

PHENOMENOLOGICAL DUALITY\*

Haim Harari

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

and

Weizmann Institute of Science, Rehovot, ISRAEL

ABSTRACT

A phenomenological description of hadronic two-body reactions is presented. It assumes two-component duality and the dominance of the peripheral s-channel resonances. A successful explanation of dip phenomena in elastic and inelastic processes follows. The relation between the present description and previous models is discussed.

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## I. Introduction

Significant progress has been made in the last few years on several different fronts of hadron dynamics. In the purely theoretical domain, better understanding of the properties and limitations of dual resonance models has been achieved.<sup>1</sup> At the same time, new ideas have been applied to the phenomenological analysis of hadronic processes, and several new models have been proposed and compared with experiment.<sup>2</sup> It is somewhat unfortunate, however, that these two areas of development have remained largely disconnected from each other. Thus, certain lessons of the phenomenological work (such as the absolute necessity of cut terms or absorption corrections) are often ignored in theoretical discussions of dual resonance models. On the other hand, several successful qualitative aspects of duality (such as the two-component conjecture or exchange degeneracy) are sometimes contradicted by specific phenomenological models.

It is therefore important to try to summarize the lessons that we have learned both from the work on duality and from the phenomenological studies, and to construct a qualitative picture of hadron reactions which will incorporate ideas from these two fields of investigation. In this talk we shall present such a picture. Many of the ideas used in our description are borrowed from earlier models but the picture as a whole differs from all previous models. We shall specify the differences between our approach and previous approaches when we discuss the experimental evidence for the validity of our ideas.

We believe that the qualitative model presented here accounts for many systematic features of the data, and that it has a good chance of being an approximately true description of hadronic two-body reactions. More quantitative work is obviously needed, however, and we hope that it will be done in the near future.

## II. The Framework: Two Component Duality

We shall use the usual assumptions of two-component duality, namely — we assume<sup>3,4</sup> that the imaginary part of any hadronic two-body amplitude can be expressed as the sum of two terms:

$$\text{Im } A(s, t) = R(s, t) + P(s, t)$$

From the s-channel point of view,  $R(s, t)$  represents the contribution of s-channel resonances while  $P(s, t)$  is given by the nonresonant background. The resonance dominance assumption can be true in a local sense (in the neighborhood of the resonance energy) only for the imaginary part, while in the real part, distant resonances can be important. This is why we apply the two component assumption in its simple form only to the imaginary part of the amplitude.

From the t-channel point of view,  $R(s, t)$  represents the contributions of "ordinary" t-channel exchanges, namely — single pole exchanges as well as pole-Pomeron cuts. We neglect the contribution of double-particle-exchanges although at energies of a few BeV they may be appreciable.<sup>5</sup>  $P(s, t)$  represents the diffractive (Pomeron exchange) part of the amplitude.

Among the many successful features of two component duality we shall mention only a few well-known results which will be relevant to our discussion. If  $R(s, t)$  represents s-channel resonances and  $P(s, t)$  represents the Pomeron we immediately get three classes of simple predictions.<sup>3,6</sup>

(i) Imaginary parts of amplitudes which show no resonances should be dominated by Pomeron exchange. This is best illustrated by the well-known behavior of total hadronic cross sections (Figure 1) which are more or less constant in energy for exotic s-channel quantum numbers while they decrease with energy when s-channel resonances are present. The t-channel description of the "flat"

total cross section for, say,  $K^+p$  scattering, requires cancellations at all energies between the allowed "ordinary" exchanges ( $\rho$ ,  $\omega$ ,  $f^0$ ,  $A_2$ , etc.). This is the famous exchange degeneracy<sup>7</sup> prediction of duality<sup>8,3</sup>. Since we allow substantial contributions of pole-Pomeron cuts, the exchange degeneracy between, say,  $\rho$  and  $A_2$  should be interpreted here as an equality between the combined contribution of the  $\rho$ -pole and the  $\rho \otimes P$  cut on one hand and the  $A_2$ -pole and the  $A_2 \otimes P$  cut on the other hand. In most simple models for the construction of pole-Pomeron cuts such an equality would necessitate separate equalities between the  $\rho$ -pole and the  $A_2$ -pole and between their associated cuts. However, for our discussion it is sufficient to assume the equality between the two combined terms. We shall use this form of exchange degeneracy throughout our description of the vector and tensor exchange contributions to two-body processes.

(ii) Imaginary parts of amplitudes which have no Pomeron term should be dominated by s-channel resonances. A well-known example is the partial wave projections of the  $\pi N \rightarrow \pi N$  scattering amplitude for a well defined t-channel isospin. The  $I_t = 1$  amplitudes which do not allow Pomeron exchange are indeed dominated by the resonances<sup>6</sup> (Figure 2).

(iii) Imaginary parts of amplitudes which do not allow Pomeron exchange and show no resonances should vanish.<sup>3</sup> Any charge exchange process with exotic s-channel quantum numbers may serve as an example. The case of  $K^+n \rightarrow K^0p$  is particularly simple. The optical theorem together with isospin gives:

$$\text{Im}(K^+n \rightarrow K^0p)_{t=0} \propto \sigma_{\text{tot}}(K^+p) - \sigma_{\text{tot}}(K^+n) \quad .$$

The experimentally observed equality (see Figure 1) between the total  $K^+p$  and  $K^+n$  cross sections teaches us that  $\text{Im}(K^+n \rightarrow K^0p) \sim 0$ . From the t-channel point of

view this is, again, guaranteed by  $\rho - A_2$  exchange degeneracy (as interpreted above).

Many other aspects of the two-component conjecture have been successfully compared with experiment<sup>6</sup> and we need not repeat them here. The one outstanding difficulty is still the case of baryon-antibaryon scattering.<sup>9</sup> Since most of our presentation here involves meson-baryon processes we shall ignore this difficulty. We stress, however, that a convincing resolution for it is still missing.

### III. The "Elastic Puzzle" and the "Inelastic Puzzle"

An obvious challenge to any phenomenological theory of hadronic two-body reactions is the seemingly erratic behavior of dips in elastic and inelastic angular distributions.

In elastic scattering several processes exhibit dips around  $|t| \sim 0.6 \text{ GeV}^2$  at energies of a few BeV's while others do not exhibit any such dips (Figure 3). There is a one-to-one correspondence between the presence of such dips and the presence of s-channel resonances in the same processes.<sup>10</sup> The explanation of these dips is therefore an obvious challenge to two-component duality. We will refer to it as the "elastic puzzle". Another feature of elastic scattering which requires explanation are the polarizations<sup>11</sup> which exhibit interesting systematic features (Figure 4).

In inelastic scattering, the  $|t| \sim 0.6$  dips are even more strange, since they do not seem to follow any simple pattern or correlations. Processes such as  $\pi^- p \rightarrow \pi^0 n$ ,  $\pi N \rightarrow \pi \Delta$  and  $\gamma p \rightarrow \pi^0 p$  exhibit such dips while reactions like  $\pi N \rightarrow \eta N$ ,  $KN \rightarrow K\Delta$ ,  $\pi N \rightarrow \omega N$  do not show them. A coherent explanation of this "inelastic puzzle" is certainly required from any successful phenomenological description.<sup>12</sup>

The dual absorptive model<sup>13,14</sup> that we shall outline in the following sections offers a simple qualitative explanation of all of these dip puzzles.

#### IV. Duality and Peripheral s-Channel Resonances

We have assumed that the first component of the imaginary part of the amplitude can be described either as a sum of s-channel resonances or as a combination of "ordinary" t-channel exchanges. In all simple t-channel models, any structure in the angular distribution (dips, bumps) occurs at approximately fixed t-values at all energies. This follows from the fact that the singularities in t control the amplitude in such models. If we now assert that the same amplitude can be viewed as a sum of contributions of s-channel resonances, we have to explain how a collection of such resonances is capable of reproducing a given structure at fixed t-values at all energies. There are several ways in which this can happen.<sup>15</sup> We shall briefly mention here two different possibilities for the simple case of  $\pi\pi$  scattering.

The simplest way in which a sum of s-channel resonances can reproduce a certain structure at a fixed t-value at all energies is if every single prominent resonance produces such a structure by itself. This is, of course, an extremely strong assumption, but it is not a priori excluded by any theoretical principle or experimental fact. In Figure 5 we see how this may happen in  $\pi\pi$  scattering. The  $\rho$ , f and g mesons have their first zero between  $|t| = 0.25$  and  $|t| = 0.31$ . If the full  $\pi\pi$  scattering amplitude is actually given to a good approximation by these states, the differential cross section will have dips at a fixed t-value around  $|t| \sim 0.3$ . A more accessible process is  $\pi N$  scattering in which every prominent  $N^*$  actually exhibits zeroes at approximately fixed t-values (Figure 6), in both helicity

amplitudes.<sup>16</sup> These two examples seem to indicate that the simple possibility of demanding that every prominent resonance possesses the same structure in  $t$ , may actually occur in nature.

An alternative possibility of producing structure at fixed- $t$  is offered by the Veneziano formula.<sup>17</sup> In this case, several (equally important) resonances contribute at any given energy. Every one of them produces zeroes at a variety of  $t$ -values but the sum of all resonances at any given energy is constrained to produce structure at a fixed  $t$ -value. In  $\pi\pi$  scattering the Veneziano formula calls for a  $\rho + \sigma$  combination at the  $\rho$ -mass, an  $f + \rho'$  combination at the  $f$ -mass, etc. All of these combinations produce zeroes at  $|t| \sim 0.5$ . This possibility is perhaps less attractive from a simple-minded  $s$ -channel point of view, but that is, of course, not a sufficient reason to discard it.

As indicated above, however, processes such as  $\pi N \rightarrow \pi N$  and also  $\gamma N \rightarrow \pi N$  hint that the first possibility is approximately realized by nature. What constraints does this impose on the quantum numbers of the prominent  $s$ -channel resonances? The angle in which a given resonance produces a zero in the amplitude depends only on the spin of the resonance (and the spins of the initial and final particles). The  $t$ -value of the same zero is, of course, related to the angle by the resonance mass. Hence, a set of resonance zeroes at fixed- $t$  at all energies must impose a relation between the spins and masses of the prominent resonances. Some trivial algebra shows that the relation is of the form:

$$\ell \propto m = \sqrt{s} .$$

In the case of  $\pi N$  scattering it is actually true that the prominent resonances lie in the neighbourhood of a curve of the form  $\ell \propto \sqrt{s}$  (Figure 7).

We shall therefore assume here that the first component of the imaginary part of any hadronic two-body amplitude is dominated by resonances and that the prominent resonances obey a relation of the form  $l \propto \sqrt{s}$ , thus enabling every one of them to produce a similar structure as a function of  $t$ .

Since the resonances do not dominate the real part in a local way, it does not follow from our assumption that the real part is also dominated by the  $l \propto \sqrt{s}$  partial waves. The partial wave expansion of the real part at a given energy may show substantial contributions of  $l \leq \sqrt{s}$  partial waves, as a result of contributions of distant resonances.

### V. Absorption

We now temporarily leave the subject of duality and briefly discuss some properties of the absorption model. The basic idea of the absorption model was first applied to hadron reactions almost ten years ago. Since then, many different versions of this model have been proposed,<sup>18,19,20,21</sup> with varying degrees of success. The fundamental idea of the model is, however, common to all of these versions. It states that inelastic hadronic two-body reactions are dominated by the most peripheral impact parameters within the interaction radius. The contributions of lower partial waves (or smaller impact parameters) are supposed to be small because of the strong competition among the many open channels, which "absorb" part of the amplitude for these partial waves. If we assume that at energies of a few BeV the range of the strong interaction is of order  $r \sim 1$  fermi, the absorption model tells us that impact parameters  $b \sim r$  or partial waves  $l \sim qr$  (where  $q$  is the c. m. momentum) dominate hadronic two-body inelastic amplitudes (Figure 8).

This condition ( $l \sim qr$ ) resembles the condition which we have deduced in



the previous section from the assumption that all prominent s-channel resonances must produce structures at fixed t-values at all energies. The only difference between the usual statement of the absorption model and our statement of the previous section is this: All versions of the absorption model assume that the  $l \sim qr$  partial waves dominate the imaginary as well as the real part of the amplitude, while we concluded that only the imaginary part is dominated by the peripheral partial waves (because we reached this "peripheral dominance" through "resonance dominance" which applies locally only to the imaginary part).

This conflict between the absorption model descriptions of the real part and our interpretation is interesting from another point of view. At asymptotic energies a well defined relation exists between the energy dependence and the phase of an amplitude.<sup>22</sup> In particular, if

$$\begin{array}{l} \text{Im } A(\nu, t) \rightarrow \nu^{\alpha(t)} \\ t \text{ fixed,} \\ \nu \rightarrow \infty \end{array}$$

then

$$\begin{array}{l} \text{Re } A(\nu, t) \rightarrow \nu^{\alpha(t)} \cdot \left. \begin{array}{l} \left( \tan \frac{\pi\alpha(t)}{2} \text{ (for crossing-odd)} \right) \\ \left( \cot \frac{\pi\alpha(t)}{2} \text{ (for crossing-even)} \right) \end{array} \right\} \\ t \rightarrow \text{fixed,} \\ \nu \rightarrow \infty \end{array}$$

These results remain true in the presence of logarithmic terms and we can therefore interpret  $\alpha(t)$  as some "effective" pole describing the phenomenological energy dependence of the amplitude, which is presumably due to poles and cuts.

If  $\text{Im } A(\nu, t)$  is dominated by the  $l \sim qr$  partial waves, and if the relation between the phase and the energy dependence is obeyed, it is not always possible to have the  $l \sim qr$  partial waves dominate the real part! This point is ignored in most absorption models, but it should obviously be considered as a necessary constraint.

We shall therefore assume here that the imaginary parts of all inelastic two-body amplitudes (or more precisely — the first component  $R(s, t)$  of all hadronic two-body amplitudes) are dominated by the  $\ell \sim qr$  partial waves, while the real parts may or may not be dominated by the same partial waves. We further assume that the asymptotic relation between the energy dependence and the phase is obeyed.

### VI. The Rules of the Model

We are now ready to summarize the rules of our model:<sup>13</sup>

1. The imaginary part of any hadronic amplitude is given by a sum of two terms:  $R + P$ .

2. The R-component is dominated by the peripheral partial waves and is therefore given<sup>20,21,13</sup> by a function of the form  $g(t) J_{\Delta\lambda}(r\sqrt{-t})$  where  $\Delta\lambda$  is the total s-channel helicity flip and  $g(t)$  is a smooth function such as  $e^{at}$ .

3. The P-component has substantial contributions from all  $\ell \leq qr$  partial waves and is given by a smooth function of  $t$  (say, of the form  $e^{ct}$ ).<sup>13</sup> We assume that the P-term approximately conserves the s-channel helicity.<sup>23</sup>

4. The real part of the non-Pomeron component is unknown. At infinite energy it presumably reaches the form  $g(t) J_{\Delta\lambda}(r\sqrt{-t}) \tan \frac{\pi\alpha}{2}$  or  $g(t) J_{\Delta\lambda}(r\sqrt{-t}) \cot \frac{\pi\alpha}{2}$ , depending on the crossing properties. On the basis of empirical data we find that this asymptotic form is actually achieved at energies of a few BeV for  $\Delta\lambda = 1$  amplitudes in the case of vector or tensor meson exchange.<sup>13,14</sup> We suspect that for  $\Delta\lambda = 0$  amplitudes in the same processes, the asymptotic phase is achieved very slowly (perhaps logarithmically), but for the purpose of our qualitative discussion in this talk, this will not be very crucial. We shall therefore leave the real part of the non-Pomeron  $\Delta\lambda = 0$  amplitude as an unknown.

5. We assume that the real part of the Pomeron exchange amplitude is negligible. This is true at  $t \sim 0$  but probably inadequate at large  $t$ -values. A detailed quantitative study should allow for a real contribution for the Pomeron, at least for  $|t| \geq 0.5 \text{ GeV}^2$ . However, we believe that our qualitative conclusions will not be affected by neglecting such a term.

Our assumptions are summarized in Table 1, and the relevant functions are schematically shown in Figure 9.

### VII. Experimental Test: Elastic Differential Cross-Sections

Our first test<sup>13</sup> of the model will be to "solve" the "elastic puzzle". The elastic differential cross section has contributions of the forms P·P, P·R and R·R. At energies of a few BeV, the last term is presumably already small and we shall therefore assume that:

$$\frac{d\sigma}{dt}_{el} \sim P^2 + 2P \cdot R$$

where P, R are respectively the imaginary parts of the Pomeron and non-Pomeron components of the  $\Delta\lambda = 0$  amplitude. (The absence of a significant real part or helicity flip in the Pomeron term is assumed.) R should be approximately given by  $e^{at} J_0(r\sqrt{-t})$ , while P is assumed to be structureless.

In exotic processes such as  $K^+ p$  and  $pp$  elastic scattering,  $R \sim 0$  and we expect a featureless differential cross section. In processes such as  $K^- p$ ,  $\bar{p}p$ ,  $\pi^+ p$  and  $\pi^- p$  elastic scattering,  $s$ -channel resonances are allowed and  $R \neq 0$ . We therefore expect a dip around the minimum point of  $J_0(r\sqrt{-t})$ , namely — around  $|t| \sim 0.6$ . This is observed in the data (Figure 3) and explains the "elastic puzzle". Furthermore, the fact that R decreases with energy relative to P, explains why

the dip structure disappears at higher energies.

A comparison of the particle and antiparticle elastic differential cross sections is even more interesting. Consider the case of  $K^+p$  and  $K^-p$ . In our approximation:

$$\frac{d\sigma}{dt}(K^+p) \sim P^2$$

$$\frac{d\sigma}{dt}(K^-p) \sim P^2 + 2P \cdot e^{at} J_0(r\sqrt{-t}) .$$

The structure of  $J_0(r\sqrt{-t})$  (Figure 9a) predicts that at  $t = 0$ ,  $\frac{d\sigma}{dt}(K^-p) > \frac{d\sigma}{dt}(K^+p)$ ; at  $|t| \sim 0.2$  they become equal (the famous crossover phenomenon); around  $|t| \sim 0.6$ ,  $\frac{d\sigma}{dt}(K^-p)$  shows a dip,  $\frac{d\sigma}{dt}(K^+p)$  continues to be structureless, and the difference between the two curves is maximal; at larger  $t$ -values the two differential cross sections should again approach each other and perhaps show a second crossover.

All of these features<sup>13</sup> are actually seen in the data for  $K^-p$  and  $K^+p$  elastic scattering, as well as in the  $\bar{p}p$  and  $pp$  data.

A recent beautiful measurement of  $\frac{d\sigma}{dt_{el}}(K^\pm p)$  at  $p_L = 5 \text{ GeV}/c$ <sup>24</sup> actually enables us to study our predictions from a slightly different angle.<sup>25</sup> Using our approximation we can extract the R and P amplitudes directly from the data. The R-amplitude is shown in Figure 10. P(t) (not shown) is featureless. We can explicitly transform the R(t) and P(t) amplitudes to an impact parameter representation and see whether or not R is indeed dominated by the peripheral partial waves, and whether P is dominated by all  $\ell \leq r$  waves. Figures 11 and 12 show<sup>25</sup> that this is actually the case, thus providing very strong evidence for the validity of our approach.

VIII. Experimental Test: Elastic Polarizations

The polarization in elastic  $\pi^\pm p$  and  $K^\pm p$  scattering is given by:

$$\mathcal{P} \frac{d\sigma}{dt} = \text{Im} \left[ A_{\Delta\lambda=0}^* A_{\Delta\lambda=1} \right]$$

Since the dominant term in  $A_{\Delta\lambda=0}$  is the Pomeron term  $P(t)$ , we can use the approximation:

$$\mathcal{P} \frac{d\sigma}{dt} \sim P(t) \cdot \text{Re} A_{\Delta\lambda=1} .$$

Using our assumed form<sup>13</sup> for  $\text{Re} A_{\Delta\lambda=1}$  (see Table 1) we predict:

$$\left( \mathcal{P} \frac{d\sigma}{dt} \right)_+ + \left( \mathcal{P} \frac{d\sigma}{dt} \right)_- \sim P(t) \cdot e^{at} J_1(r\sqrt{-t}) \cot \frac{\pi\alpha(t)}{2}$$

$$\left( \mathcal{P} \frac{d\sigma}{dt} \right)_+ - \left( \mathcal{P} \frac{d\sigma}{dt} \right)_- \sim P(t) \cdot e^{at} J_1(r\sqrt{-t}) \tan \frac{\pi\alpha(t)}{2}$$

where  $\left( \mathcal{P} \frac{d\sigma}{dt} \right)_\pm$  refers to either  $K^\pm p$  or  $\pi^\pm p$  elastic scattering. Since  $P(t)$  is structureless, the entire structure in the polarization sum or difference is given by  $\text{Re} A_{\Delta\lambda=1}$  and the experimental values of these combinations of  $\mathcal{P} \frac{d\sigma}{dt}$  should follow the features of Figure 9c, d. The data in Figure 4 are consistent with such a behavior. (In Figure 4 the combinations of polarizations rather than of  $\mathcal{P} \frac{d\sigma}{dt}$  are plotted, but this does not change any of the qualitative features.)

IX. Experimental Test:  $|t| \sim 0.6$  Dips in Inelastic Reactions

A glance at Table 1 and Figure 9 reveals that, within the framework of our model, dips around  $|t| \sim 0.6$  in inelastic differential cross sections<sup>12,14</sup> are likely to be connected with the zeroes of the  $J_1$  function. When the dominant helicity amplitude is the  $\Delta\lambda = 0$  (or  $\Delta\lambda = 2, 3, \dots$ ) amplitude, no dip is expected at

$|t| \sim 0.6$  (unless the real parts of these amplitudes somehow produce such a structure). When  $\Delta\lambda = 0$  and  $\Delta\lambda = 1$  are equally important, the  $|t| \sim 0.6$  dip in  $\left[ J_1(r\sqrt{-t}) \right]^2$  will be filled by the bump in  $\left[ J_0(r\sqrt{-t}) \right]^2$ . We therefore conclude that the only simple way of producing a  $|t| \sim 0.6$  dip is when the  $\Delta\lambda = 1$  amplitude actually dominates. But this is not enough. If this amplitude is even under crossing (as in the  $A_2$ -exchange case) the dip will not be observed because of the  $\cot \frac{\pi\alpha(t)}{2}$  factor in the real part (see Table 1). Only if the  $\Delta\lambda = 1$  amplitude dominates, and it is odd under crossing, we should expect a  $|t| \sim 0.6$  dip. This situation is summarized in Table 2.

What remains to be done in order to predict the presence or absence of  $|t| \sim 0.6$  dips in an inelastic two-body reaction is therefore to decide which s-channel helicity amplitude dominates. This cannot be done in a model independent way. We will therefore use here several assumptions which are outside the scope of our dual absorptive description. Our predictions here are therefore less reliable and any failure may be blamed either on our model or on our additional assumptions (or both). We will return to this point in the next section.

Our additional assumptions are:

1. The exchange of  $I = 0$  vector and tensor mesons ( $\omega$  and  $f^0$ ) tends to conserve the s-channel helicity in the baryon vertex. There are several pieces of evidence for this — (i) In  $\pi N$  elastic scattering the  $f^0$  term approximately conserves the helicity.<sup>26</sup> (ii) In NN elastic scattering the  $\omega$  and  $f^0$  terms seem to conserve the helicity.<sup>27</sup> (iii) The magnetic coupling of a vector meson to the nucleon is presumably related to the helicity flip. The isoscalar magnetic moment of the nucleon is small, and if we assume vector dominance — the  $\omega$  should be coupled to the nucleon mainly through the electric coupling.<sup>20,21</sup>

2. The exchange of  $I = 1$  vector and tensor mesons ( $\rho$  and  $A_2$ ) is dominated by helicity flip in the baryon vertex. This is supported by the following facts:

(i) In  $\pi^- p \rightarrow \pi^0 n$  and  $\pi p \rightarrow \eta n$  a shoulder is observed in  $\frac{d\sigma}{dt}$  at  $t \sim 0$ . This indicates that the  $\Delta\lambda = 1$  amplitude is larger than the  $\Delta\lambda = 0$  term. (ii) Vector dominance arguments indicate that the magnetic  $\rho NN$  coupling is somewhat larger than the electric coupling.<sup>20,21</sup> (iii) The same type of argument calls for a  $\rho N\Delta$  coupling of the M1 type,<sup>28,29</sup> corresponding to a single helicity flip between the N and the  $\Delta$ .

3. The helicity flip in the meson vertex is unique whenever only pseudoscalar mesons are involved. It is also unique in the case of photoproduction. In the case of vector meson production it is a priori not unique but we shall assume that it follows the photoproduction pattern, and that the s-channel helicity of the produced vector meson is predominantly  $\pm 1$  for vector and tensor meson exchange.

These three assumptions are definitely not exact. They may even be wrong. However, without them we cannot proceed with our test. We therefore take the risk of making these assumptions, and apply them to 15 reactions in which we can isolate the contributions of vector and/or tensor meson exchange. Table 3 shows for every reaction the dominant helicity change in the baryon and meson vertex according to the above assumptions. On the basis of these, the answer to the question "does  $\Delta\lambda = 1$  dominate?" can be determined for every process. Using the pattern of Table 2 we can then predict in every case whether or not we should expect a  $|t| \sim 0.6$  dip. In all 15 cases our prediction<sup>14</sup> agrees with the data.

A few remarks should be added here with respect to specific entries in the table.

(i) The energy dependence of some processes in the table (mainly  $\gamma N \rightarrow \pi N$  and  $\pi N \rightarrow VN$ ) does not follow the simple  $\nu^{\alpha(t)}$  rule with  $\alpha(t) = 0.5 + t$ . Our

assumptions on the real parts of these processes are therefore dubious, since we use the usual form of the trajectory.

(ii) Our discussion of  $\pi^+ p \rightarrow \rho^+ p$  assumes that we can isolate the  $\omega$ -exchange term. This can be done by eliminating  $\pi$ -exchange, as in the combination 
$$\frac{d\sigma}{dt}(\pi^+ p \rightarrow \rho^+ p) + \frac{d\sigma}{dt}(\pi^- p \rightarrow \rho^- p) - \frac{d\sigma}{dt}(\pi^- p \rightarrow \rho^0 n).$$

(iii) The reactions  $\gamma p \rightarrow \pi^+ n$ ,  $\pi^- \Delta^{++}$  involve a large  $\pi$ -exchange term and a possible  $A_2$  exchange contribution. These may obscure our conclusion concerning the absence of a dip in the  $\rho$ -exchange term in these reactions.

#### X. Discussion of the Experimental Tests and Comparison with Other Models

The qualitative success of our experimental tests indicates that there is at least some truth in our description. It is extremely important, however, to try to analyze which of our assumptions or "rules" are really tested by the data, and to what extent. It is also interesting to see how our model is related to other phenomenological models and how we can distinguish between different models.

A superficial classification of all previous models may group them into two classes. The first ("class I") include the Regge pole model in its various versions and the "weak cut" models<sup>19</sup>. The second ("class II") include absorption models such as the "strong cut" model<sup>20</sup> and the Dar-Weisskopf model.<sup>21</sup>

Our prescription is in qualitative agreement with class I models, on the imaginary and real parts of the  $\Delta\lambda = 1$  amplitude. We disagree with these models on  $\text{Im} A_{\Delta\lambda=0}$ . Our prediction for the elastic differential cross section as well as for  $\gamma p \rightarrow \eta p$  are crucial tests which strongly favor our approach. Our predictions for  $\gamma N \rightarrow \pi^\pm N$ ,  $\pi^\pm \Delta$  and  $\pi N \rightarrow \omega N$ ,  $\omega \Delta$  are also different from those of class I models, but here the possibility of other exchanges obscures the issue and a much



more detailed analysis is necessary. In particular, the density matrix elements for  $\pi N \rightarrow \omega N$  and  $\pi N \rightarrow \omega \Delta$  could provide us with detailed quantitative tests of the various models. At present, it is possible to fit, but not to predict, these density matrix elements with our model. It is not clear whether this will still be the case when better data are available. However, one must remember that in this processes, our "rules" of Section VI as well as our extra assumptions of Section IX are tested. We would first abandon the latter, in case of conflict, since they are less general. Regardless of any possible difficulty in  $\pi N \rightarrow \omega N, \omega \Delta$  we claim, however, that all class I models are ruled out in their present form at least by the elastic differential cross section. The "weak cut" model<sup>19</sup> could be resurrected if "overabsorption" is allowed, namely — if it will have "strong cuts" for  $\Delta\lambda = 0$  amplitudes (without abandoning its exchange degeneracy and  $\alpha$ -factors).

Our model agrees with class II models on the imaginary parts of  $\Delta\lambda = 0$  and  $\Delta\lambda = 1$  but disagrees with such models on the real part of  $\Delta\lambda = 1$ . Consequently, the elastic polarizations as well as  $\pi N \rightarrow \eta N, \eta \Delta, K^- p \rightarrow \bar{K}^0 n, K \Delta$ , and  $K^+ n \rightarrow K^0 p, K \Delta$  provide us with an opportunity to distinguish between our approach and these approaches. The data clearly favors our model. In addition class II models<sup>20,21</sup> do not assume exchange degeneracy. As a result, in spite of their ability to account for the difference between, say,  $K^- p$  and  $K^+ p$  elastic scattering, they cannot explain the dips in  $K^- p, \pi^\pm p$  and  $\bar{p} p$  elastic scattering and their absence in  $K^+ p$  and  $pp$  elastic scattering.

In summary — all previous models contradict our description in one respect or the other. However, many of our assumptions are actually borrowed from these models, and we suspect that we would not be able to construct the present picture, had we not enjoyed the insight provided by the previous models. We now believe

that the correct description of hadron two-body processes must incorporate (i) Duality (including exchange degeneracy), (ii) Absorption effects which are sometimes very strong, at least in the imaginary part of the non-Pomeron component, (iii) Correct asymptotic phase even at intermediate energies, at least for  $\Delta\lambda = 1$  amplitudes.

Consequently, if we are forced to choose between "weak cut" models with Regge  $\alpha$ -factors and "strong cut" models without  $\alpha$ -factors, we will probably prefer "strong cuts" with  $\alpha$ -factors, since we believe in strong absorption and in exchange degeneracy. We realize, however, that we are still far from having a correct method of calculating the cut contributions and we suspect that using the Regge pole term as a Born approximation is unjustified and possibly misleading.

#### XI. Open Problems

We conclude by listing several problems which are still open.

(1) We should probably move towards a more quantitative approach in our model. However, it is not absolutely clear that this is already desirable at the present stage. There are several reasons for this. In the absence of a theory of strong interactions, all phenomenological models are necessarily approximate. They should not try to explain every single detail of the data since it is apriori clear that no simple model will succeed in doing so. The purpose of the phenomenological work, as we see it, is to search for simple principles which account for systematic features, and which can perhaps, one day, give us clues as to the substructure of hadrons or the properties of the strong interactions. This does not require that a successful model will account for all minor details of all processes. On the more practical level, it is clear that many effects (such as

double-particle-exchange) are ignored in our approach, but should not be ignored in quantitative work at energies of a few BeV. An overenthusiastic quantitative model may thus seem to fail because of minor effects which are perhaps irrelevant to the main goal of this kind of work. The ideal approach is, presumably, to try to use our model in data-fitting but to keep in mind the fact that it is more important to give a description of all hadron processes within, say, 20% than to build a model which explains one or two processes within 5%.

(2) We have used a value of  $r \sim 1$  fermi throughout our discussion. Two questions arise here. (i) Is  $r$  different for different particles? We suspect that the answer is positive and that in  $\pi\pi$  scattering  $r$  is smaller than in NN scattering. A good way of testing this would be a precise determination of the crossover point in elastic  $\pi^\pm p$  scattering and in elastic  $pp$  and  $\bar{p}p$  scattering. We suspect that the  $\pi^\pm p$  crossover occurs at a larger  $t$ -value. (ii) Is  $r$  constant with energy? Does it grow like  $\log s$ ? like  $(\log s)^{\frac{1}{2}}$ ? The shrinkage of  $pp$  elastic scattering at high energies indicates that  $r$  grows. That may mean that crossover points and dips will move towards smaller  $t$ -values. It will be extremely interesting to see if this is the case.

(3) We assumed that the imaginary part of the non-Pomeron amplitude is dominated by the  $b \sim r$  impact parameters as shown schematically in Figure 8 and as exhibited by the  $K^\pm p$  data in Figure 12. This  $b$ -dependence can be roughly characterized by three quantities — the radius  $r$ , the width  $\Delta$  of the peripheral domain, and the strength at the maximum. How do these quantities change with energy? If  $r$  increases, does  $\Delta$  increase? Is the  $\Delta/r$  ratio constant in energy, in case that both increase? We hope to report soon on a detailed study of these problems. <sup>30</sup>

(4) The real part of  $A_{\Delta\lambda = 0}$  was left as a question mark in Table 1. We should be able to learn about it from  $K^*-K^{**}$  exchange reactions in which  $\Delta\lambda = 0$  dominates, and possibly from the polarization in  $\pi^- p \rightarrow \pi^0 n$ .

(5) Finally, it is clear that all of the ideas discussed here must have an impact on the description of processes with many particles in the final state. Absorption effects must play an important role there; the impact parameter description of such processes for different multiplicities may be as useful as in the case of two-body reactions; duality and exchange degeneracy are certainly relevant; factorization of poles is probably not very relevant (although correct) because of the large cut contributions, etc. We are looking forward to the application of these and other aspects of two-body phenomenology, to the description of multiparticle final states.

#### Acknowledgements

We wish to thank Adam Schwimmer for many helpful discussions.

Table 1

	Non-Pomeron		Pomeron	
	Im	Re	Im	Re
$\Delta\lambda = 0$	"J <sub>0</sub> "	?	e <sup>at</sup>	0
$\Delta\lambda = 1$	"J <sub>1</sub> "	"J <sub>1</sub> " tan $\frac{\pi\alpha}{2}$ (odd) "J <sub>1</sub> " cot $\frac{\pi\alpha}{2}$ (even)	0	0

The approximate form of the imaginary and real parts of the two components of  $\Delta\lambda = 0$  and  $\Delta\lambda = 1$  amplitudes. "J<sub>0</sub>", "J<sub>1</sub>" denote functions possessing the general characteristic of J<sub>0</sub>(r√-t) and J<sub>1</sub>(r√-t) at |t| < 1 BeV<sup>2</sup> (zeroes, minima, maxima, etc.).

Table 2

Does $\Delta\lambda = 1$ dominate?	Crossing properties	$d\sigma/dt$	dip at $ t  \sim 0.6$
NO		$J_0^2 + J_2^2 + ? + \dots$	NO
YES	ODD	$J_1^2 (1 + \tan^2 \frac{\pi\alpha}{2}) = J_1^2 / \cos^2 \frac{\pi\alpha}{2}$	YES
YES	EVEN	$J_1^2 (1 + \cot^2 \frac{\pi\alpha}{2}) = J_1^2 / \sin^2 \frac{\pi\alpha}{2}$	NO
YES	Equal amount of odd and even	$J_1^2 (\tan \frac{\pi\alpha}{2} - \cot \frac{\pi\alpha}{2})^2 = 4J_1^2 / \sin^2 \pi\alpha$	NO

The presence or absence of  $|t| \sim 0.6$  dips in inelastic differential cross sections is determined by whether or not the  $\Delta\lambda = 1$  s-channel helicity amplitude dominates, and by its properties under crossing.

Table 3

	Exchanged Particle	$\Delta\lambda_B$	$\Delta\lambda_M$	Does $\Delta\lambda = 1$ dominate?	Crossing	$ t  \sim 0.6$ dip
$\left\{ \begin{array}{l} \pi^- p \rightarrow \pi^0 n \\ \pi p \rightarrow \pi \Delta \end{array} \right\}$	$\rho$	1	0	Yes	-	Yes
$\left\{ \begin{array}{l} \gamma p \rightarrow \pi^0 p \\ \pi^+ p \rightarrow \rho^+ p \end{array} \right\}$	$\omega$	0	1	Yes	-	Yes
$\left\{ \begin{array}{l} \pi^- p \rightarrow \eta n \\ \pi N \rightarrow \eta \Delta \end{array} \right\}$	$A_2$	1	0	Yes	+	No
$\left\{ \begin{array}{l} K^- p \rightarrow \bar{K}^0 n, \bar{K} \Delta \\ K^+ n \rightarrow K^0 p, K \Delta \end{array} \right\}$	$\rho, A_2$	1	0	Yes	$\pm$	No
$\{ \gamma p \rightarrow \eta p \}$	$\rho$	1	1	No		No
$\left\{ \begin{array}{l} \gamma p \rightarrow \pi^+ n, \pi^- \Delta^{++} \\ \pi N \rightarrow \omega N, \omega \Delta \end{array} \right\}$	$\rho$	1	1	No		No

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Figure Captions

- Fig. 1: Total cross sections for  $K^+p$ ,  $K^+n$ ,  $pp$  and  $pn$  scattering are more or less "flat" as a function of energy while those for  $K^-p$ ,  $K^-n$ ,  $\bar{p}p$ ,  $\bar{p}n$ ,  $\pi^+p$  and  $\pi^-p$  decrease with energy. Flat cross sections correspond to exotic s-channel quantum numbers (and hence  $R = 0$ ) while decreasing cross sections correspond to nonexotic processes (and  $R \neq 0$ ). The Pomeron term itself may have some  $\log s$  factors which are not yet seen below 30 BeV. It is clear, however, that the asymptotic Pomeron dominance starts earlier in the exotic processes.
- Fig. 2: s-channel partial wave amplitudes for  $\pi N \rightarrow \pi N$  with well defined t-channel isospin. The  $I_t = 1$  amplitudes do not involve Pomeron exchange. They are strongly dominated by resonances and show relatively "clean" circles in the Argand diagram. The  $I_t = 0$  amplitudes show resonance circles superimposed on a substantial nonresonant background, presumably corresponding to the Pomeron contribution (from reference 6).
- Fig. 3: Schematic presentation of the main features of the elastic differential cross sections for  $K^\pm p$ ,  $\pi^\pm p$ ,  $pp$  and  $\bar{p}p$ . Exotic s-channel quantum numbers ( $K^+p$  and  $pp$ ) correspond to "smooth"  $\frac{d\sigma}{dt}$ . Nonexotic channels ( $K^-p$ ,  $\pi^\pm p$ ,  $\bar{p}p$ ) show structure somewhere around  $|t| \sim 0.6 \text{ BeV}^2$ . This figure was prepared on the basis of data compiled in reference 10.
- Fig. 4: Sums and differences of polarizations in elastic  $K^\pm p$  and  $\pi^\pm p$  scattering (data from reference 11). While the  $\pi^+p$  and  $\pi^-p$  polarizations do not resemble the  $K^+p$  and  $K^-p$  polarizations (not shown here), their sums and differences are surprisingly similar.

- Fig. 5: The  $t$ -dependence of the  $\rho^0$ ,  $f^0$  and  $g^0$  contributions to  $\pi^+\pi^-$  elastic scattering at the energy values of these resonances. The first zero of the amplitude is always around  $|t| \sim 0.25 - 0.30 \text{ BeV}^2$ .
- Fig. 6: A "map" of the zeroes of the prominent  $N^*$ -states in  $\pi N \rightarrow \pi N$  for both  $s$ -channel helicity amplitudes. The helicity non-flip amplitude shows zeroes around  $|t| \sim 0.2$  while the helicity flip amplitude has zeroes at  $|t| \sim 0.5$ . A slightly different version of these observations has already been made by Dolen, Horn and Schmid in reference 16.
- Fig. 7: The same "prominent" nucleon resonances of Figure 6 are shown on a Chew-Frautschi plot. A curve of the form  $J + \frac{1}{2} = qr$  where  $q$  is the c. m. energy and  $r = 1$  fermi is also shown. This is, presumably, the  $\ell \propto \sqrt{s}$  curve mentioned in the text.
- Fig. 8: All versions of the absorption model assume that contributions from impact parameters  $b \sim r$  are dominant in inelastic amplitudes while  $b < r$  contributions are largely absorbed.
- Fig. 9: Schematic presentation of functions of the types mentioned in Table 1.  
 (a) " $J_0$ "( $r\sqrt{-t}$ ), (b) " $J_1$ "( $r\sqrt{-t}$ ), (c) " $J_1$ "( $r\sqrt{-t}$ )  $\tan \frac{\pi\alpha(t)}{2}$  , (d) " $J_1$ "( $r\sqrt{-t}$ )  $\cot \frac{\pi\alpha(t)}{2}$  .  $r \sim 1$  fermi;  $\alpha(t) \sim 0.5 + t$ .
- Fig. 10: The non-Pomeron component  $R(t)$  determined from the  $K^\pm p$  data of reference 24. The figure is taken from reference 25.

Fig. 11: Partial wave projections (or impact parameter representation) of the function of Figure 10. The dashed area between the two curves represent the uncertainties in the data. This figure is taken from reference 25.

Fig. 12: Partial wave projections (or impact parameter representation) for  $P(t) = \sqrt{\frac{d\sigma}{dt}(K^+p)}$  and  $P(t) + R(t) = \sqrt{\frac{d\sigma}{dt}(K^-p)}$ . The difference between the two curves is consistent with the curve of Figure 11. This figure is taken from reference 25.

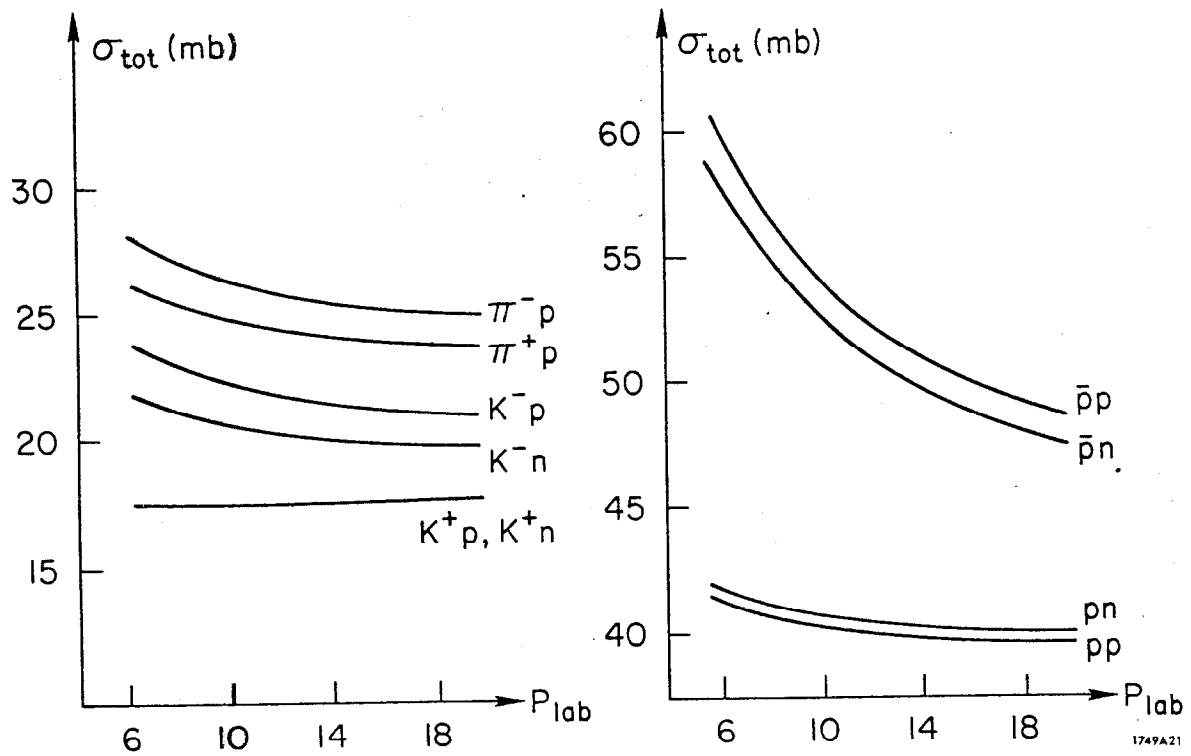


Fig. 1

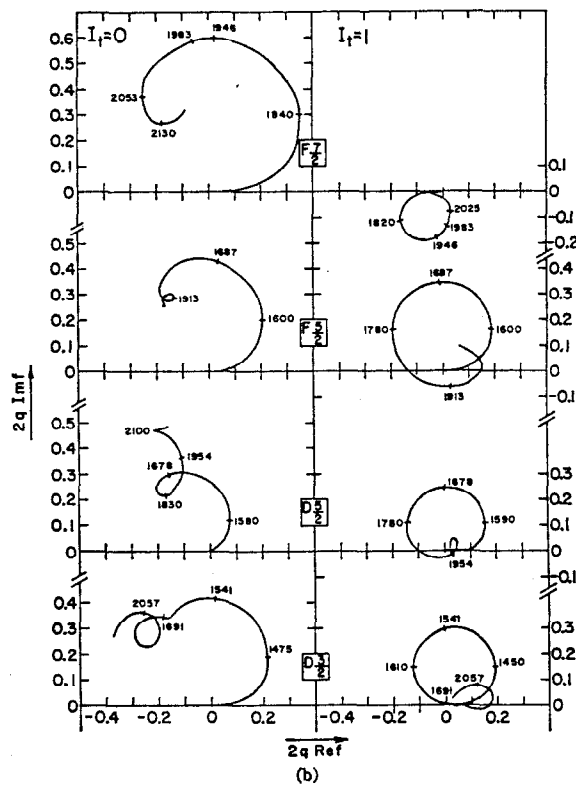
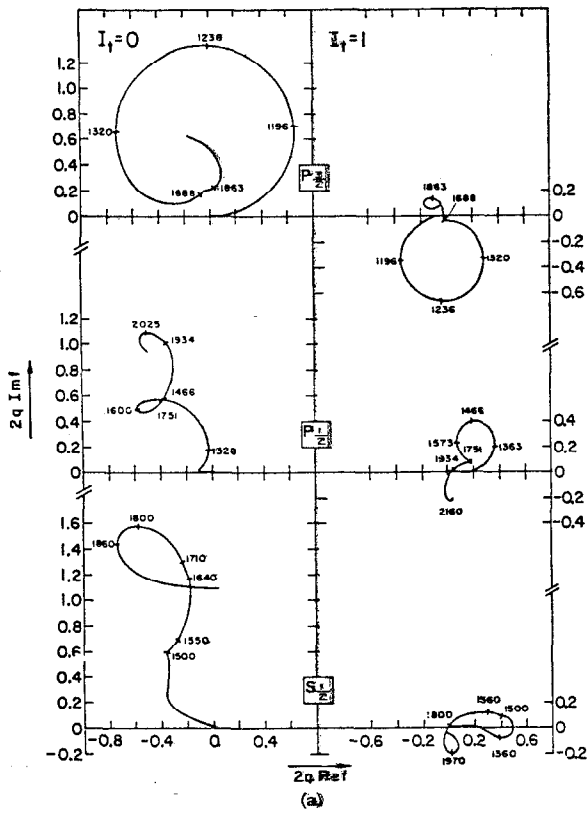


Fig. 2

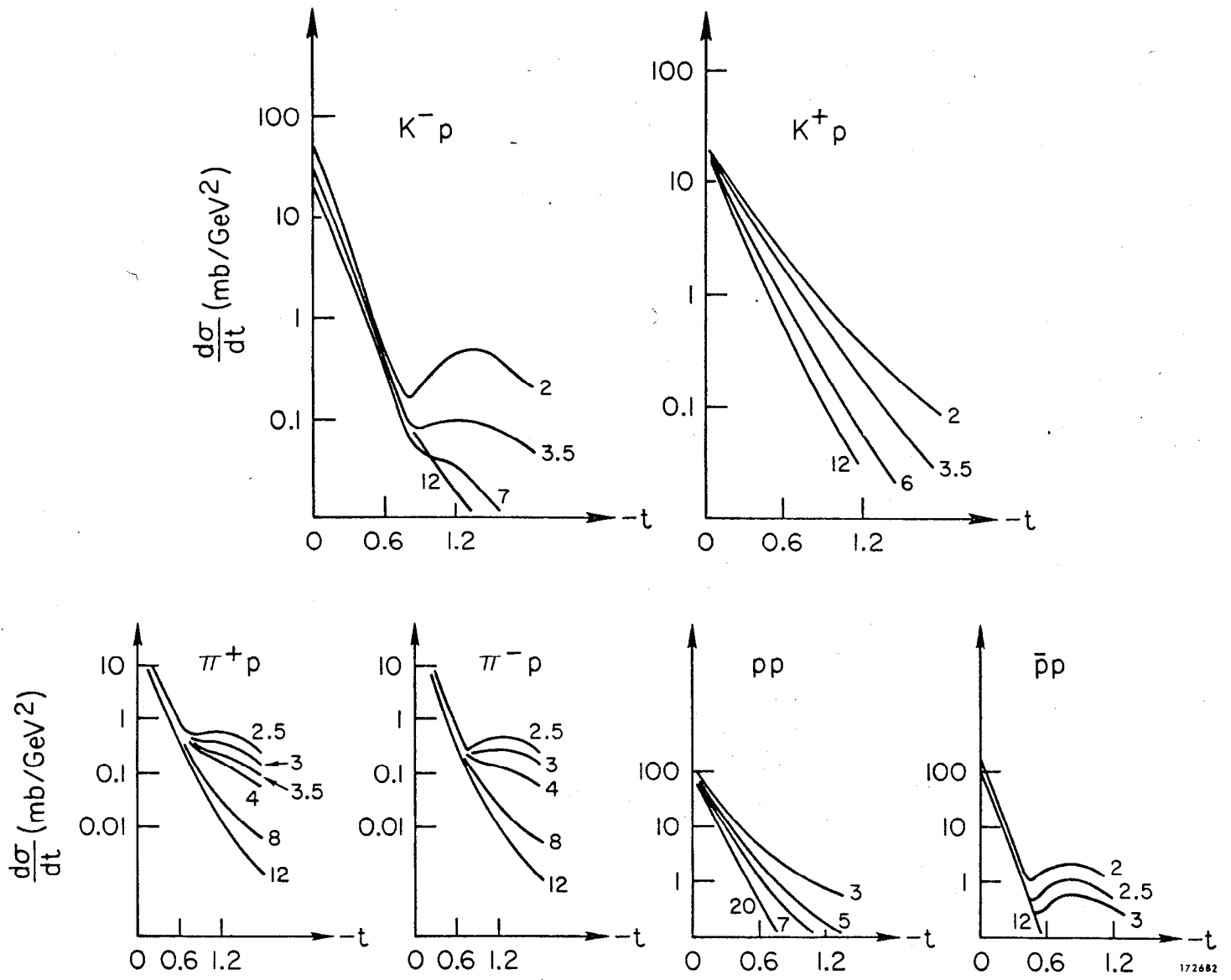


Fig. 3



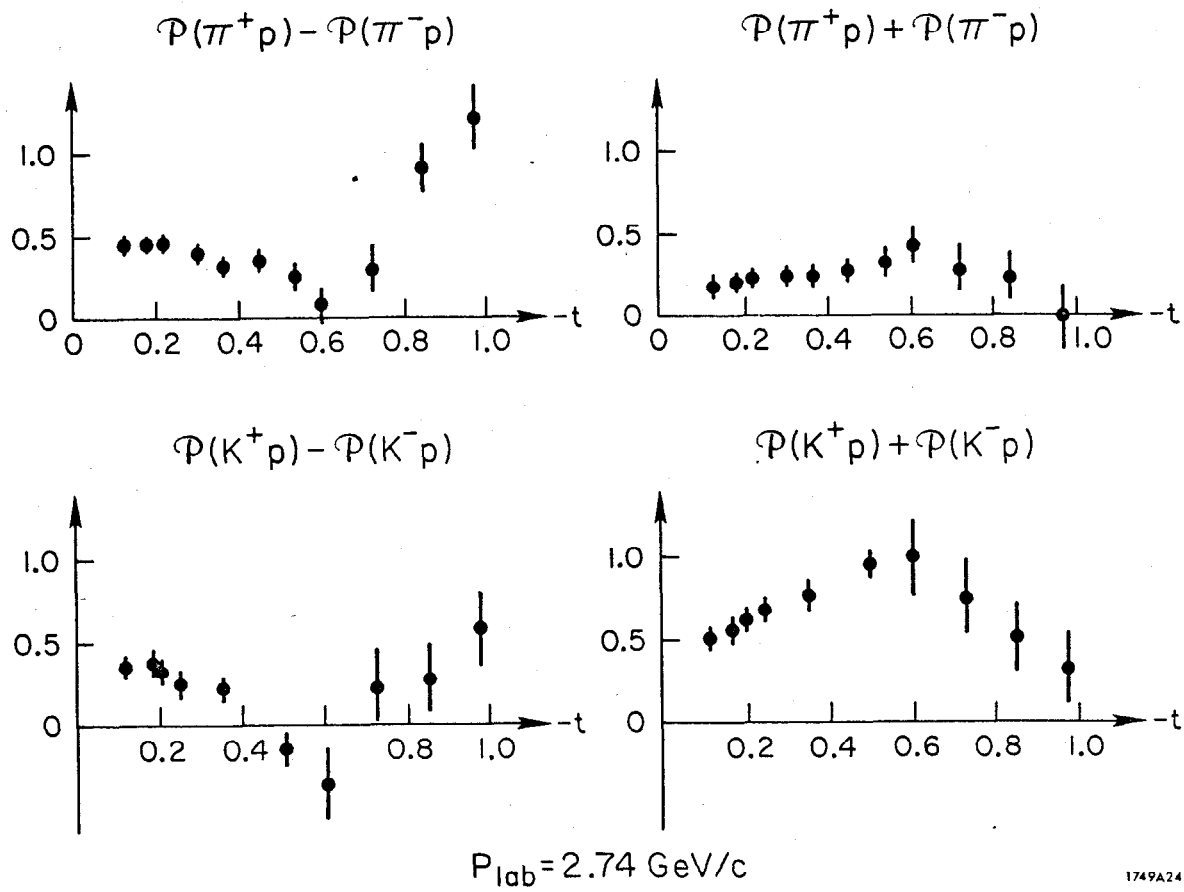
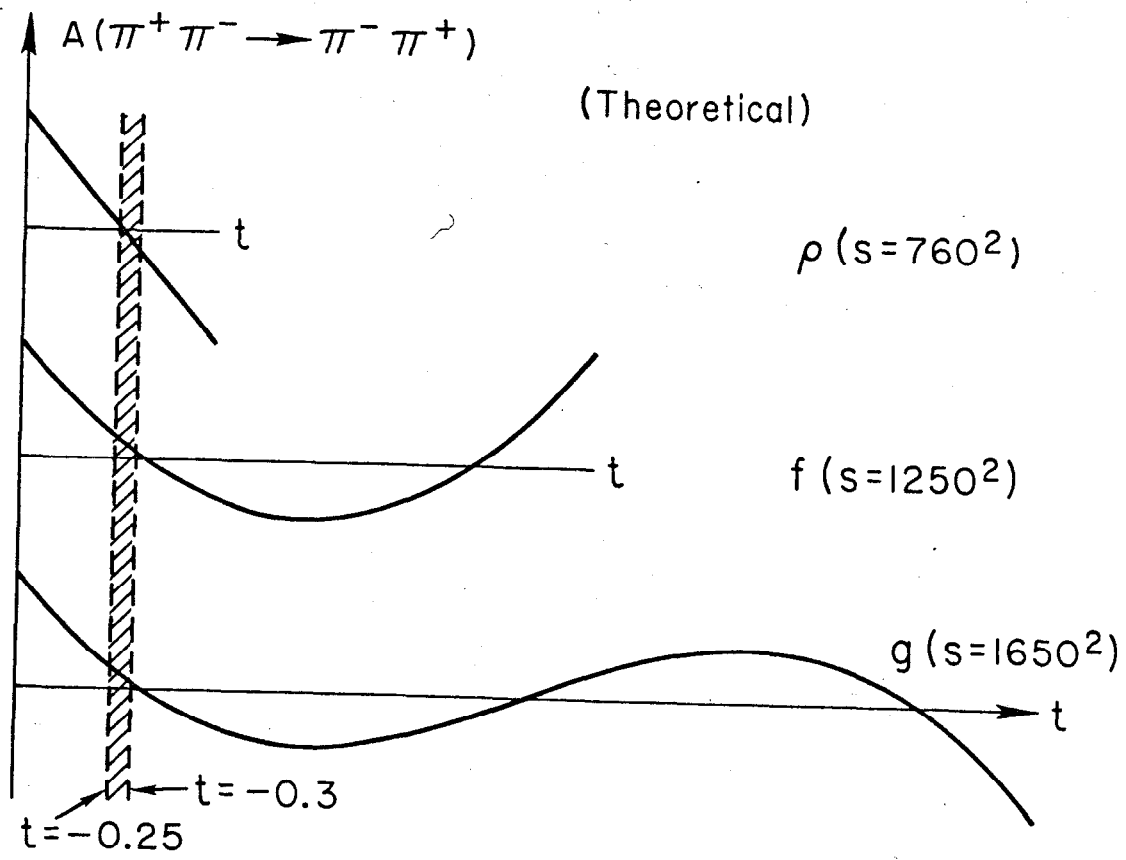
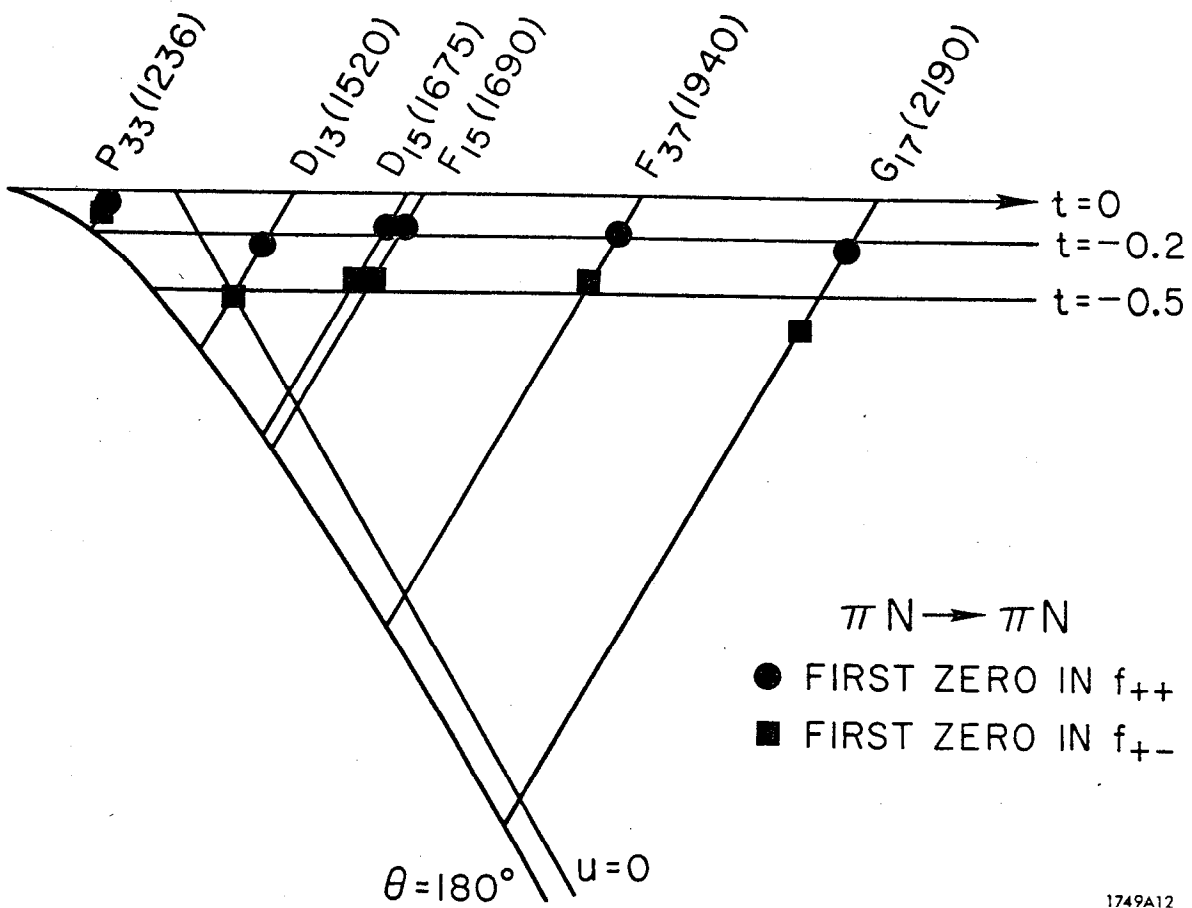


Fig. 4



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Fig. 5



1749A12

Fig. 6

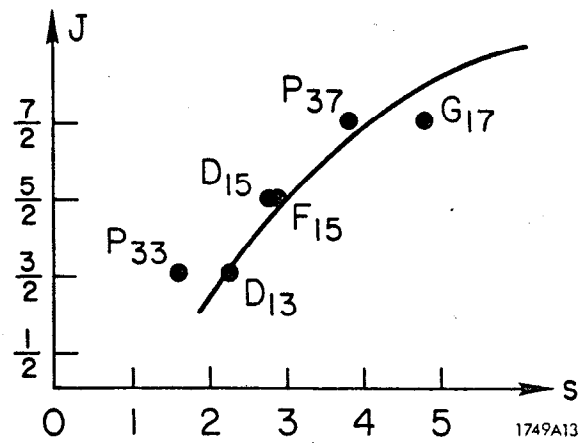


Fig. 7

GEOMETRICAL-OPTICAL MODELS

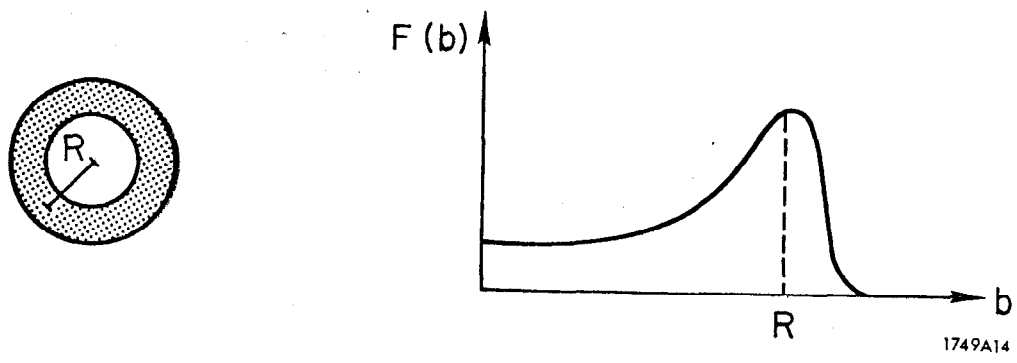
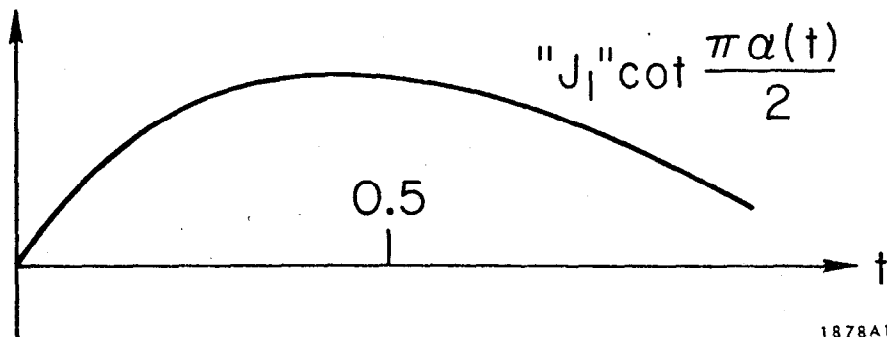
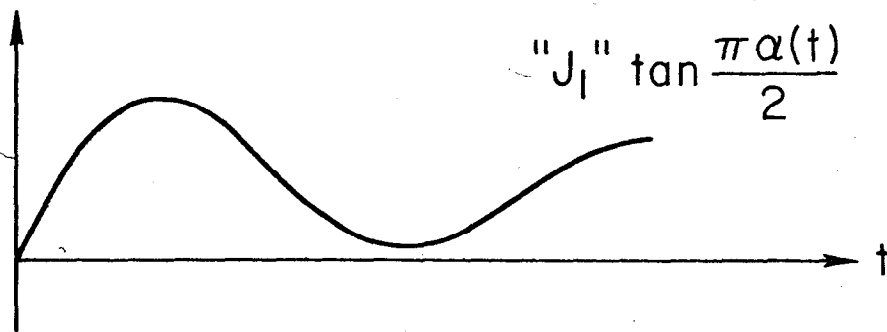
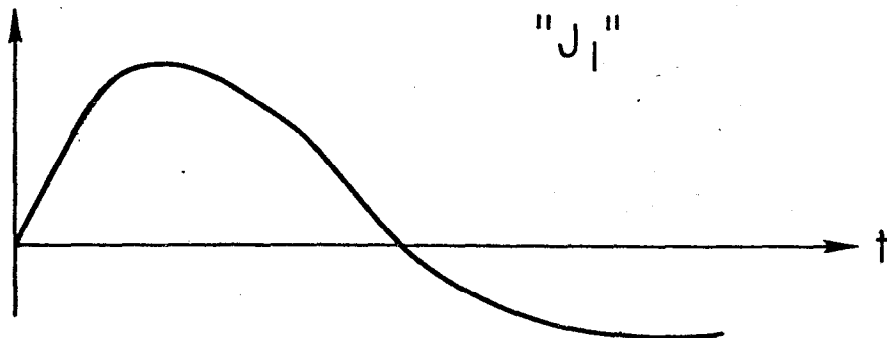
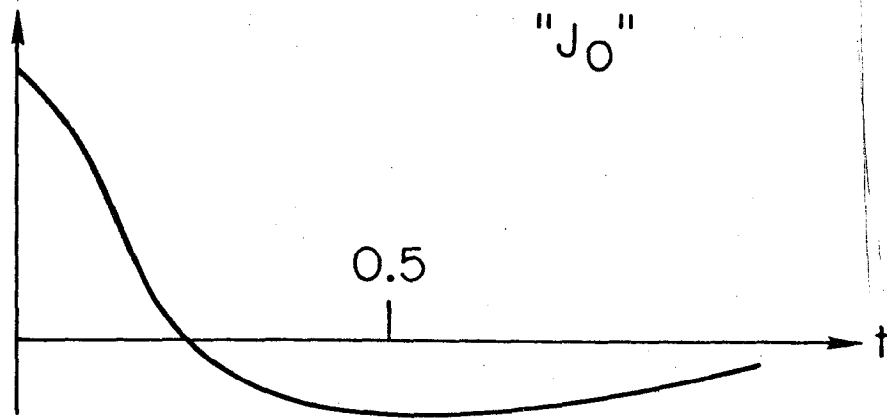


Fig. 8



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Fig. 9

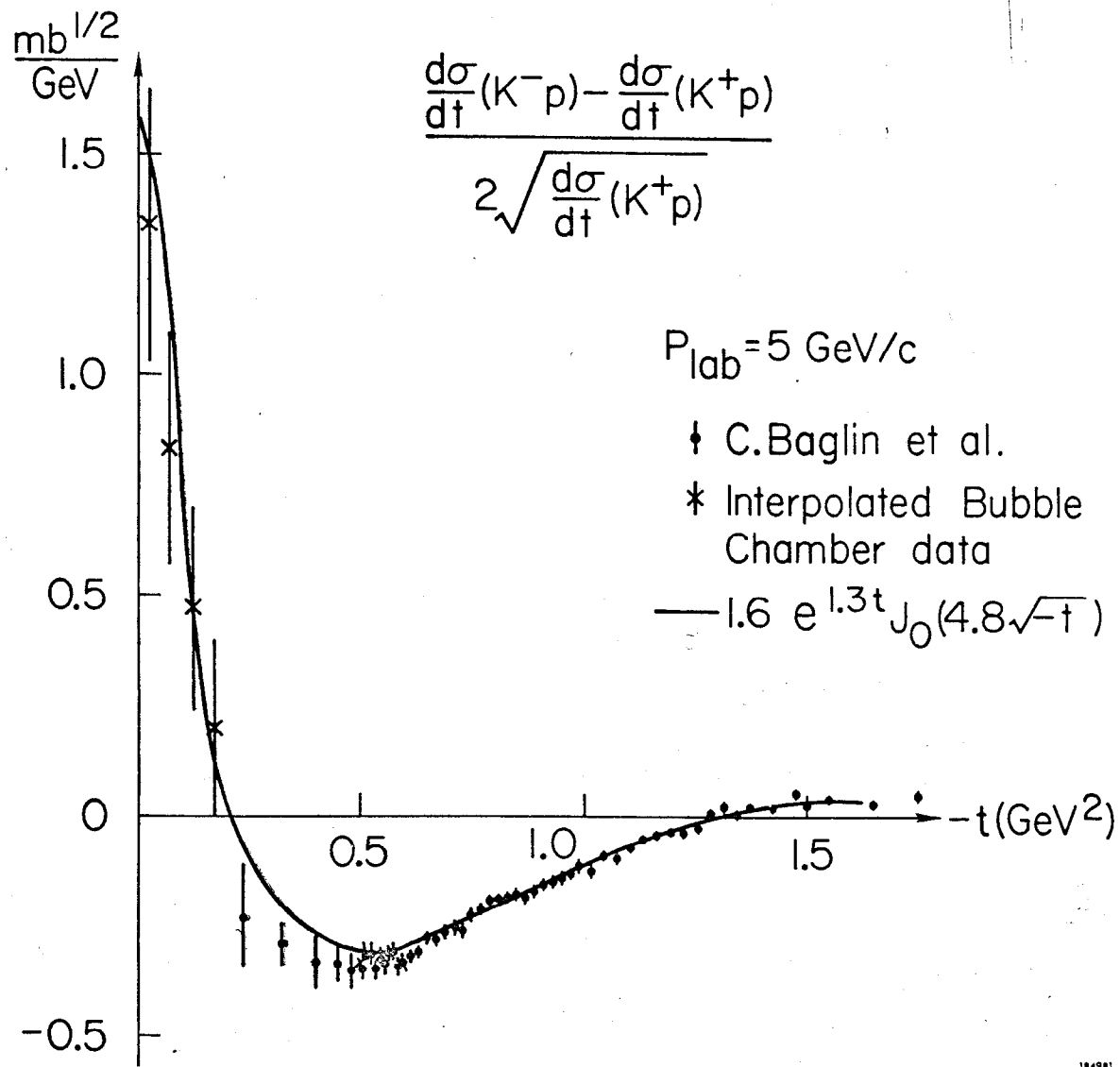


Fig. 10

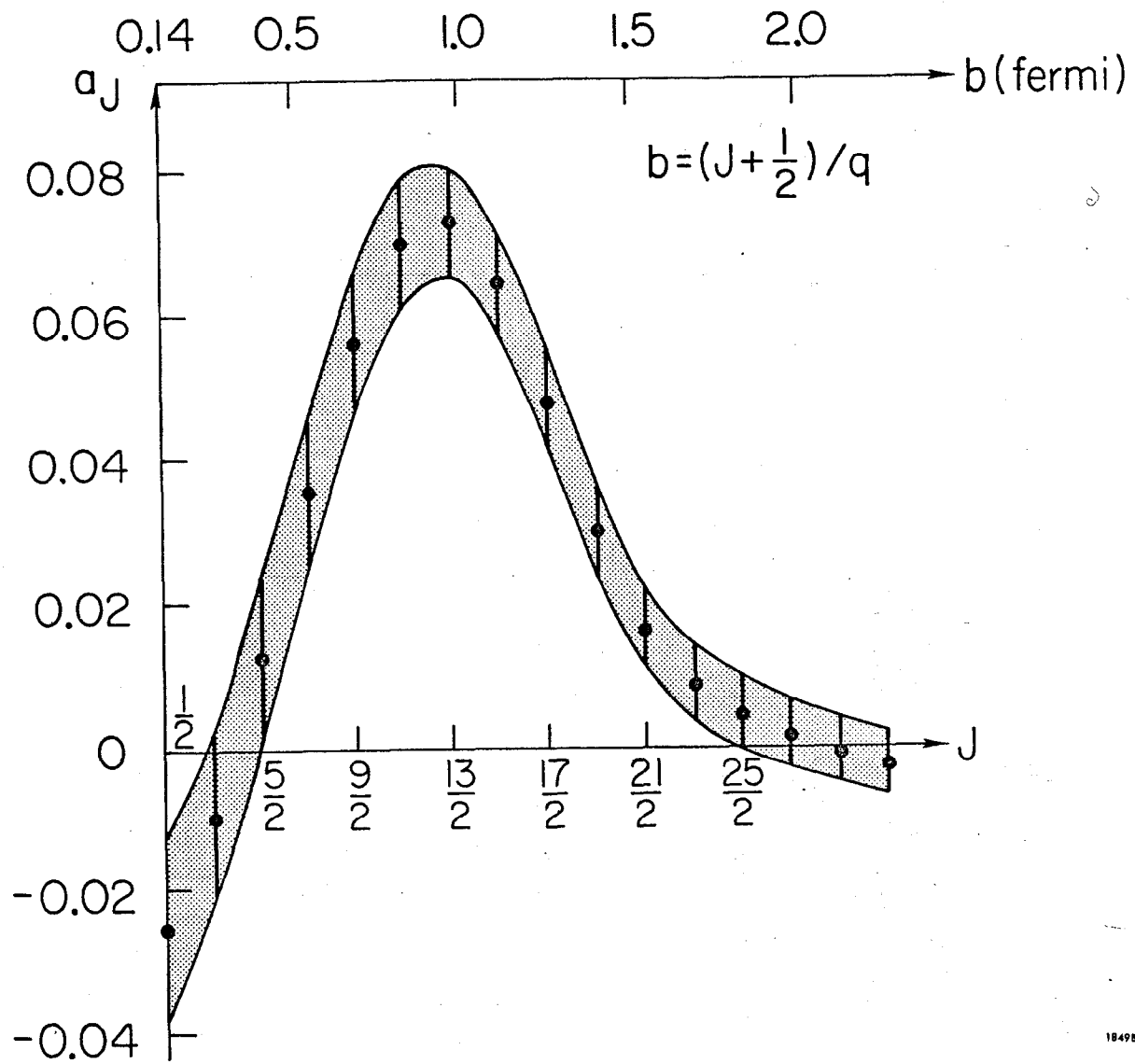
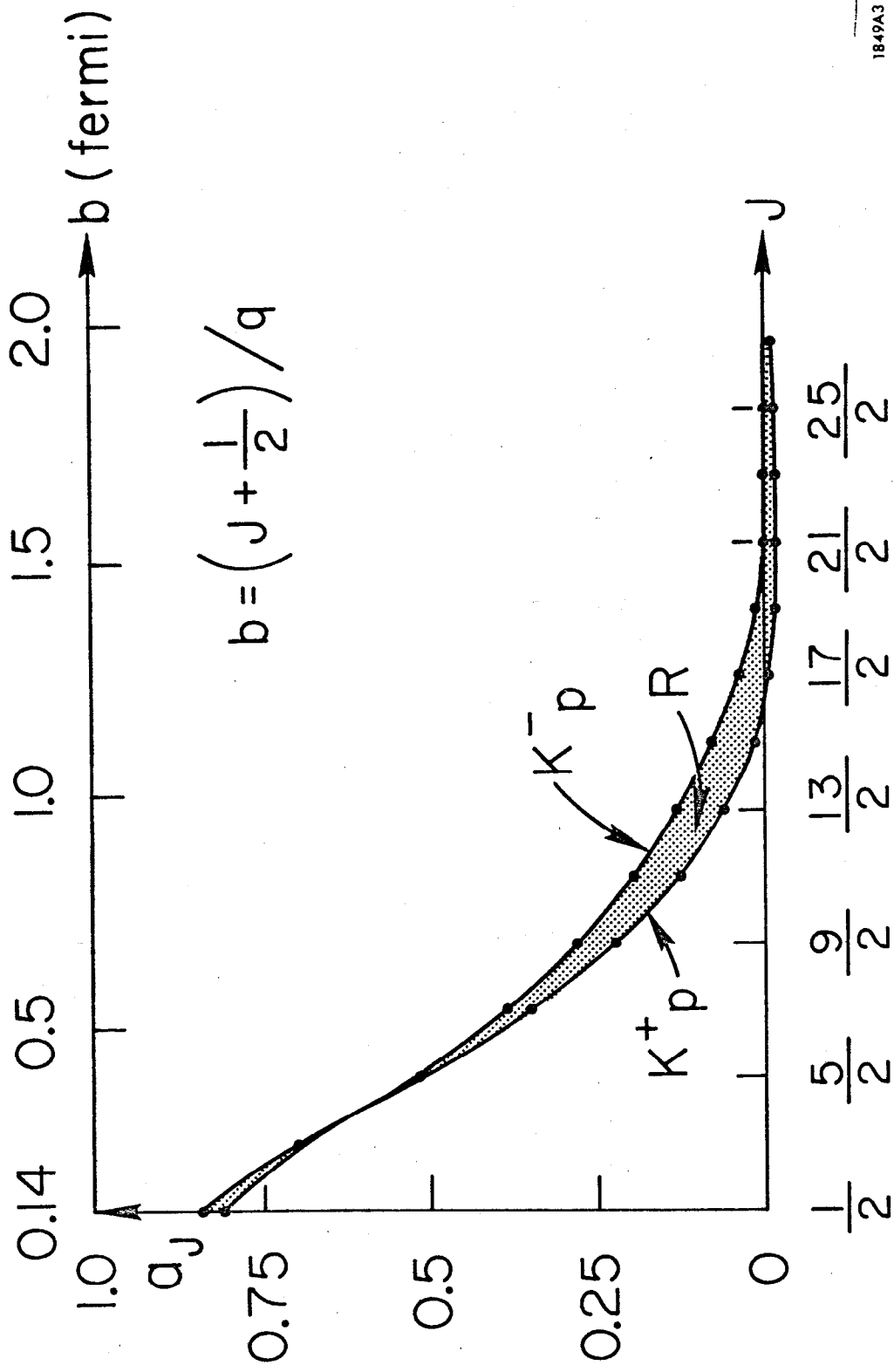


Fig. 11





1849A3

Fig. 12