easily tested consequences of the model could all be derived formally in the gluon model using the Bjorken limit with naive canonical values for the equal time commutators. In this paper we show that <u>all</u> the old results of generalized parton models can be formally derived in renormalizable quark models. We also present some new results which depend essentially on the assumption that none of the partons travels backwards in the infinite momentum frame; it turns out that these results can be rederived formally if the interaction between the quarks is due to a scalar or pseudoscalar field but not if it is due to a vector field (the conventional gluon model)[2].

In perturbation theory the formal arguments used in this paper are invalid[5,6] and scale invariance is broken by logarithmic terms. Although they are not excluded by the data we shall assume that such terms are absent and that therefore arguments based on perturbation theory may be irrelevant. In this sense perhaps "Nature reads books on free field theory"[7].

We will not dwell on the experimental implications of the results which have been reviewed elsewhere[8]. After completing this work we received an elegant preprint from Gross and Treiman[9] who have independently rederived the "old" parton results in the gluon model.[10] They have actually gone further and derived the explicit form of the light cone expansion in the presence of a vector interaction.

Formal derivation of all "old" parton model results

Inelastic electron and neutrino scattering processes in which only the final lepton is observed are described by the tensors:

$$
\begin{aligned}
W^{\gamma}_{\mu\nu} &= \overline{\sum}\int \frac{d^4x}{4\pi}\, e^{iq\cdot x} < P \,|\left[J^{\gamma}_{\mu}(x), J^{\gamma}_{\nu}(0)\right]|\, P > \\
&= -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W^{\gamma}_1 + \left(P_\mu - \frac{q_\mu \nu}{q^2}\right)\left(P_\nu - \frac{q_\nu \nu}{q^2}\right)\frac{W^{\gamma}_2}{M^2} \\
W^{\nu,\bar{\nu}}_{\mu\nu} &= \overline{\sum}\int \frac{d^4x}{4\pi}\, e^{iq\cdot x} < P \,|\left[J^{\mp}_{\mu}(x), J^{\pm}_{\nu}(0)\right]|\, P > \\
&= -g_{\mu\nu}\, W^{\nu,\bar{\nu}}_1 + \frac{P_\mu P_\nu W^{\nu,\bar{\nu}}_2}{M^2} - \frac{i\epsilon_{\mu\nu\alpha\beta}P^{\alpha} q^{\beta} W^{\nu,\bar{\nu}}_3}{2M^2} \\
&\quad + \frac{q_\mu q_\nu W^{\nu,\bar{\nu}}_4}{M^2} + \frac{(q_\mu P_\nu + q_\nu P_\mu) W^{\nu,\bar{\nu}}_5}{2M^2} + \frac{i(q_\mu P_\nu - q_\nu P_\mu) W^{\nu,\bar{\nu}}_6}{2M^2}
\end{aligned}
\tag{1}
$$

where $\nu = q\cdot P$, J^{γ}_{μ} is the electromagnetic current, J^{+}_{μ} ($J^{-}_{\mu} = (J^{+}_{\mu})^{+}$) is the current which couples to the neutrino (antineutrino) current, $\overline{\sum}$ indicates an average over the spin states of the target and the states are normalized to 2E per unit volume. We assume the conventional Cabibbo current and work in the approximation $\theta_C = 0$, i. e. our results apply to the structure functions for the production of non-strange final states. It is easy to generalize to the case $\theta_C \neq 0$; the results are given in reference 8. With $\theta_C = 0$ the isovector nature of the weak current gives:

$$
W_i^{\nu\, {p \atop n}} = W_i^{\bar{\nu}\, {n \atop p}} \; . \tag{2}
$$

Bjorken's scaling hypothesis[11], which we assume to be correct, is

$$\lim_{\substack{\nu \to \infty \\ \omega \text{ fixed}}} W_1(\nu, q^2) = F_1(\omega)$$

$$\lim_{\substack{\nu \to \infty \\ \omega \text{ fixed}}} \frac{\nu W_i(\nu, q^2)}{M^2} = F_i(\omega), \quad (i \neq 1) \tag{3}$$

$$(\omega = 2\nu/-q^2) .$$

The derivation of relations for the F_i starts from the scaling limit[11] of the Cornwall-Norton sum rules[12]:

$$L^n_{xx} = 2 \int_1^\infty \frac{F_1^{\mp}}{\omega^{n+2}} d\omega \qquad (n = 0, 1, 2, 3\ldots)$$

$$L^n_{zz} = \int_1^\infty (2F_1^{\mp} - \omega F_2^{\mp}) \frac{d\omega}{\omega^{n+2}} \qquad (n = 1, 2, 3\ldots)$$

$$L^n_{yx} = i \int_1^\infty F_3^{\pm} \frac{d\omega}{\omega^{n+2}} \qquad (n = 0, 1, 2\ldots) \tag{4}$$

$$L^n_{oz} = \int_1^\infty (F_5^{\mp} - \omega F_2^{\mp}) \frac{d\omega}{\omega^{n+2}} \qquad (n = 1, 2, 3\ldots)$$

$$L^n_{oz} - L^n_{oo} - L^n_{zz} = 4 \int_1^\infty \frac{F_4^{\mp}}{\omega^{n+3}} d\omega \qquad (n = 1, 2, 3\ldots)$$

$$L^n_{\mu\nu} = \lim_{P_0 \to \infty} i \left(\frac{i}{2P_0}\right)^{n+1} \int d^4x\, \delta(x_0) < P_z \left| \left[\frac{\partial^n J^+_\mu(x)}{\partial t^n}, J^-_\nu(0) \right] \right| P_z >$$

where the upper (lower) sign holds for n even (odd) and

$$F_i^{\pm} = F_i^{\nu} \pm F_i^{\bar{\nu}} . \tag{5}$$

In the electromagnetic case:

$$L_{xx}^n = 4 \int_1^{\infty} \frac{F_1^{\gamma} d\omega}{\omega^{n+2}} \qquad (n = 1, 3, 5, \ldots)$$

$$L_{zz}^n = 2 \int_1^{\infty} (2F_1^{\gamma} - \omega F_2^{\gamma}) \frac{d\omega}{\omega^{n+2}} \tag{6}$$

$$L_{\mu\nu}^n = \lim_{P_0 \to \infty} i \left(\frac{i}{2P_0}\right)^{n+1} \int d^4x \, \delta(x_0) < P_z \left| \left[\frac{\partial^n J_\mu^\gamma(x)}{\partial t^n}, J_\nu^\gamma(0) \right] \right| P_z > .$$

When n = 0 equations 4 give all the sum rules which relate integrals over the F_i to numbers.[13,14,15] For $n \geq 1$ we must specify the interaction Hamiltonian which we take to be a sum of renormalizable interactions:

$$\mathscr{H}_I = g_S \phi_S \bar{\psi} \psi + i g_P \phi_P \bar{\psi} \gamma_5 \psi + g_V B_\mu \bar{\psi} \gamma^\mu \psi . \tag{7}$$

We consider first the case $g_V = 0$. The equal time commutators in Eq. (4) and (6) are formally constructed using $\mathscr{H}_I$. Only those parts which are components of tensors of rank n + 1 or higher can grow rapidly enough to contribute when we take the limit $|\vec{P}| \to \infty$. Using the fact that the equal time commutators never introduce inverse powers of the masses or fields it is easy to show (as we do explicitly in Appendix 1) that the only possible tensor operators whose matrix elements grow like $|\vec{P}|^{n+1}$ and have the appropriate dimensions are

$$\begin{gathered} \bar{\psi} \gamma_{\alpha_1} \partial_{\alpha_1} \cdots \partial_{\alpha_{n+1}} \psi \\ \bar{\psi} \sigma_{\alpha_1 \alpha_2} \partial_{\alpha_3} \cdots \partial_{\alpha_{n+2}} \psi \end{gathered} \tag{8}$$

where the second operator does not contribute if we consider spin averaged matrix elements (there are no appropriate operators whose matrix elements grow faster than $|\vec{P}|^{n+1}$).

Therefore that part of the equal time commutators which contributes in Eqs. (4) and (6) is the same as in a free field theory of massless quarks in the case $g_V = 0$. Hence the structure functions are related in the same way as in free field theory. This gives the Callan-Gross relation $\sigma_L/\sigma_T = 0$, or

$$2F_1 = \omega F_2 \,, \tag{9}$$

since the quarks have spin $\frac{1}{2}$.[16] It implies that in the deep inelastic region the axial currents are conserved (chiral symmetry) so that:

$$\begin{aligned} 2F_1 &= F_5 \\ F_4 &= 0 \,. \end{aligned} \tag{10}$$

The first of these relations actually follows from Eq. (9) and the inequalities satisfied by the F_i.[8,17] (It is interesting to note that the inequalities imply that if either Eq. (9) or Eq. (10) is satisfied then the T violating structure function F_6 is zero[17].) Furthermore, we obtain the two relations[18]:

$$12(F_1^{\gamma p} - F_1^{\gamma n}) = F_3^{\nu p} - F_3^{\nu n} \tag{11}$$

$$F_1^{\nu p} + F_1^{\nu n} \leq \frac{18}{5}(F_1^{\gamma p} + F_1^{\gamma n}) \,. \tag{12}$$

Previously we had derived these relations in the parton model and the moment $\int \frac{d\omega}{\omega^3}$ of them in models with the interaction in Eq. (7).[1]

In the case $g_V \neq 0$ it seems at first that the previous argument might fail since the free field commutator

$$\left[\phi_\mu, \phi_\nu\right] = -\left(g_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{M_V^2}\right)\Delta \tag{13}$$

can introduce inverse powers of M_V which spoil our dimensional reasoning. That this is not the case is almost obvious since we know that the $\partial_\mu \partial_\nu \Delta$ term in the propagator is irrelevant when we calculate Feynman diagrams because the vector field is coupled to a conserved current. In Appendix 2 we show that vector field theory can easily be formulated in such a way that the troublesome term in the commutator is absent. Therefore the operators which can contribute have the forms given in Eq. (8) except that ∂_α can anywhere be replaced by B_α.

In calculating the equal time commutators the non-canonical operator $\ddot{B}_\mu$ must be replaced by canonical operators using the field equations ($\ddot{B}_\mu = \nabla^2 B_\mu + g_V \bar{\psi} \gamma_\mu \psi$) whenever it is encountered. However, we note that the interaction g_V can be set equal to zero in this replacement since it introduces terms involving at least four fields ψ which cannot contribute in the limit $|\vec{P}| \to \infty$. Therefore that part of the equal time commutators which contributes in Eqs. (4) and (6) is the same as in a field theory of massless quarks interacting with an external massless C number vector field. This observation was also made independently by Gross and Treiman[9] who used it to derive the explicit form of the light cone expansion in the case $g_V \neq 0$.

With $g_V \neq 0$ it is well known that Eq. (9) still obtains.[16,19] According to our prescription the effective parts of the equal time commutators are chirally symmetric. This gives all the other results above except Eq. (11). However it is easy to show that this equation still holds when $g_V \neq 0$, as we do in Appendix 1.

New Results

We consider the explicit forms of two of the n = 1 sum rules of Eqs. (4) and (6) obtained using Eq. (7):

$$\int F_2^{\gamma} \frac{d\omega}{\omega^2} = \lim_{P_0 \to \infty} \frac{1}{2P_0^2} < P_z \mid \bar{\psi}(0)(i\gamma_z \partial_z + g_V \gamma_z B_z) Q^2 \psi(0) \mid P_z >$$

$$\int (F_2^{\nu p} + F_2^{\nu n}) \frac{d\omega}{\omega^2} = \lim_{P_0 \to \infty} \frac{1}{2P_0^2} < P_z \mid \bar{\psi}(0)(i\gamma_z \partial_z + g_V \gamma_z B_z)(4B + 2Y)\psi(0) \mid P_z > \tag{14}$$

where Q, B and Y are the usual 3×3 SU(3) matrixes and $Q^2 = 2B/3 + Y/6 + I_3/3$. An important point in the following is that the matrices B, B-Y and $2B + Y \pm 2I_3$ make positive semidefinite contributions whenever they appear on the right hand side of Eqs. (14)[1] (this was used in deriving Eq. (12)). If we call one of these matrixes λ then we can consider a structure function F_2^{λ} defined in terms of $\bar{\psi}\gamma_\mu \lambda \psi$ just as F_2^{γ} is defined in terms of $\bar{\psi}\gamma_\mu Q\psi$. In the analogue of Eq. (14) for F_2^{λ}, Q^2 will be replaced by $\lambda^2 \propto \lambda$. The left hand side is positive semidefinite (since $F_2 \geq 0$) and hence the right hand side must be so also. In parton language this corresponds to the fact that the contribution of each type of parton to F_2 is positive semidefinite.

Next we note that with our normalization:

$$\lim_{P_0 \to \infty} \frac{1}{2P_0^2} < P_z \mid \bar{\psi}(0)\, i\gamma_z \partial_z B\psi(0) \mid P_z > = \lim_{P_0 \to \infty} \frac{1}{6P_0^2} < P_z \mid \theta_{zz} - \theta^g_{zz} - \mathscr{L} \mid P_z >$$

$$= \tfrac{1}{3}(1 - \epsilon) \qquad (0 \leq \epsilon \leq 1) \tag{15}$$

where $\theta_{\mu\nu}$ is the energy momentum tensor, $\theta^{g}_{\mu\nu}$ the energy momentum tensor of a free gluon and $\mathscr{L}$ the Lagrangian density. In Eq. (14) we used the fact $< P_z | \theta^{g}_{zz} | P_z > \geq 0$ and ϵ is actually the contribution of θ^{g}_{zz}.

Provided $g_V = 0$ (i. e. the interaction is due to scalar or pseudoscalar fields only) we can combine Eqs. (14) and (15) to obtain

$$\epsilon = 1 + \int \left[\tfrac{3}{4} (F_2^{\nu p} + F_2^{\nu n}) - \frac{9}{2} (F_2^{\gamma p} + F_2^{\gamma n}) \right] \frac{d\omega}{\omega^2} . \tag{16}$$

This result has been derived independently by Fritzsch and Gell-Mann[7] in the particular case that there are no gluons at all and $\epsilon = 0$. Using the SLAC-MIT data[20] extrapolated to infinity assuming Regge behaviour and the value of the total neutrino cross section obtained at CERN[21] Eq. (16) gives $\epsilon \geq 0.52 \pm 0.38$.

Eqs. (16) and (12) together give the absolute upper bound:

$$(F_2^{\gamma p} + F_2^{\gamma n}) \frac{d\omega}{\omega^2} \leq \frac{5}{9} \tag{17}$$

which is satisfied by the data (unless quite unexpected behaviour occurs at unexplored ω). We can also obtain the lower bound (still assuming $g_V = 0$):

$$\int F_2^{\gamma p, \gamma n} \frac{d\omega}{\omega^2} \geq \frac{1}{9} (1 - \epsilon) . \tag{18}$$

This provides a possible test of the indication that $\epsilon \neq 0$ which has the advantage of involving electromagnetic data alone.[22] However, the left hand side is certainly $> 1/9$ for the proton and very likely also for the neutron.[20]

These results (Eqs. (16)-(18)) are true in any quark parton model in which all the partons travel in the same direction when the proton has infinite momentum and ϵ is just the fraction of the proton's momentum carried by the gluons (they are therefore true in every particular parton model which has previously been considered but their generality does not seem to have been noticed before). We might be tempted to interpret the fact that we were unable to derive these results when $g_V \neq 0$ by saying that partons can travel backwards in this case. However, doubt is cast on this interpretation by the fact that the sum rules for F_1, F_2 and F_3 [14,13,15] are independent of the interaction yet, in parton language, they depend on the assumption that certain combinations of quarks and antiquarks travel forwards.

Conclusions

From a practical point of view we have not progressed far beyond reference 1. Theoretically, however, it does seem remarkable that we can formally rederive all the old parton model results in interacting field theory. Neutral scalar and pseudoscalar fields play no role in the appropriate infinite momentum commutators (or, equivalently, in the leading terms in the light cone expansion). In this case we have therefore "derived" the parton model since the process is described by the same one body operator as in free field theory. The vector field enters in such a way that the old free field (parton) theory results are unchanged.

The new results do depend on the interaction. They require that $g_V = 0$ and are therefore untrue in the conventional vector gluon model. Unfortunately, they cannot be used to establish that $g_V = 0$. However, we think it is interesting that we can obtain an absolute upper bound on the data in this case (Eq. 17)(it is unfortunate that this was not known before the data were obtained). Other results which depended on the interaction would be very interesting.

The only other obvious application of these techniques is to the case of polarized targets. Bjorken derived a sum rule for the scattering of polarized electrons from polarized targets some years ago.[23] It is easy to derive a similar relation for neutrino scattering from polarized targets and relations between the structure functions can also be obtained in this case. However, such experiments are so remote that it does not seem worth stating the results. By the time they are carried out the ideas discussed here will either be already accepted or long since forgotten. In fact neutrino experiments at NAL will not only be able to test the scaling hypothesis but also the quark algebra in the near future since the predicted value of the F_3 sum rule[15] (which is the easiest sum rule to test) depends essentially on the nonintegral baryon number attributed to the quark fields (the value 6 changes to 2 in the Sakata or Fermi-Yang models; Eqs. (11) and (12) also depend critically on the quark algebra[8]).

Appendix 1

In this appendix we prove some assertions in Section 2. We wish to find that part of

$$O_{\mu\nu} = \int \left[\frac{\partial^n J^a_\mu(x)}{\partial t^n}, J^b_\nu(0) \right] \delta(x_0) d^4x$$

whose matrix elements

$$< \vec{P} | O_{\mu\nu} | \vec{P} >$$

can grow like $|\vec{P}|^{n+1}$ as $|\vec{P}| \to \infty$. Note that $O_{\mu\nu}$ is not a second rank Lorentz tensor despite its appearance. It is a sum of terms each of which is a component of a Lorentz tensor of the form:

$$T_{\alpha_1\alpha_2\ldots\beta_1\ldots\gamma_1\ldots} = \bar{\psi}(0) A_{\alpha_1\alpha_2\ldots} \psi(0) \bar{\psi}(0) B_{\beta_1\ldots} \psi(0) \bar{\psi} C_{\gamma_1\ldots} \psi \bar{\psi} \ldots$$

The highest rank tensor which can be built from the γ matrices is $\sigma_{\mu\nu}$ so that in the case $g_V = 0$ T has the symbolic structure:

$$T \sim (\bar{\psi}\psi)^N (\gamma_\mu)^R (\partial_\nu)^P M^Q$$

$$N \geq 1, \quad 2N \geq R \geq 0$$

$$P \geq 0, \quad Q \geq 0$$

$$(M^Q = M^S_{\text{quark}} M^T_{\substack{\text{scalar}\\ \text{gluon}}} M^U_{\substack{\text{P-scalar}\\ \text{gluon}}} \phi^V_S \phi_P^{Q-S-T-U-V}; \ S, T, U, V \geq 0).$$

The indices are positive semidefinite because the commutation relations and field equations never introduce inverse powers of the masses or fields.

We consider the tensor T after any explicit factors $\epsilon_{\mu\nu\alpha\beta}$ and $g_{\mu\nu}$ have been removed so that the rank r and dimension n + 3 are given by

$$r = R + P$$

$$n + 3 = 3N + P + Q$$

$$r = R + n + 3 - 3N - Q \leq n + 3 - N - Q \leq n + 2 \,.$$

Note that $\sigma_{\mu\nu}$ can give at most one power of $|\vec{P}|$ to that the only two solutions with $r \geq n+1$ whose matrix elements grow like $|\vec{P}|^{n+1}$ are:

$$\bar{\psi}(0)\gamma_{\alpha_1}\partial_{\alpha_2}\cdots\partial_{\alpha_{n+1}}\lambda^c\psi(0)$$

$$\bar{\psi}(0)\sigma_{\alpha_1\alpha_2}\partial_{\alpha_3}\cdots\partial_{\alpha_{n+2}}\lambda^c\psi(0) \,.$$

In the case $g_V \neq 0$, the only change is that derivatives ∂_α may be replaced by the vector field B_α.

We now wish to show that Eq. (11) obtains if $g_V \neq 0$. A necessary and sufficient condition is that

$$< p \,|\left[\frac{\partial^n J_x^\gamma(\vec{x},0)}{\partial t^n}, J_x^\gamma(0)\right]|\, p > - < n \,|\left[\frac{\partial^n J_x^\gamma(\vec{x},0)}{\partial t^n}, J_x^\gamma(0)\right]|\, n >$$

$$\approx \frac{1}{3i} < p \,|\left[\frac{\partial^n J_y^+(\vec{x},0)}{\partial t^n}, J_x^-(0)\right]|\, p >$$

$$= \frac{1}{6i} < p \,|\left[\frac{\partial^n J_y^+(\vec{x},0)}{\partial t^n}, J_x^-(0)\right] + \left[J_x^+(0), \frac{\partial^n J_y^-(\vec{x},0)}{\partial t^n}\right]|\, p >$$

where here and below the symbol $\approx$ indicates that the terms in the spin averaged matrix elements which may grow like $|\vec{P}|^{n+1}$ when $\vec{P} \to \infty$ along the z axis are equal. The only part of $\partial^n J_i^a/\partial t^n$ which can contribute is a bilinear in ψ which we write as $\overline{\psi} A_i \lambda^a \psi$. Using the chiral symmetry, the necessary and sufficient condition may now be written:

$$i\psi^+(0)\left[\gamma_0 A_x, \gamma_0\gamma_x\right]\psi(0) \approx -\psi^+(0)\left[\gamma_0 A_y, \gamma_0\gamma_x\gamma_5\right]\psi(0)$$

where we have chosen A_i to correspond to the n^{th} time derivative of the vector current. In constructing the part of A_i which contributes (using the Dirac equation for massless quarks) we may put $\partial_x = \partial_y = B_x = B_y = 0$ since $\vec{P} \to \infty$ along the z axis. Therefore each term in the effective part of A_i has either the γ matrix structure $\sim \gamma_i$ or $\sim \gamma_i\gamma_0\gamma_z$. The necessary and sufficient condition is satisfied in the first case (trivially) and in the second Q. E. D.

Appendix 2

In this appendix we prove some almost obvious properties of theories with neutral vector fields coupled to conserved currents. Although many related discussions are contained in the literature (see e.g. reference 24) and our results may be well known, we could not find the theory formulated anywhere in exactly the desired form.

We consider the Lagrangian density

$$\mathscr{L} = -\tfrac{1}{2}(\partial_\mu B_\nu)(\partial^\mu B^\nu) + \frac{M_V^2}{2} B_\mu B^\mu - g_V B_\mu J^\mu$$

which gives

$$(\Box + M_V^2) B_\mu = g_V J_\mu \, .$$

This is entirely equivalent to the usual equations of motion in the case $\partial_\mu J_\mu = 0$ provided we impose the subsidiary condition:

$$\partial_\mu B_\mu = 0 \, .$$

Proceeding to quantise this theory in the traditional way the free field commutation relations

$$\left[B_\mu(x), B_\nu(x') \right] = -i g_{\mu\nu} \Delta(x-x', M_V^2)$$

or, in momentum space:

$$\left[b_\mu(\vec{k}), b_\nu^+(\vec{k}') \right] = -g_{\mu\nu} k_0 \delta^3(\vec{k}-\vec{k}') \, ,$$

indicate that we are faced with a theory with an indefinite metric. It is convenient to introduce the "vector" and "scalar" operators:

$$V_\mu = b_\mu - \frac{k_\mu(k\cdot b)}{M_V^2}$$

$$S_\mu = \frac{k_\mu(k\cdot b)}{M_V^2}$$

which satisfy

$$\left[V_\mu(\vec{k}),\ V_\nu^+(\vec{k}')\right] = \left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{M_V^2}\right) k_0 \delta^3(\vec{k}-\vec{k}')$$

$$\left[S_\mu(\vec{k}), S_\nu^+(\vec{k}')\right] = -\frac{k_\mu k_\nu}{M_V^2}$$

$$\left[S_\mu(\vec{k}),\ V_\nu^+(\vec{k}')\right] = 0 \ .$$

Since $k_0 \geq M_V$ we may associate a positive metric with the creation and annihilation operators V_μ^+ and V_μ^- but we must associate a negative metric with S_μ. The subsidiary condition ensures that the negative metric part of the Hilbert space never enters into the calculation of physical quantites. In exact analogy with quantum electrodynamics, the subsidiary condition restricts the states $|\psi>$ allowed in the theory which are required to satisfy

$$\left(\partial_\mu B^\mu(x)\right)^- |\psi> = 0$$

or

$$S_\mu(\vec{k})^- |\psi> = 0.$$

(The separation of $\partial_\mu B^\mu$ into positive and negative frequency parts is relativistically invariant since $(\Box + M_V^2)\partial_\mu B^\mu = 0$.) Having imposed this condition initially, transitions to states for which it is not satisfied are impossible since current conservation ensures that

$$\left[S_{\mu}, H_{I}\right] = 0 .$$

It is easy to see that the energy is positive for the allowed states. We therefore have a consistent theory (which is equivalent to the usual one) in which the various components of B_{μ} commute with each other and with ψ at equal times.

References and Footnotes

1. C. H. Llewellyn Smith, Nuclear Physics B17, 277 (1970).

2. Two arguments are often advanced in favour of models with a vector interaction:

   a) The $SU(3) \times SU(3)$ symmetry breaking is very simple, being due to the quark masses only.

   b) In the naive quark model it is easy to understand why the Q-$\overline{Q}$ state is bound but not the Q-Q state.

   Two unconvincing arguments against a vector interaction are:

   a) One might guess that the mesons with the same J^P as the interaction would show the least trace of nonet symmetry since the ninth member can mix with the gluon[3]. This suggests an interaction due to a 0^- meson.[3] R. P. Feynman (private communication) has independently used the same argument to suggest that the force is 1^+ (which has a 0^- component when the gluon is off mass shell); we reject this possibility because the interaction is not renormalizable and scaling would be unlikely to obtain.

   b) Interpreted in terms of the quark model, the analysis of Gell-Mann, Oakes and Renner[4] gives:

   $$\frac{M_p + M_n}{M_p + M_\lambda} \simeq \frac{M_\pi^2}{M_K^2}$$

   where $M_{p,n,\lambda}$ are the bare quark masses. This result would be distasteful if we assumed that heavy quarks exist. In quark language it depends on two assumptions:

   1. That the quantities $\langle 0 | \overline{\psi} \gamma_5 \lambda^i \psi | \pi/K \rangle$ are approximately $SU(3)$ invariant.
   2. That the interaction is due to a vector gluon.

The first assumption is true in (heavy) quark models which lead to $f_\pi \simeq f_K$ in a natural way[3]. We might therefore be tempted to reject the second assumption.

3. C. H. Llewellyn Smith, Phys. Lett. 28B, 335 (1969), Ann. of Phys. 53, 521 (1969).

4. M. Gell-Mann, R. J. Oakes and B. Renner, Phys. Rev. 175, 2195 (1968).

5. S. L. Adler and W. K. Tung, Phys. Rev. Letters 22, 978 (1969).

6. R. Jackiw and G. Preparata, Phys. Rev. Letters 22, 975 (1969).

7. H. Fritzsch and M. Gell-Mann, CALT-68-297 (to be published in Proceedings 1971 Coral Gables Conference).

8. C. H. Llewellyn Smith, "Neutrino Reactions at Accelerator Energies", SLAC-PUB in preparation, to be published in Physics Reports (preliminary draft circulated March 8, 1971; this draft quoted all the results of the present paper).

9. D. J. Gross and S. B. Treiman, Princeton University preprint, May 1971 (to be published).

10. Gross and Treiman quote reference 8 to the effect that we had obtained all the "old" parton results in quark models with scalar gluons; in fact it is asserted in reference 8 that the results were obtained in models with the interaction Hamiltonian of Eq. (7).

11. J. D. Bjorken, Phys. Rev. 179, 1547 (1969).

12. J. M. Cornwall and R. E. Norton, Phys. Rev. 177, 2584 (1969).

13. S. L. Adler, Phys. Rev. 143, 1144 (1966).

14. J. D. Bjorken, Phys. Rev. 163, 1767 (1967).

15. D. J. Gross and C. H. Llewellyn Smith, Nucl. Phys. B14, 337 (1969).

16. C. Callan and D. J. Gross, Phys. Rev. Letters 22, 156 (1969).

17. M. G. Doncel and E. De Rafael, to be published in Nuovo Cimento.

18. Actually we obtain the results in the form of an infinite number of moments e.g.

$$\int_0^1 x^{2n+1} \phi(x)\, dx = 0 , \qquad n = 0,1,2,\ldots$$

$$\phi(x) = 12(F_1^{\gamma p} - F_1^{\gamma n}) - (F_3^{\nu p} - F_3^{\nu n}) \qquad (x = 1/\omega) .$$

Unless $\phi(x)$ changes sign an infinite number of times in $0 \leq x \leq 1$, we can construct a polynomial $\psi(x^2)$ with the same zeros so that $\phi\psi \geq 0$. The moment condition gives $\int_0^1 x\phi\psi\, dx = 0$ and hence $\phi \equiv 0$ in this case. (I am indebted to A. suri for a discussion which produced this proof.)

19. Actually Callan and Gross[16] derived Eq. (9) for the F_i^{γ}. In reference 15 it was derived for $F_i^{\nu} + F_i^{\bar{\nu}}$. It follows for $F_i^{\nu} - F_i^{\bar{\nu}}$ by a combined use of the result for $F_i^{\nu} + F_i^{\bar{\nu}}$ and the sum rules for F_2[13] and F_1[14]. (This result has not previously been stated explicitly in the literature as far as we know.)

20. E. D. Bloom et al., SLAC-PUB-796 (Report to the Kiev Conference) and SLAC-PUBs 815 and 907 (submitted to Phys. Rev. Letters).

21. I. Budagov et al., Phys. Letters 30B, 364 (1969).

22. It is frequently asserted that the integrals in Eq. (17) must be $\geq 2/9$ if there are no gluons but this depends on assuming particular properties for the parton distributions.

23. J. D. Bjorken, Phys. Rev. 148, 1467 (1966), and Dl, 1376 (1970).

24. S. N. Gupta, Proc. Phys. Soc. 64, 695 (1951). (I am grateful to J. S. Bell for bringing this reference to my attention when commenting on Appendix 2.)