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COMPARISON OF ABSORPTIVE ONE PION EXCHANGE MODEL WITH MEASUREMENTS OF $\pi^- p \to \pi^+ \pi^- n$ *

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ABSTRACT

The results of a wire spark chamber experiment studying the reaction $\pi^- p \to \pi^+ \pi^- n$ at 15 GeV/c are compared with the predictions of the absorptive one pion exchange model. The structure at small values of momentum transfer observed in the data is well described by the model.

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The absorptive one pion exchange model[1,2] (OPEA) makes some particular predictions for the small momentum transfer behavior of the reaction $\pi^- p \rightarrow \pi^+ \pi^- n$, quite different from those of the elementary one pion exchange model[3,4] (OPE) or form factor modified[5,6,7] OPE models. Specifically, a sharp rise is predicted in the forward differential cross section for transversely polarized rho mesons; this is also expected in the vector dominance model[8] (VDM). Furthermore, it has been pointed out that the behavior of $\pi^- p \rightarrow \pi^+ \pi^- n$ should be dominated at small momentum transfers by the propagator of the exchanged particle and by the minimal t-dependence required by angular momentum conservation.[2] Any other t-dependence is expected to be smooth and slowly varying. It is then possible to make definite predictions about the structure of the dipion matrix elements and the differential cross section for $-t < m_\pi^2$. In the special parameterization of P. K. Williams[9] the predictions of the model for the small t behavior of the density matrix elements permit a detailed comparison with our measurements of the reaction $\pi^- p \rightarrow \pi^+ \pi^- n$.

A detailed study of the reaction $\pi N \rightarrow \pi\pi N$ at small t is of further interest, because this process can in principle yield information on the elastic $\pi\pi$ cross section by extrapolating to the pion pole. However, the Chew-Low[4] method does not give a specific recipe for the extrapolation, and consequently a model is needed either to select a suitable variable, expected to be a smooth function of t, or even to prescribe a certain t-dependence. Recently the absorption model, as suggested by Kane and Ross,[10] has been used by Chan et al.,[11] to obtain the extrapolated $\pi\pi$ cross section. Since the model makes some striking predictions for the momentum transfer region $-m_\pi^2 \leq -t \leq m_\pi^2$, it is important to test the model in the physical region.

We have studied the reaction $\pi^- p \rightarrow \pi^+\pi^- n$ at 15 GeV using a wire spark chamber spectrometer at SLAC. In a recent publication[12] we have presented the results of our experiment on the $\pi^+\pi^-$ differential cross section and density matrix elements. In this report we compare the data with the predictions of the absorption model.

Pronounced structure in the density matrix elements in the helicity (H) or Gottfried-Jackson (GJ) reference frame[13] for small values of -t is expected in the absorption model. Below we list a number of predictions for the small t region ($-t < m_\pi^2$) given by M. Ross et al.[14] Prior to this experiment they have not been directly verified.

1. ρ_{11}^{GJ} and ρ_{11}^{H} $d\sigma/dt$ will have sharp forward peaks
2. ρ_{00}^{GJ} and $d\sigma/dt$ will have forward turnovers
3. ρ_{11}^{GJ} $d\sigma/dt$ will have no structure
4. $\rho_{1-1}^{H}/\rho_{11}^{H}$ behaves like $-t/m_\pi^2$
5. $\rho_{1-1}^{GJ} \approx 0$.

In Figs. 1 and 2a we present our measurements of the above quantities in the momentum transfer interval $0 \leq -t \leq .15$ GeV^2 for the dipion mass region $.665 \leq m_{\pi\pi} \leq .865$ GeV. In the presence of S-wave, it is not possible to determine directly the longitudinal (i.e., ρ_{00}) and transverse (i.e., $2\rho_{11}$) contributions to the ρ meson cross section, and hence the density matrix elements shown have been obtained by using additional information on the S-wave background, as discussed previously.[12] Our matrix elements are normalized such that $\rho_{00} + 2\rho_{11} + \rho_{00}^{S} = 1$, where ρ_{00}^{S} is the relative amount of S-wave.

Our results confirm all 5 qualitative predictions listed above. In order to make a more quantitative comparison between our data and the absorption model, we use the parameterization of P. K. Williams,[9] who predicts the magnitude and momentum transfer dependence of the pure rho density matrix elements. However, the measurable dipion matrix elements depend on the interference with the S-wave; the amount of S-wave and its relative phase are free parameters. In the context of this model the helicity amplitudes are assumed to possess a common t-dependence characterized by a form factor in addition to their individual dependence of $(-t)^{n/2}$ required by angular momentum conservation, where n is the net helicity change.[10] The form factor is supposed to represent the collimating effect of absorption and in the case under discussion is chosen to be of the form[9] $\exp\left(A(t-m_\pi^2)\right)$. By retaining only the nucleon helicity flip contribution and evaluating the amplitudes at the pion pole, the subsequent relations in the helicity frame can be obtained[9]:

$$\frac{d^2\sigma}{dt\,dm_{\pi\pi}^2} = \frac{\gamma(m_{\pi\pi})}{s^2 \cdot (t-m_\pi^2)^2} \cdot \xi(t) \cdot \exp\left(2A(t-m_\pi^2)\right)$$

with

$$\xi(t) = 1 + (x/2 + \epsilon)\cdot\delta + \delta^2$$

$$\rho_{00}-\rho_{11} = \frac{1}{2}\cdot(x\delta - \delta^2 - 1)/\xi(t)$$

$$\rho_{1-1} = \delta/\xi(t)$$

$$\mathrm{Re}\,\rho_{10} = \frac{1}{2}\left(\frac{1}{2}x\delta\right)^{1/2}\cdot(\delta-1)/\xi(t)$$

$$\mathrm{Re}\,\rho_{0S} = \cos(\phi)\cdot\delta\cdot\left(\frac{1}{2}x\epsilon\right)^{1/2}/\xi(t)$$

$$\mathrm{Re}\,\rho_{1S} = \frac{1}{2}\cos(\phi)\cdot(\delta-1)\cdot(\delta\epsilon)^{1/2}/\xi(t)$$

where $\delta = -t/m_\pi^2$ and $\gamma(m)$, A, ϵ, ϕ are free parameters.

The parameter x is related to the ratio of longitudinal to transverse rho mesons and assumes in P. K. Williams' model the value $x = (m_{\pi\pi}/m_\pi)^2$. The square of the total CMS energy for $\pi N \rightarrow \pi\pi N$ is denoted by s.

The parameters to be fitted are cos ϕ, ϵ, A, and $\bar{\gamma}$, where $\bar{\gamma}$ is the average value of $\gamma(m_{\pi\pi})$ in our mass interval. Note that $\rho_{00}-\rho_{11}$, ρ_{1-1} and Re ρ_{10} depend on only one free parameter, ϵ. In addition, this dependence is quite weak, since ϵ, a parameter describing the amount of S-wave background, is small compared to x. The interference terms of S and P wave, Re ρ_{0S} and Re ρ_{1S}, are scaled by cos ϕ, an S-P mixing parameter. The magnitude of $d^2\sigma/dtdm^2$ is given by $\gamma(m_{\pi\pi})$, whereas the slope of $d\sigma/dt$ is determined by the parameter A.

The resulting best fit is shown in Fig. 2 together with our measurement of $d\sigma/dt$ and the density matrix elements in the helicity system.

Since our sample of events from the reaction $\pi^- p \rightarrow \pi^+\pi^- n$ contains a (12±2)% contamination from $\pi^- p \rightarrow \pi^+\pi^-\Delta^0$, as discussed in Ref. 12, the actual fit shown in Fig. 2 was obtained allowing for a fixed spin nonflip contribution of this size. The parameters for the nonflip amplitudes were obtained from a study of our $\pi^+\pi^-\Delta^0$ events.

We are able to obtain a satisfactory fit to the data with a χ^2 of 65 for 73 degrees of freedom; the rich structure observed in the data is well described by the model. Specifically the change of sign of Re ρ_{10} and Re ρ_{1S} at $-t = m_\pi^2$ in the data is a simple consequence of the assumed t-dependence and the requirement of parity conservation for the amplitudes at the pion pole.[9]

The fit was performed in a restricted momentum transfer region, $0 \leq -t \leq .15\ \mathrm{GeV}^2$, since the model is not expected to describe the data adequately for larger values of -t. We obtain the following results for the free parameters: $\phi = 0 \pm 0.2$, $\epsilon = 1.8 \pm 0.1$, $\bar{\gamma} = 145 \pm 7$ and $A = 5.0 \pm 0.3$. We also fit with x a free parameter and the result is compatible with the value given in the model.[9]

The vector dominance model of Cho and Sakurai[15] also correctly predicts the ratio of transverse to longitudinal rho cross section. In addition they predict the observed structure in Re ρ_{10}.

As was pointed out above, the density matrix elements ρ_{00} and ρ_{11} cannot be measured directly, but have to be determined by making assumptions on the S-wave background. This model fit gives a determination of $d\sigma/dt$ for the S-wave (from $\rho^S_{00} = \epsilon\delta / \xi(t)$), which is in good agreement with the one used in Ref. 12.

Having determined the free parameters of P. K. Williams' model, we can calculate the on-shell $\pi\pi$ cross section $\sigma_{\pi\pi}$ as an additional check on the results.

$$\sigma_{\pi\pi} = \frac{\pi(4M^2 P_L^2/s^2)}{(G^2/4\pi)\, m_\pi^2\, P_1 m_{\pi\pi}}\, \bar{\gamma}\left(\frac{x}{2} + \epsilon - 2\right)$$

where P_L is the laboratory momentum of the incident pion, $G^2/4\pi = 14.6$ is the π-N coupling constant, P_1 is the momentum of the outgoing pion in the $\pi\pi$ CMS and M is the proton mass. We obtain an average cross section for the mass interval $.665 \leq m_{\pi\pi} \leq .865$ GeV of $\sigma_{\pi\pi} = (85 \pm 15)$ mb including our expected normalization error.[12] By taking the observed form[12] of the $\pi\pi$ mass distribution into account we derive a $\pi\pi$ cross section $\sigma_{\pi\pi} = (118 \pm 20)$ mb at $m_{\pi\pi} = .765$ GeV, which is consistent with the S and P wave unitary limit.

It is important to note that our data strongly indicate a nonzero forward cross section for $\pi^- p \to \pi^+\pi^- n$. In contrast to our results it has been often assumed in Chew-Low extrapolations that the differential cross section vanishes at t=0. However, our results indicate that $\pi\pi$ scattering cross section studies require the use of nonevasive extrapolation methods.[9,10,16,17]

To conclude, we have shown that the qualitative predictions of the absorptive one pion exchange model for the region of very small momentum transfer, $-t \leq m_\pi^2$, are in very good agreement with our experimental data. Furthermore, the special form of the model by P. K. Williams provides a good quantitative description of the density matrix elements and the differential cross section out to larger values of t ($-t \lesssim 10\ m_\pi^2$). The rich structure observed in the data can be explained in the absorption model by the minimum t-dependence, $(-t)^{n/2}$, required by angular momentum conservation.

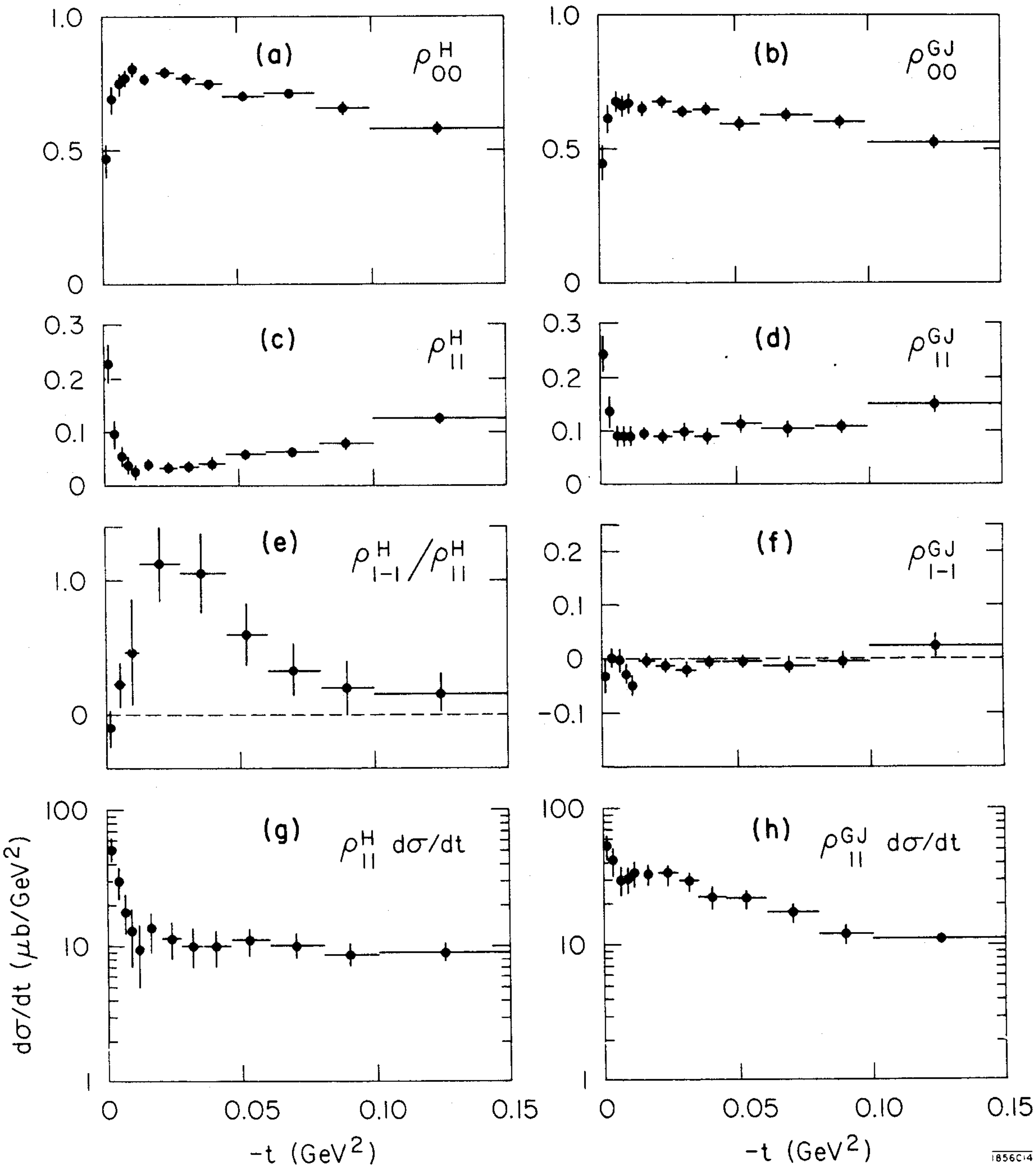
REFERENCES

1. K. Gottfried and J. D. Jackson, Nuovo Cimento 34, 735 (1964).
2. F. Henyey et al., Phys. Rev. 182, 1579 (1969).
3. C. Goebel, Phys. Rev. Letters 1, 337 (1958).
4. G. F. Chew and F. E. Low, Phys. Rev. 113, 1640 (1959).
5. H. P. Dürr and H. Pilkuhn, Nuovo Cimento 40, 899 (1965).
6. J. Benecke and H. P. Dürr, Nuovo Cimento 56, 269 (1968).
7. G. Wolf, Phys. Rev. 182, 1538 (1969).
8. J. J. Sakurai, Ann. Phys. (N.Y.) 11, 1 (1960) and Proc. of Int. School of Physics (Academic Press, New York, 1963), p. 41; D. S. Beder, Phys. Rev. 149, 1203 (1966).
9. P. K. Williams, Phys. Rev. D1, 1312 (1970).
10. G. L. Kane and M. Ross, Phys. Rev. 177, 2353 (1969).
11. L. Chan et al., Phys. Rev. D2, 583 (1970).
12. F. Bulos et al., Report No. SLAC-PUB-884, Stanford Linear Accelerator Center, submitted to Phys. Rev. Letters (1970).
13. K. Gottfried and J. D. Jackson, Nuovo Cimento 33, 309 (1964).
14. M. Ross et al., Nucl. Phys. B23, 269 (1970).
15. C. F. Cho and J. J. Sakurai, Phys. Rev. D2, 517 (1970).
16. C. D. Froggart and D. Morgan, Phys. Rev. 187, 2044 (1969).
17. J. D. Kimel, Phys. Rev. D2, 862 (1970).

FIGURE CAPTIONS

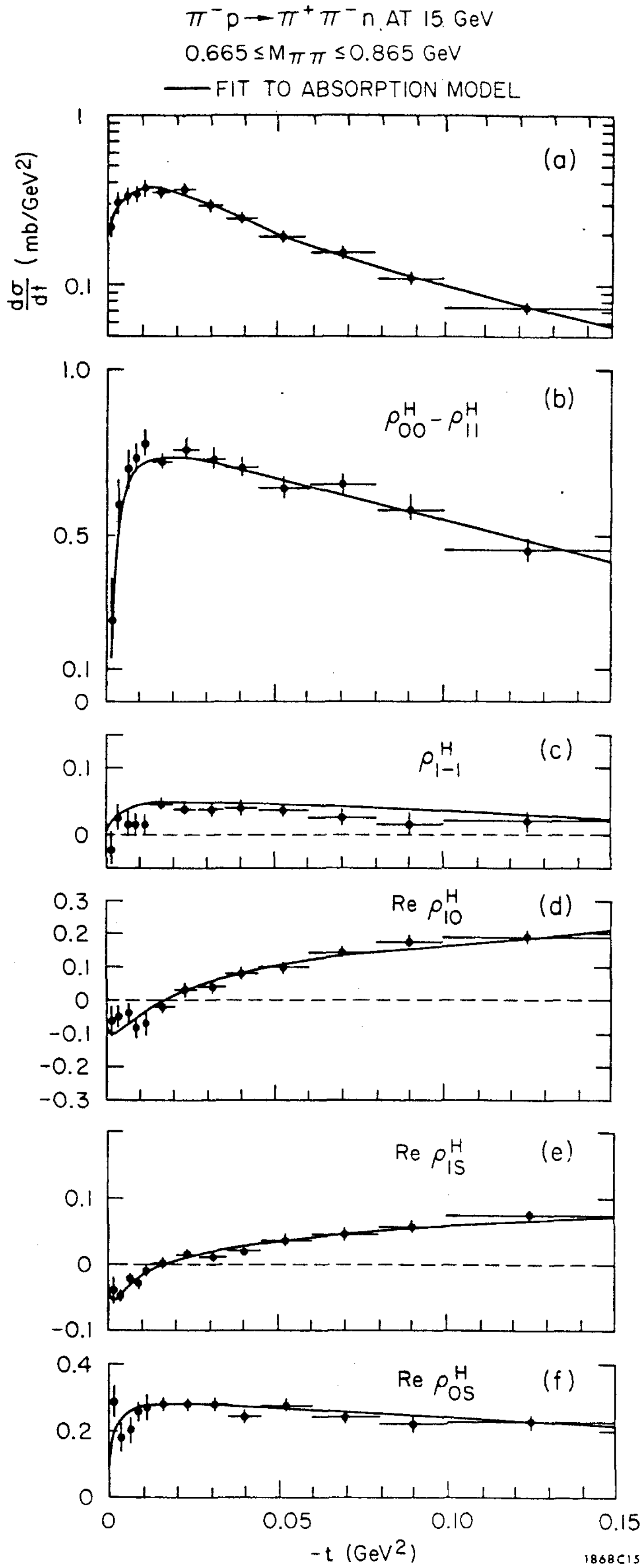
1. Density matrix elements expected to exhibit particular behavior for $-t < m_\pi^2$ in the helicity (H) and Gottfried-Jackson (GJ) frames as a function of momentum transfer for $\pi^- p \rightarrow \pi^+ \pi^- n$ in the dipion mass interval $.665 \leq m_{\pi\pi} \leq .865$ GeV. a) ρ_{00}^{H}, b) ρ_{00}^{GJ}, c) ρ_{11}^{H}, d) ρ_{11}^{GJ}, e) $\rho_{1-1}^{H}/\rho_{11}^{H}$, f) ρ_{1-1}^{GJ}, g) $\rho_{11}^{H}\, d\sigma/dt$, h) $\rho_{11}^{GJ}\, d\sigma/dt$.

2. The differential cross section, $d\sigma/dt$, and density matrix elements in the helicity frame for $\pi^- p \rightarrow \pi^+ \pi^- n$ with $.665 \leq m_{\pi\pi} \leq .865$ GeV as a function of t. The solid line represents a fit to the absorption model in the parameterization of P. K. Williams with additional terms allowing for a small nonspin-flip background. a) $d\sigma/dt$, b) $\rho_{00}^{H}-\rho_{11}^{H}$, c) ρ_{1-1}^{H}, d) Re ρ_{10}^{H}, e) Re ρ_{1S}^{H}, f) Re ρ_{0S}^{H}.



(a)
ρ^{H}_{00}
(b)
ρ^{GJ}_{00}
(c)
ρ^{H}_{11}
(d)
ρ^{GJ}_{11}
(e)
$\rho^{H}_{1-1}/\rho^{H}_{11}$
(f)
ρ^{GJ}_{1-1}
(g)
ρ^{H}_{11} dσ/dt
(h)
ρ^{GJ}_{11} dσ/dt
dσ/dt (μb/GeV^2)
-t (GeV^2)
-t (GeV^2)
1856C14

Fig. 1



$\pi^- p \to \pi^+\pi^- n$ AT 15 GeV
$0.665 \leq M_{\pi\pi} \leq 0.865$ GeV
— FIT TO ABSORPTION MODEL
$\frac{d\sigma}{dt}$ (mb/GeV^2)
(a)
(b)
$\rho^H_{00} - \rho^H_{11}$
(c)
ρ^H_{1-1}
(d)
Re ρ^H_{10}
(e)
Re ρ^H_{1S}
(f)
Re ρ^H_{0S}
1
0.1
1.0
0.5
0
0.3
0.2
-0.1
-0.2
-0.3
0.4
0.05
0.10
0.15
-t (GeV^2)
1868C15

Fig. 2