SLAC-PUB-907
April 1971
(TH) and (EXP)

BEHAVIOR OF THE ELECTROMAGNETIC INELASTIC STRUCTURE FUNCTIONS OF THE PROTON

J. I. Friedman, H. W. Kendall

Physics Department and Laboratory for Nuclear Science
Massachusetts Institute of Technology*
Cambridge, Massachusetts

E. D. Bloom, D. H. Coward, H. DeStaebler
J. Drees**, C. L. Jordan, G. Miller, R. E. Taylor

Stanford Linear Accelerator Center***
Stanford, California

ABSTRACT

The question of scaling of $2M_pW_1$ and νW_2 as a function of ω is discussed. Scaling is verified for a large kinematic range. Also, a new scaling variable which reduces to ω in the Bjorken limit is introduced which extends the scaling region. The behavior of σ_T and σ_S are also discussed as a function of ν and q^2. Various weighted sum rules of νW_2 are evaluated.

(Submitted to Phys. Rev. Letters.)

* Work supported in part by the Atomic Energy Commission under Contract No. AT(30-1)2098.

** Present address: Bonn University; Bonn, Germany.

*** Work supported by the U. S. Atomic Energy Commission

In the previous letter[1], new data taken at SLAC on inelastic electron-proton scattering at laboratory scattering angles of 18^o, 26^o and 34^o were reported. In that letter, hereafter referred to as I, these and 6^o and 10^o data obtained earlier at SLAC[2] were used to determine separately the absorption cross sections, σ_T and σ_S, for virtual photons with transverse and longitudinal polarizations, respectively. From these cross sections the structure functions W_1 and W_2 may be found directly. The definitions of the quantities and their relationships, having been previously discussed in detail[3], are noted in I.

If a single photon is exchanged between the electron and proton, then W_1 and W_2 will be functions of two kinematic variables, q^2 and ν, where q is the four momentum transfer from the electron and ν is the energy loss of the electron in the laboratory. From the results of the experiment at 6^o and 10^o combined with the assumption of a predominantly transverse electromagnetic interaction (that is, the ratio $\sigma_S/\sigma_T = R$ is small) it was found that νW_2 depended only on the ratio of q^2 and ν over a substantial range of the data[3]. This property is called "scaling", in the variable $\omega \equiv 2 M_p \nu / q^2$. Bjorken had predicted the possibility of this behavior in the asymptotic kinematic region reached by letting q^2 and ν go to infinity with ω held constant[4].

The separate determinations of σ_T and σ_S reported in I yield an average value of R equal to $0.18 \pm .10$ which supports the earlier assumption that led to scaling of the 6^o and 10^o data. In this letter we discuss the validity of scaling behavior in the light of these experimental determinations of R and, additionally, over the wider range of q^2 covered by the large angle data. In Fig. 1 the shaded area labelled "separation region" contains the kinematic locations of the points available for the separation studies. Within this region, W_1 and W_2 are separately determined without any assumption about the relative contributions from the transverse and longitudinal components of the cross sections. To investigate possible scaling behavior elsewhere in the full kinematic region of our data including the data at 6° and 10°, bounded by the heavy line in Fig. 1, values of R have been obtained by extrapolation of the measured values. In order to determine the sensitivity of our knowledge of νW_2 to variations in the method of extrapolation, we have employed three parametrizations of R, all consistentent with the measured values[1]. These are: $R = 0.18$, $R = 0.031 (q^2/M_p^2)$, and $R = q^2/\nu^2$. It has been found that the conclusions reported below concerning scaling behavior are insensitive to the choice among these forms.

Throughout the remainder of this paper a constant value of $R = 0.18$ has been assumed over the full kinematic region of measurement. With this assumption, each cross section yields values for W_1 and W_2. To test for scaling behavior it is useful to plot νW_2 for fixed ω as a function of q^2, or equivalently, as a function of W, the mass of the unobserved and final hadronic state. (W, q^2 and ω are related by $W^2 = 2M_p\nu + M_p^2 - q^2 = q^2(\omega-1) + M_p^2$ where M_p is the proton mass.) For constant ω, scaling behavior is exhibited in such a plot if νW_2 is independent of W(or q^2). Values of νW_2 are shown in Fig. 2, calculated from interpolations of radiatively corrected spectra measured at 6°, 10°, 18°, 26°, and 34°. The plots are presented for representative values of ω; $\omega = 1.5$, 2, 3, 6, and 12. Scaling behavior is not expected where there are observable resonance "bumps" because resonances occur at fixed W, not at fixed ω, nor is it expected for small q^2, because νW_2 cannot depend solely on ω in this limit. By inspecting the plots in Fig. 2 and other similar graphs we have come to a number of conclusions regarding the validity of scaling in several kinematic regions. These conclusions are summarized below; the regions, with the kinematic variables, are shown in Fig. 1.

1) For $4 < \omega < 12$

For $W > 2.0$ GeV and $q^2 > 1.0$ GeV^2, νW_2 is a constant within experimental errors and hence "scales" in ω (or, indeed, in any other variable). The range of kinematics for the measurements included in this test covers q^2 from 1 to 7 GeV^2 and values of W between 2 and 5 GeV.

2) For $\omega < 4$

In this region of ω the number of measurements of νW_2 above the resonance region is considerably increased by the large angle data. The experimental values of νW_2 scale for $W > 2.6$ GeV, but νW_2 appears to increase as W decreases below 2.6 GeV. This region covers kinematic ranges of W between 2.6 GeV and 4.9 GeV, and of q^2 between 2 GeV^2 and 20 GeV^2.

3) For $\omega > 12$

There are relatively few points above $q^2 = 1$ GeV^2 and no points above $q^2 = 2$ GeV^2, making it difficult to determine any variation of νW_2 with changing q^2. There are no measurements of R in this region, and the values of νW_2 are especially sensitive to variations in R. The large angle data have a maximum ω of 8 and influence the values of νW_2 for $\omega > 12$ only through the values of R determined in the low ω region. Scaling cannot be tested critically in this region, since the uncertainty in R prevents the large ω behavior of νW_2 from being known with assurance. If $R = 0.18$ is assumed, then for $q^2 > 0.8$ GeV^2, νW_2 decreases slightly as ω increases. However, for larger values of R, consistent with the extrapolated values, νW_2 is constant. Preliminary analysis of more recent data does not resolve these questions[5].

These conclusions, based on data which extend the q^2 range of previous νW_2 measurements and which include measurements of R, confirm the scaling behavior of νW_2 as a function of ω for $q^2 > 1$ GeV^2 as indicated by the earlier 6^o and 10^o data. Though these studies cover an extensive kinematic region, we would, however, give greater emphasis to conclusions based on data from the separation region in Fig. 1 where the analysis relies on interpolations between measured values of R.

As a byproduct of studying the behavior of the large angle data at small ω we discovered that for the whole range of ω the scaling region is extended from $W \approx 2.6$ GeV down to $W \approx 1.8$ GeV (which is approaching a resonance bump) if a new variable $\omega' \equiv \omega + a/q^2$ is used instead of ω. The constant a was determined to be 0.95 ± 0.07 GeV^2 by fitting the data with $W > 1.8$ GeV and $q^2 > 1$ GeV^2.[6] The quoted error on a was derived from the covariance matrix of the fit and does not include systematic errors or any contribution from the uncertainty in R. The statistical significance of a is greatly reduced in a fit to the data for $W > 2.6$ GeV implying that functions of either ω or ω' give satisfactory statistical fits to the data in this kinematic range. In what follows we use $a = M^2 = 0.88$ GeV^2 which gives $\omega' = \omega + M_p^2/q^2 = 1 + W^2/q^2$. Regarding ω' we note that: a) in the Bjorken limit ω' becomes equal to ω so the two variables have the same asymptotic properties, and b) in the kinematic region covered by our measurements, q^2 and ν may not be large in the sense of the Bjorken limit and parametrization in terms of some variable other than ω might have physical significance.

The question naturally arises whether the other structure function, W_1, also exhibits scaling behavior. A study of our results shows that, within errors, W_1 scales as a function of ω (or ω') over the same kinematic range as νW_2.

Figure 3a shows $2M_pW_1$ and νW_2 as functions of ω for $W \geq 2.6$ GeV, and Fig. 3b shows these quantities as functions of ω' for $W \geq 1.8$ GeV. The data presented in both figures are for $q^2 > 1\ GeV^2$ and use $R = 0.18$ in the evaluation of the points. The observe d scaling behavior in ω and ω' is impressive for both structure functions over a large kinematic region.

Since W_1 and W_2 are related by

$$\frac{2M_pW_1}{\nu W_2} = \frac{\omega}{1+R}\left[1+\frac{2M_p}{\omega\nu}\right] = \frac{\omega'}{1+R}\left[1-\frac{a}{\omega' q^2}+\frac{2M_p}{\omega'\nu}\right] \qquad (1)$$

it can be seen that scaling in W_1 accompanies scaling in νW_2 only if R has the proper functional form to make the right-hand sides of the equations functions of ω (or ω'). In the Bjorken limit, it is evident that νW_2 and W_1 will mutually scale if R is a constant or a function of ω (or ω'). The measured values of R are small, and are not sufficiently precise to determine its functional form.

Parametric fits of νW_2 to the values shown in Figs. 3a and 3b give

$$\nu W_2 = F(\omega) = \left(1-\frac{1}{\omega}\right)^3\left[1.274 + 0.5989\left(1-\frac{1}{\omega}\right) - 1.675\left(1-\frac{1}{\omega}\right)^2\right] \qquad (2)$$

and

$$\nu W_2 = F(\omega') = \left(1-\frac{1}{\omega'}\right)^3\left[0.6453 + 1.902\left(1-\frac{1}{\omega'}\right) - 2.343\left(1-\frac{1}{\omega'}\right)^2\right] \qquad (3)$$

For either ω or ω' less than two, νW_2 can be satisfactorily fit with a single cubic term, a result consistent with the threshold behavior of νW_2 predicted by models[7] which relate it to the elastic form factor. $2M_pW_1$ may be satisfactorily fit by $(1 - 1/\omega')^3$ for $1.2 < \omega' < 1.5$ and by $(1 - 1/\omega')^4$ for $1.5 < \omega' < 5$.

An equivalent but intuitively different picture of inelastic scattering is revealed in terms of the functions σ_T and σ_S which can be thought of as the total absorption cross sections for virtual photons of mass q^2 having transverse and longitudinal polarizations, respectively. In the limit as $q^2 \to 0$ one has

$\sigma_S \to 0$, and $\sigma_T \to \sigma_{\gamma p}$, the total photoabsorption cross section. Figure 4 shows the cross sections σ_T and σ_S plotted for constant q^2 as functions of W^2. The dashed lines indicate the W^2 dependence of $\sigma_{\gamma p}$. For $q^2 \leq 3$ GeV^2 the cross sections are consistent with a constant or a slowly falling energy dependence similar to the behavior of $\sigma_{\gamma p}$. For larger q^2, σ_T shows a rising energy dependence resembling a threshold-type behavior. This rising behavior of σ_T at high energy is unique among the various total cross sections that have been measured. The q^2 dependence of σ_T, shown in Fig. 5, shows no pure power law behavior but varies in the region of the present data between $1/q^2$ and $1/q^6$ as indicated by the straight lines shown in Fig. 5. The point $\omega' = 5$ roughly separates the threshold region of νW_2 from the flat, structureless region. The rising energy dependence of σ_T for large q^2 reflects the rising behavior of νW_2 for $\omega' < 5$. The $1/q^2$ dependence is correlated with the constancy of νW_2 for $\omega' > 5$, and the $1/q^6$ asymptotic dependence as ω' approaches unity corresponds to the asymptotic limit of the threshold behavior of νW_2 obtained using Eq. 3:

$$\lim_{\substack{q^2 \to \infty \\ W^2 \text{ constant}}} \nu W_2 = \frac{K}{4\pi^2 \alpha} (\sigma_T + \sigma_S) \propto \lim_{\substack{q^2 \to \infty \\ W^2 = \text{constant}}} \left(\frac{1}{1 + q^2/W^2} \right)^3 \propto \frac{1}{q^6}$$

The new data permit further investigation of sum rules involving νW_2 reported with the 6° and 10° data.[3] Using R = 0.18, interpolations of both the small and large angle data were used to determine νW_2 at a constant value of $q^2 = 1.5$ GeV^2. The evaluation of the integral in the Gottfried sum rule,[8] based on a non-relativistic point-like quark model of the proton gives

$$\int_1^{20} \frac{d\omega}{\omega} \nu W_2 = 0.78 \pm 0.04 \quad (4)$$

when integrated over the range of our data.

We have also evaluated the Callan-Gross[9] sum, which is related to the equal-time commutator of the current and its time derivative and which is also equal to the mean square charge per parton in parton models.[10] For this integral we find

$$\int_1^{20} \frac{d\omega}{\omega^2} \nu W_2 = 0.172 \pm 0.009 \quad (5)$$

which is about one half the value predicted on the basis of a simple quark model of the proton, and is also too small for a proton described by a quark model with three "valence" quarks, in a sea of quark-antiquark pairs.

Recently, Bloom and Gilman[11] have proposed a constant q^2 finite energy sum rule based on scaling in ω'[12] that equates an integral over νW_2 in the resonance region with the corresponding integral over the asymptotic expression for νW_2. They have pointed out that the applicability of the sum rule to spectra which have prominent resonances is indicative of a substantial non-diffractive component in νW_2. The sum rule requires that J_1 equal J_2 with

$$J_1 \equiv \left(\frac{2M_p}{q^2}\right) \int_0^{\nu_m} d\nu (\nu W_2)_{exp}$$

and

$$J_2 \equiv \left(\frac{2M_p}{q^2}\right) \int_0^{\nu_m} d\nu\, F(\omega') = \int_1^{\omega'_m} d\omega'\, F(\omega')$$

where $(\nu W_2)_{exp}$ is given by interpolation, at fixed q^2, of the 6° and 10° data with $R = 0.18$ and where $F(\omega')$ is given by Eq. 3. The upper limit is determined by choosing a missing mass W_m which is somewhat beyond the prominent resonance bumps whence $2M\nu_m = W_m^2 - M_p^2 + q^2$. We find that in the range of q^2 from 1 to 4 GeV^2 and W_m from 2.2 to 2.5 GeV the maximum deviation of J_2 from J_1 is 9%. This result is only weakly sensitive to modest changes in R.

Parton models and diffraction models have been suggested to explain the inelastic scattering results. The scaling behavior observed in these measurements arises naturally in simple parton models in which the proton is made up of point-like constituents, and, in general, diffraction models are not inconsistent with scaling. Preliminary analysis[5] of our recent measurements of inelastic electron-deuteron scattering suggest that there are differences between the inelastic electron-proton and electron-neutron cross sections, although further theoretical studies are required before these conclusions may be verified. Such differences would point to a substantial non-diffractive component of the deep inelastic cross section for values of ω less than approximately six.

REFERENCES

1. G. Miller, E. D. Bloom, G. Buschhorn, D. H. Coward, H. DeStaebler, J. Drees, C. L. Jordan, L. W. Mo, R. E. Taylor, J. I. Friedman, G. C. Hartmann, H. W. Kendall, R. Verdier. Previous paper hereinafter referred to as Paper I.
2. E. D. Bloom, D. H. Coward, H. DeStaebler, J. Dress, G. Miller, L. W. Mo, R. E. Taylor, M. Breidenbach, J. I. Friedman, G. C. Hartmann, H. W. Kendall, Phys. Rev. Letters 23, 930 (1969).
3. M. Breidenbach, J. I. Friedman, H. W. Kendall, E. D. Bloom, D. H. Coward, H. DeStaebler, J. Drees, L. W. Mo, R. E. Taylor, Phys. Rev. Letters 23, 935 (1969).
4. J. D. Bjorken, Phys. Rev. 179, 1547 (1969).
5. E. D. Bloom, *et al*, Report presented to the XV International Conference on High Energy Physics, Kiev, U.S.S.R. (September 1970). This report is available as SLAC-PUB-796.
6. Somewhat more discussion about ω' may be found in the dissertation of G. Miller which is available as SLAC-Report No. 129, January 1971.
7. S. Drell and T. M. Tan, Phys. Rev. Letters 24, 181 (1970).
   G. West, Phys. Rev. Letters 24, 1206 (1970).
8. K. Gottfried, Phys. Rev. Letters 18, 1174 (1967).
9. C. G. Callan, Jr. and D. J. Gross, Phys. Rev. Letters 21, 311 (1968).
10. J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975 (1969). See also F. J. Gilman in Proceedings of the 4th International Symposium on Electron and Photon Interactions at High Energies, Daresbury Nuclear Physics Laboratory, 1969.

11. E. D. Bloom and F. J. Gilman, Phys. Rev. Letters 25, 1140 (1970).

12. A family of finite energy sum rules based on scaling in ω relating the deep inelastic and resonance regions was proposed earlier by H. Leutwyler and J. Stern, Phys. Letters 31B, 458 (1970).

FIGURE CAPTIONS

1 The kinematic plane in q^2 and W^2 is shown. The heavy line bounds all data points measured at 6^o, 10^o, 18^o, 26^o and 34^o. The region marked "Separation Region" includes all points where data at 3 or more angles exist. Various values of ω are indicated with $\omega = \infty$ coinciding with the $q^2 = 0$ absissa and $\omega = 1$ corresponding to elastic scattering ($W^2 = .88$). Region I indicates the region where the data are consistent with scaling in $\omega = 2M_p\nu/q^2$. Region II indicates the extension of the scaling region if the data are plotted against $\omega' = 1 + W^2/q^2$. The ranges A, B, and C in the variable ω indicated in the figure are discussed in the text.

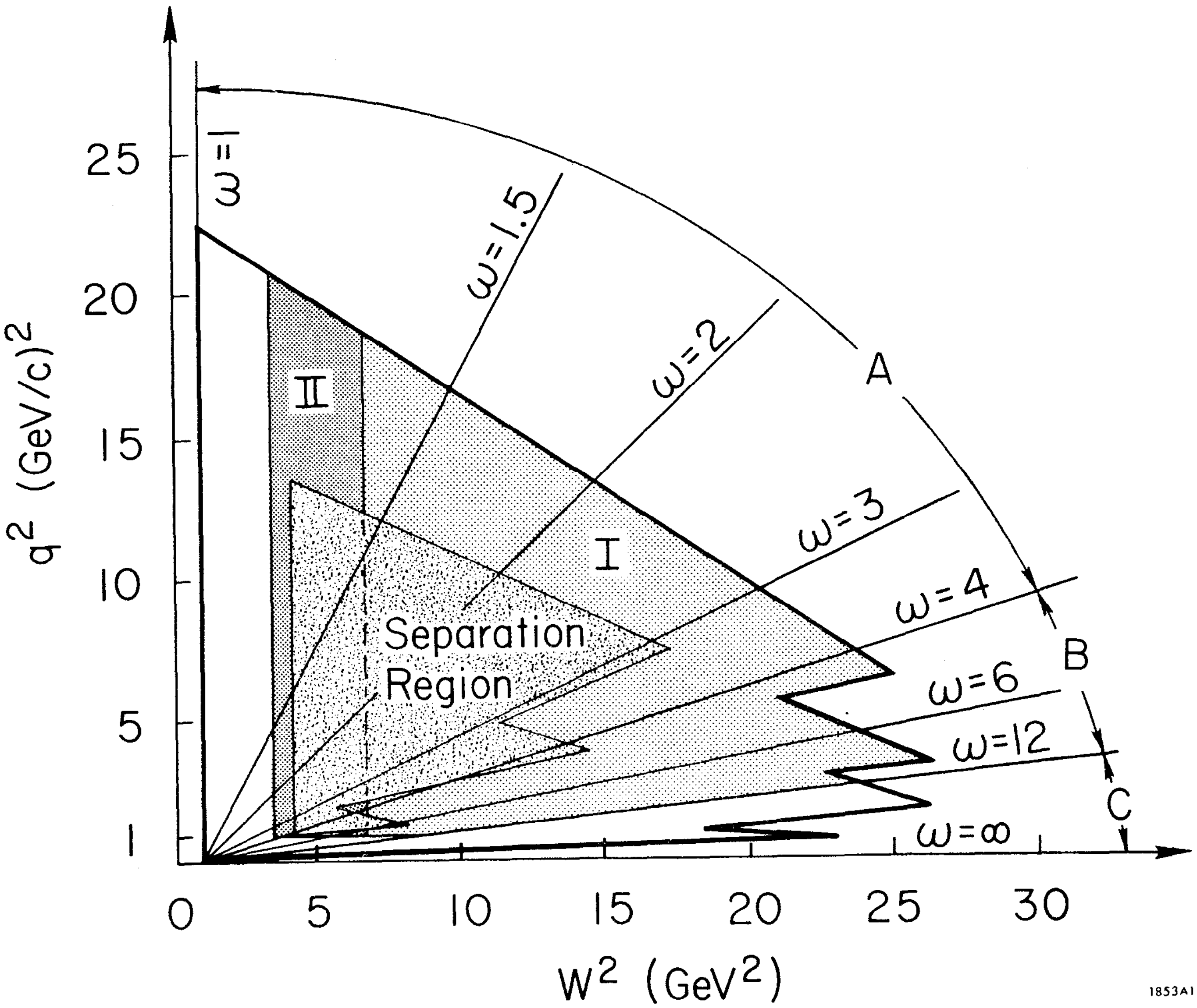
2 Interpolated data are shown for five values of $\omega = 1.5, 2, 3, 6, 12$. Scaling in ω would imply a constant value of νW_2 as W (or q^2) is varied. The curved solid line represents the fit to νW_2 as a function of ω' given in Eq. 3. Note that a graph with ω constant does not have ω' constant. The horizontal dashed line is the value of $F(\omega)$ from Eq. 2.

3a $2M_pW_1$ and νW_2 are shown as functions of ω for $R = 0.18$, $W > 2.6$ GeV and $q^2 > 1$ GeV^2.

3b $2M_pW_1$ and νW_2 are shown as functions of ω' for $R = 0.18$ and $W > 1.80$ GeV and $q^2 > 1$ GeV^2.

4 The 23 values of σ_T and σ_S given in I are shown at constant q^2 as a function of W^2 (or ν) for $q^2 = 1.5$, 3, 5, and 8 GeV^2. Also shown is the ν dependence of the total photoabsorption cross section.

5 The 23 values of σ_T given in I are shown at constant W as a function of q^2 for W = 2, 2.5 and 3.0 GeV. The solid line indicates a $1/q^2$ dependence and the dashed line represents a $1/q^6$ variation with q^2. The point $\omega' = 5$ is also indicated.



25
20
15
10
5
1
0
5
10
15
20
25
30
q^2 (GeV/c)^2
W^2 (GeV^2)
ω=1
ω=1.5
ω=2
ω=3
ω=4
ω=6
ω=12
ω=∞
A
B
C
II
I
Separation Region
1853A1

Fig. 1

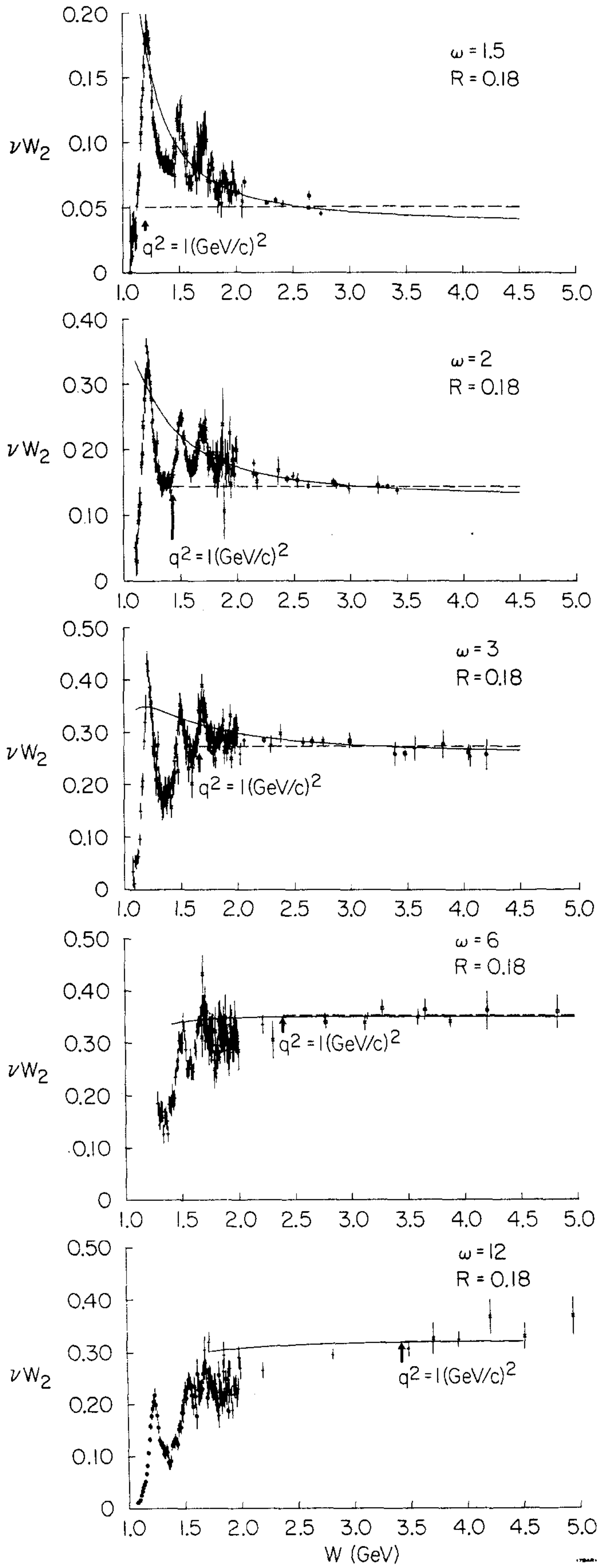
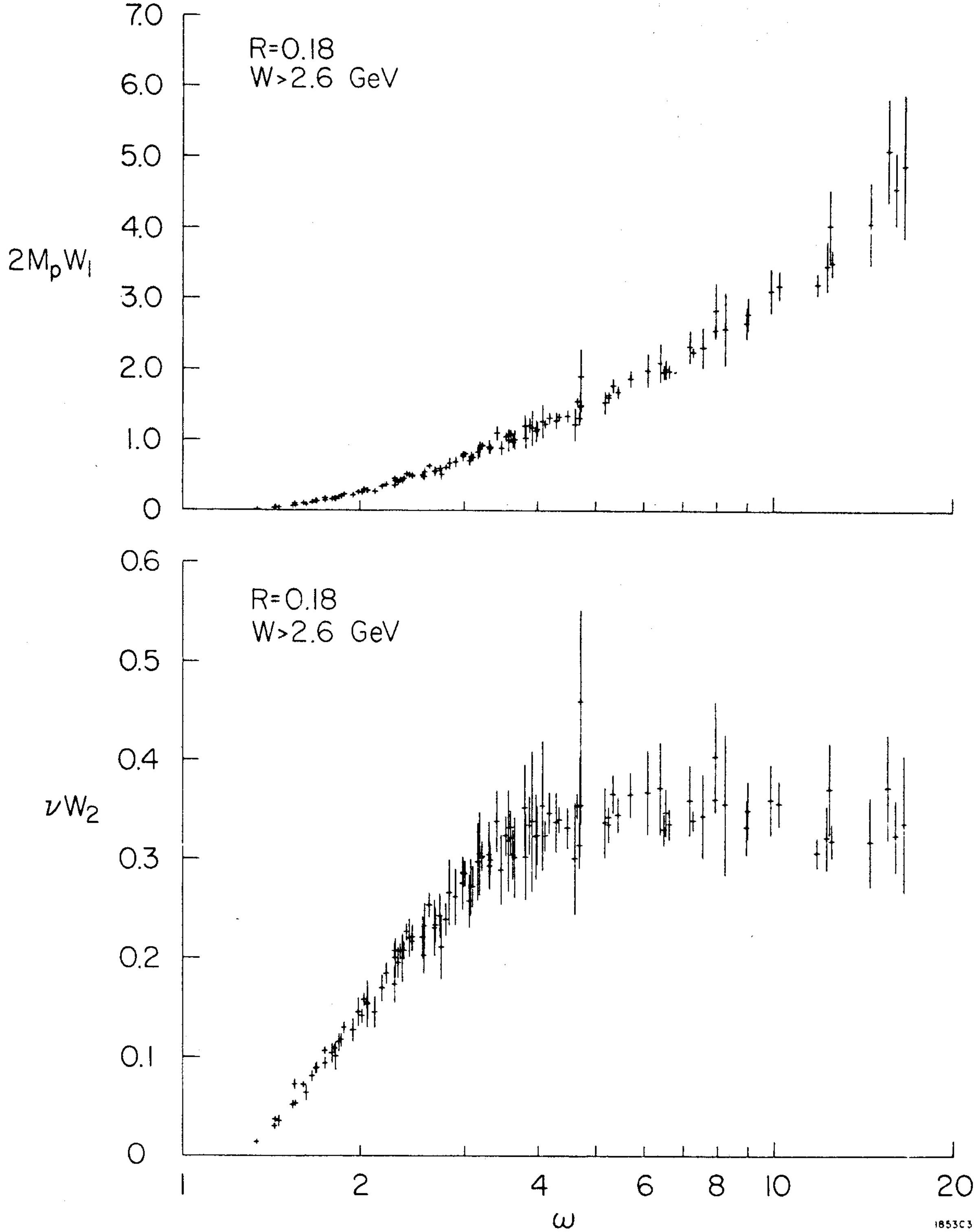


Fig. 2



7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
R=0.18
W>2.6 GeV
2MpWl
0.6
0.5
0.4
0.3
0.2
0.1
0
R=0.18
W>2.6 GeV
νW2
1
2
4
6
8
10
20
ω
1853C3

Fig. 3a

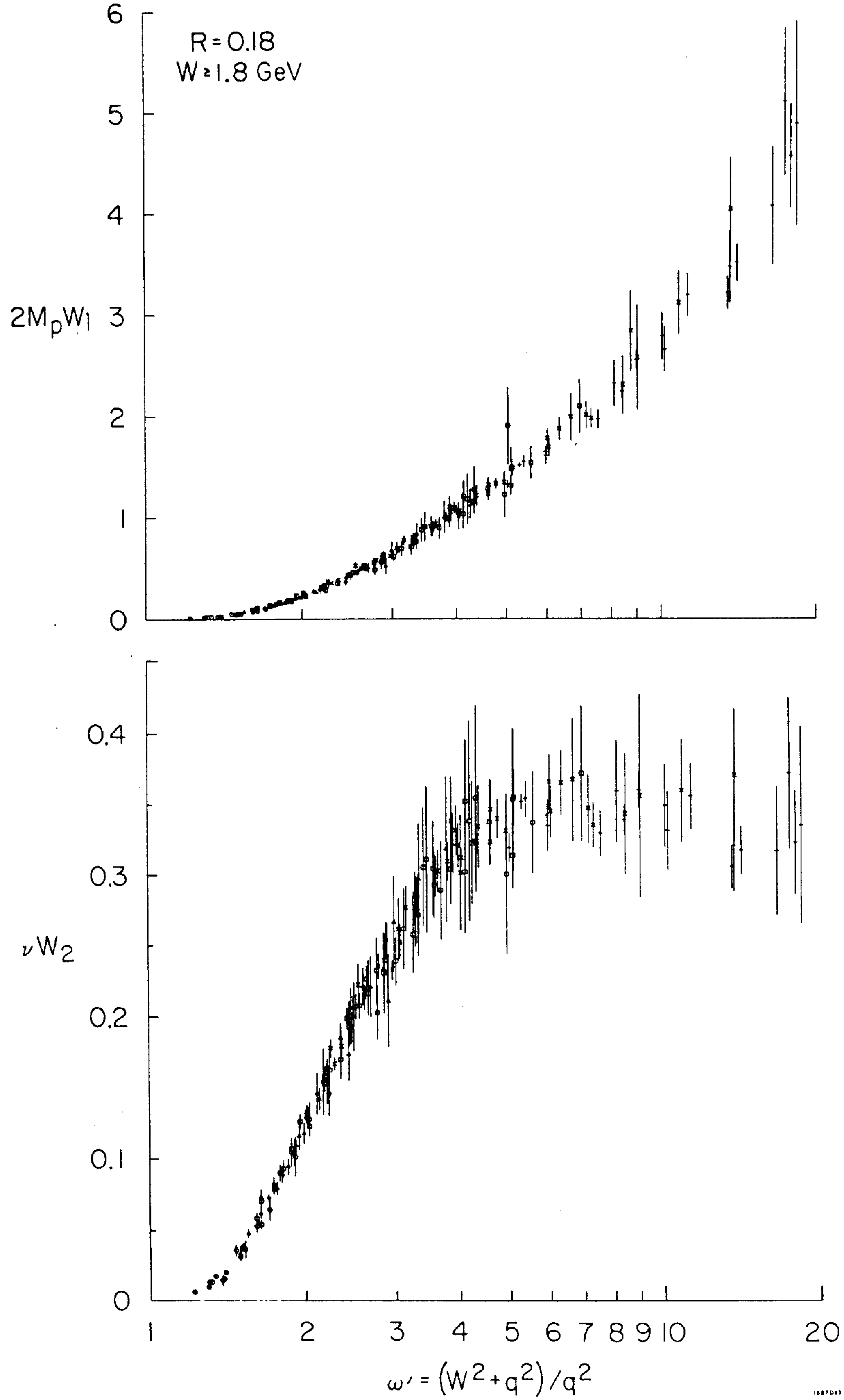
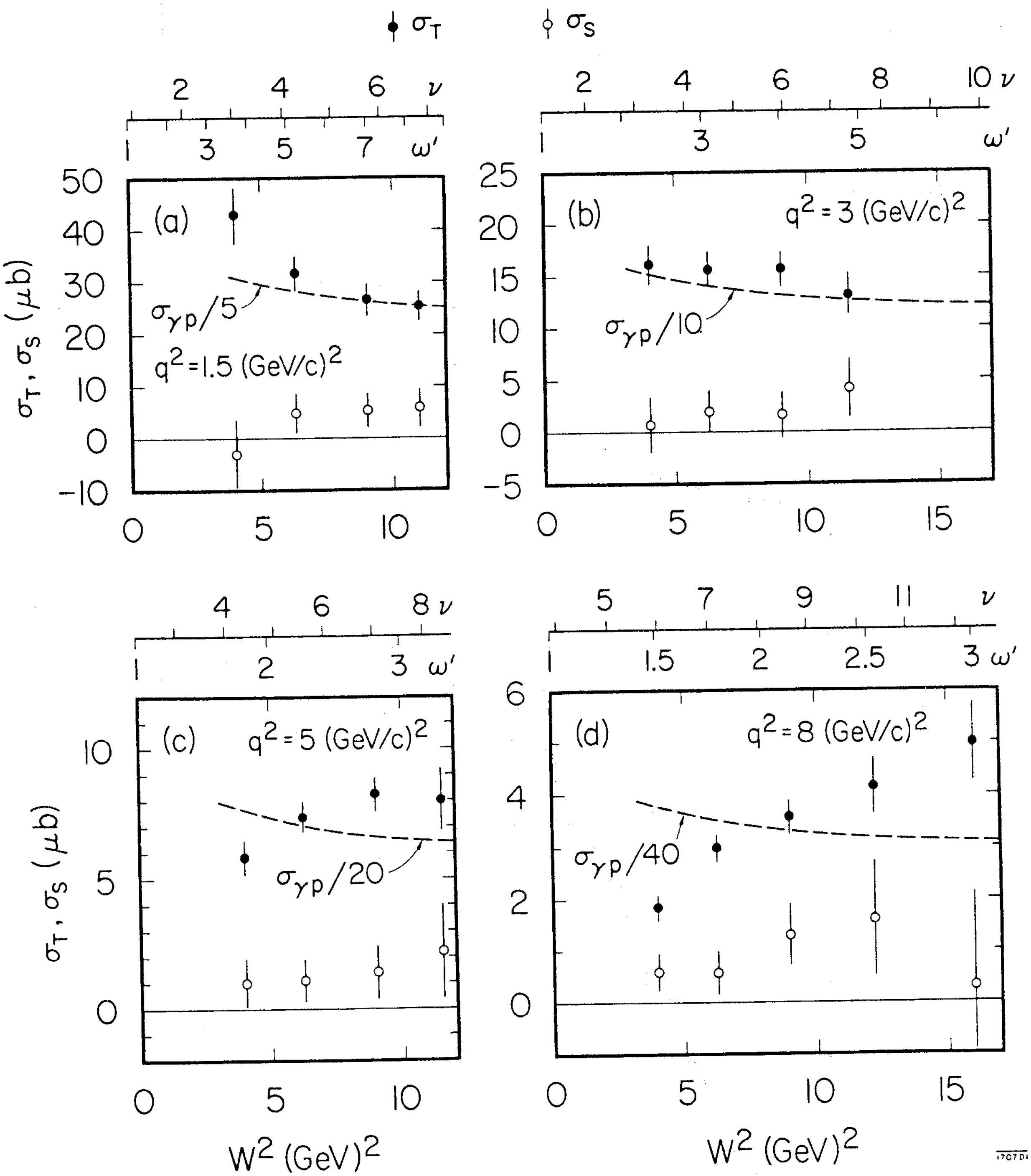

R = 0.18
W ≥ 1.8 GeV
2M_pW_1
0
1
2
3
4
5
6
νW_2
0
0.1
0.2
0.3
0.4
1
2
3
4
5
6
7
8
9
10
20
ω′ = (W² + q²)/q²

Fig. 3b



σT
σS
(a)
q² = 1.5 (GeV/c)²
σγp/5
(b)
q² = 3 (GeV/c)²
σγp/10
(c)
q² = 5 (GeV/c)²
σγp/20
(d)
q² = 8 (GeV/c)²
σγp/40
σT, σS (μb)
W² (GeV)²
ν
ω'

Fig. 4

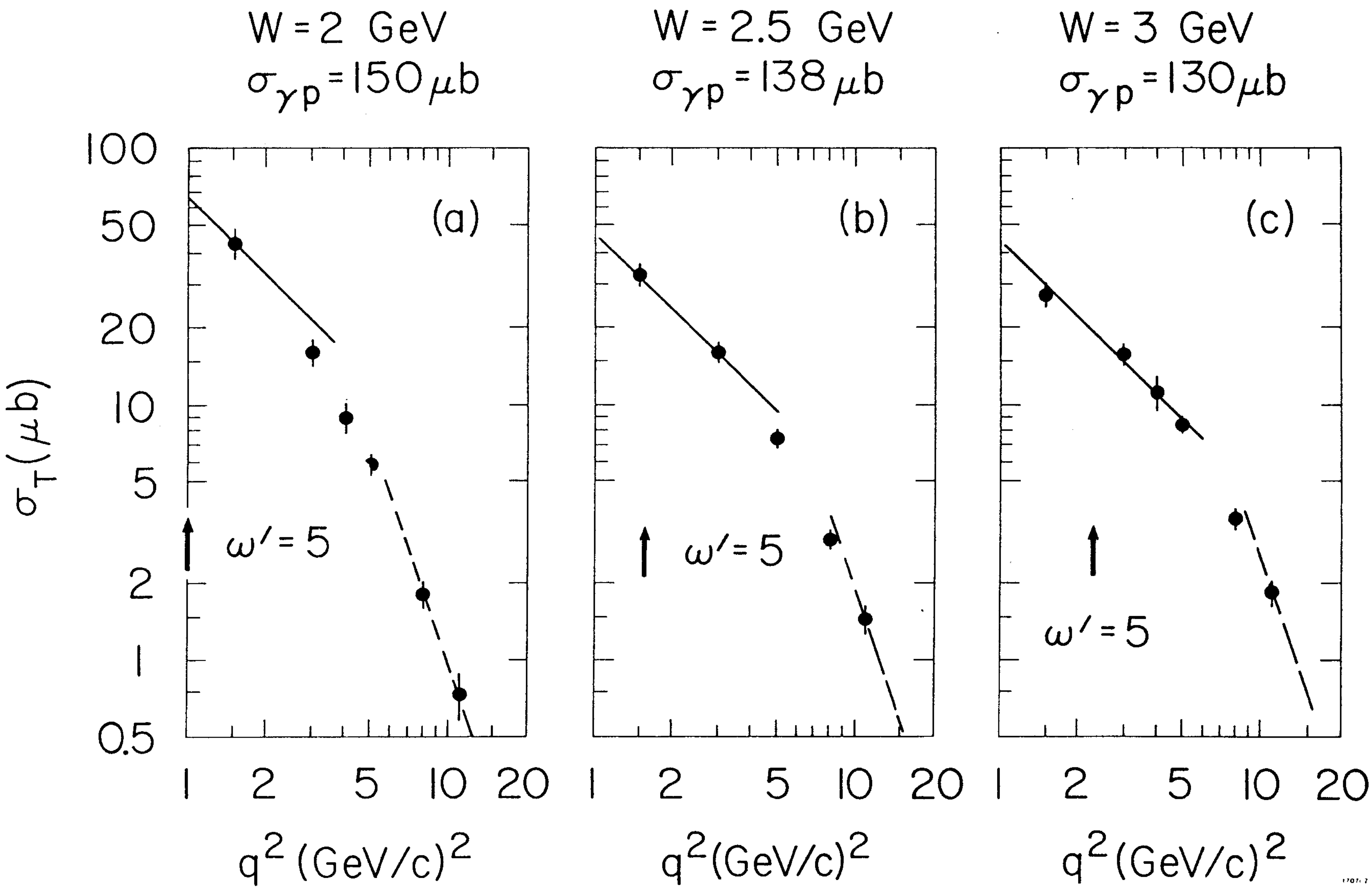

W = 2 GeV
σγp = 150 μb
W = 2.5 GeV
σγp = 138 μb
W = 3 GeV
σγp = 130 μb
σT (μb)
100
50
20
10
5
2
1
0.5
(a)
(b)
(c)
ω′ = 5
ω′ = 5
ω′ = 5
1 2 5 10 20
1 2 5 10 20
1 2 5 10 20
q2 (GeV/c)2
q2 (GeV/c)2
q2 (GeV/c)2

Fig. 5