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PARTONS*

James D. Bjorken
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

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PARTONS

Introduction

Parton is Feynman's word for supposed pointlike constituents residing within physical hadrons: the eigenstates of H_0. Their usefulness comes mainly from the simple interpretation of the scaling behavior found[1] in deep-inelastic electroproduction, and the simple and intuitive way[2] in which the local current-algebra sum rules may be interpreted. But the foundations of the concept are not that well-established, and beyond what has been done it is not clear what else there is to do with the partons. Indeed much of the original parton territory in the deep-inelastic phenomena is being co-opted by light-cone commutators,[3] but there is an area in which such more formal and precisely formulated concepts appear at present rather powerless and where prior experience is of limited value. This area concerns the question of the nature of the secondary-hadron distributions in the deep-inelastic processes of electroproduction, high-energy neutrino reactions, and e^+e^- annihilation into hadrons. Inasmuch as such processes are certain to be of experimental importance in the next few years even sloppy qualitative arguments which lean on the parton idea may be of help.

This discussion has three parts. We shall first illustrate the parton idea by using the example of quantum electrodynamics, where high energy scattering from external fields is very simply described by expanding the incoming state in terms of wave-functions for configurations of partons. The action of the external field on such configurations is very simple. We give the example of lepton-pair electroproduction which might teach some lessons regarding deep-inelastic hadron processes.

The second part concerns itself with the nature of such wave functions for hadrons. We discuss models such as Feynman's dx/x mixture of partons, multiperipheral wave-functions, and an oscillator wave function which produces something like the Veneziano formula. Finally in the third part we will try to apply these ideas to deep-inelastic processes.

I. Quantum Electrodynamics

Much of the monumental work of Cheng and Wu on high energy electrodynamics[4] can be simply reproduced by a formal theory suited to high energies.[5,6] The basic idea is to rotate the t and z coordinates onto the light-cone[7,8] and reformulate the theory in terms of the new variables $\tau = (t+z)/\sqrt{2}$ and $\zeta = (t-z)/\sqrt{2}$. Because the world lines of relativistic particles lie near the light-cone, the new time variable τ closely follows the proper evolution of the particle. In momentum space the rotated variables are

$$H = \frac{E-p_z}{\sqrt{2}} \qquad \eta = \frac{E+p_z}{\sqrt{2}} \tag{1.1}$$

and for a free particle

$$H = \frac{p_\perp^2 + m^2}{2\eta} = \frac{(\underset{\sim}{\sigma}\cdot\underset{\sim}{p}_\perp + im)(\underset{\sim}{\sigma}\cdot\underset{\sim}{p}_\perp - im)}{2\eta} \tag{1.2}$$

where the last form is appropriate for a Dirac particle.

The form of the energy-momentum relation suggests a nonrelativistic analogy[7,8]

$$\begin{aligned} H &\longleftrightarrow \text{"energy"} \\ \eta &\longleftrightarrow \text{"mass"} \end{aligned} \tag{1.3}$$

which goes deeper: Lorentz covariance implies invariance under "Galilean" transformations

$$\begin{aligned} \underset{\sim}{p}_\perp &\to \underset{\sim}{p}_\perp + \eta \underset{\sim}{v} \\ \eta &\to \eta \end{aligned} \tag{1.4}$$

Also Lorentz transformations in the z-direction are just scale-transformations

$$\begin{aligned} \eta &\to \eta e^{\omega} \\ H &\to H e^{-\omega} \\ \underset{\sim}{p}_\perp &\to \underset{\sim}{p}_\perp \end{aligned} \tag{1.5}$$

Finally, invariant phase-space is given by

$$\frac{d^3 p}{E} = d^2 p_\perp \frac{d\eta}{\eta} = d^2 p_\perp dR \tag{1.6}$$

where $R = \log \eta$ is the rapidity and merely displaces under longitudinal boosts: $R \to R + \omega$.

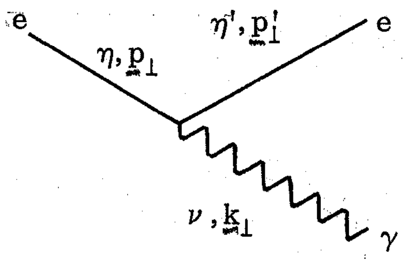
Returning to (1.2) the minimal substitution then properly introduces the electromagnetic field. A formal canonical theory exists,[5] and the wave functions of physical electrons and photons can be constructed order by order using old-fashioned Heitler perturbation theory with the simple vertices given in Figure I.

An external c-number electromagnetic field can also be introduced, and the high-energy limit of the scattering operator obtained in closed form.[6] Its action on partons is formally very simple:

1. No partons are created or destroyed.
2. No longitudinal momentum (η) is transferred by the potential.

Helicities

for $e \rightarrow e\gamma$

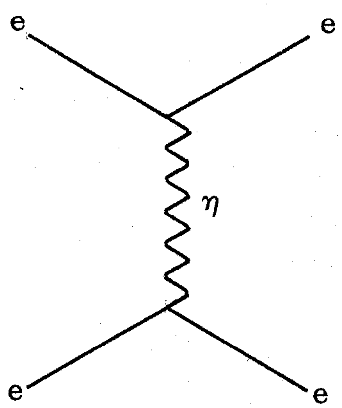


$$\pm \rightarrow \pm\pm : \quad H' = \frac{e}{\sqrt{2\nu}} \left(\frac{\mathbf{k}_\perp \cdot \boldsymbol{\epsilon}^*}{\nu} - \frac{\mathbf{p}'_\perp \cdot \boldsymbol{\epsilon}^*}{\eta'} \right)$$

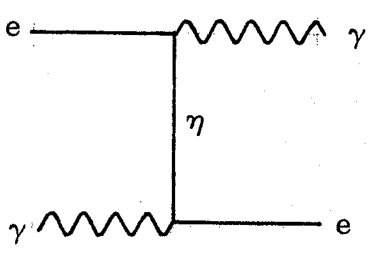
$$\pm \rightarrow \pm\mp : \quad H' = \frac{e}{\sqrt{2\nu}} \left(\frac{\mathbf{k}_\perp \cdot \boldsymbol{\epsilon}^*}{\nu} - \frac{\mathbf{p}_\perp \cdot \boldsymbol{\epsilon}^*}{\eta} \right)$$

$$\pm \rightarrow \mp\pm : \quad H' = \frac{e}{\sqrt{2\nu}} \; \frac{im}{\sqrt{2}} \left(\frac{1}{\eta} - \frac{1}{\eta'} \right)$$

$$\pm \rightarrow \mp\mp : \quad H' = 0$$



$$H' = \frac{e^2}{\eta^2} \begin{pmatrix} \text{no helicity flip} \\ \text{at either vertex} \end{pmatrix}$$



$$H' = \frac{e^2}{2\sqrt{\nu\nu'}} \; \frac{1}{2\eta} \begin{pmatrix} \text{neither electron nor photon} \\ \text{flips its helicity} \end{pmatrix}$$

Figure I: Vertices for infinite momentum QED.

3. The parton wave function in configuration space is modified only by a phase factor $e^{i\chi(\underset{\sim}{x}_\perp)}$, essentially a generalized Coulomb phase shift.

These features are the reason why the parton basis is a natural and attractive one for high-energy quantum electrodynamics.

There are many problems with the formal theory which have not been fully met, and high-order effects involving "wee" partons (those with $\eta \lesssim m$) lead to a more complex picture than what is given above.[9] Indeed most of the trouble can be traced to the question of the proper role of wee partons. Rather than trying to resolve such questions by further study of quantum electrodynamics, we shall instead speculate directly about their role in hadron processes.

Before leaving quantum electrodynamics, we consider electroproduction of a μ-pair from an external field as an example of how the parton calculations work. As shown in Figure 2 (leaving out the electron vertex) we must calculate the wave

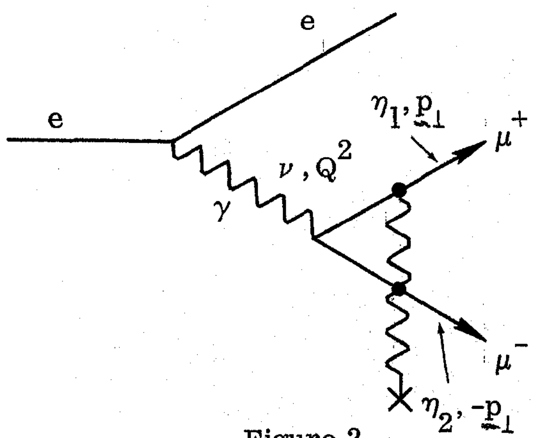


Figure 2

function $\Psi(x_1, x_2; \eta_1 \eta_2, Q)$ describing the μ-pair in the virtual photon. The forward virtual Compton amplitude is then

$$T(Q) \sim \int d^2x_1 d^2x_2 d\eta_1 \, \Psi^*(x_1, x_2, \eta_1) \left\{ e^{i[\chi(x_1)-\chi(x_2)]} - 1 \right\} \Psi(x_1, x_2, \eta_2) \tag{1.7}$$

where the longitudinal momenta η_1 and η_2 of the pair must sum to the photon longitudinal momentum ν. The wave function Ψ is easily computed from first order perturbation theory and the rules in Figure 1

$$\psi = \frac{1}{\Delta H} V \mid \gamma > \tag{1.8}$$

where, in momentum-space, V is obtained from Figure 1: $V \sim \underset{\sim}{p}_\perp/\eta_1$ or $\underset{\sim}{p}_\perp/\eta_2$ for a transverse photon. The energy denominator is

$$\Delta H = \frac{p_\perp^2 + m^2}{2\eta_1} + \frac{p_\perp^2 + m^2}{2\eta_2} - \left(\frac{-Q^2}{2\nu}\right) = \frac{Q^2}{2\nu} + \frac{\nu}{2\eta_1\eta_2}(p_\perp^2 + m^2) \tag{1.9}$$

When a Fourier transform is performed, and the longitudinal fraction of the muon $\eta_1 = \nu y$ is introduced, this all reduces to

$$\begin{aligned} T \longrightarrow \frac{i\alpha}{\pi^2} \int_0^1 dy\, y^2 \int d^2x_1 d^2x_2 \left| \underset{\sim}{\epsilon} \cdot \underset{\sim}{\nabla} K_0 \left(\sqrt{m^2 + Q^2 y(1-y)} \mid \underset{\sim}{x}_1 - \underset{\sim}{x}_2 \mid \right) \right|^2 \\ \times \left\{ e^{i[\chi(x_1)-\chi(x_2)]} - 1 \right\} \qquad (\nu \longrightarrow \infty) \end{aligned} \tag{1.10}$$

Thus in addition to being simple, this result shows that the important transverse distances in the wave-function of the μ-pair are small, of order $1/\sqrt{Q^2}$, provided neither member of the pair has a small longitudinal momentum η.

Suppose this feature, which is essentially kinematical, holds for the hadronic component of the wave function of a virtual photon. Then if electroproduction of hadrons in the "diffractive" region ($\omega = 2M\nu/Q^2 >> 1$) proceeds by a vector-dominant mechanism, we expect the size of the virtual photon to be $\sim 1/\sqrt{Q^2}$ and

1. The t-distribution for $e + p \rightarrow e + p + \rho^o$ should broaden as Q^2 increases up to a factor 4, but not more.[11] The net impact parameter decreases by a factor 2 as Q^2 increases and the photon becomes smaller.

2. Perhaps the target is less excited by a virtual photon than a real photon because the virtual γ makes a smaller hole as it goes through.

3. The cross-section of the virtual hadrons on a nucleon is up to 4 times smaller at large Q^2 than at $Q^2 = 0$ because of the decreasing impact parameter. Therefore in nuclear matter the mean free path of the hadrons in the virtual photon should be up to 4 times greater than usual. Consequently, the ratio

$$r = \frac{\sigma(e + (A) \rightarrow e + (A) + \rho^o)}{\sigma(e + p \rightarrow e + p + \rho^o)} \qquad (1.11)$$

should increase as Q^2 increases, because a thicker layer of nucleons in the nucleus contribute to the coherent production.

II. Hadron Wave Functions

In turning to lepton-hadron collisions, it is tempting to apply the same parton picture. For example, in electroproduction we expect that when both projectiles have large and opposite momenta only the Coulomb interaction between the partons in the electron (for this application, just the bare electron itself) and the partons in the hadron is important, and that at large transverse momentum transfer the scattering is incoherent. What we need, then, are answers to the following questions:

(a) What are the properties (e.g. charge, spin, SU(3)) of partons in hadron physics?

(b) What are the wave functions of hadrons in terms of the partons?

For hadron-hadron collisions, the problem is even worse. We must add a third question

(c) What is the parton-parton interaction?

The study of the deep-inelastic electroproduction structure-functions W_1 and νW_2 give some clues:

1. First of all the scaling phenomenon[12]

$$\nu W_2 \equiv F(\omega) \qquad \omega = 2M\nu/Q^2$$

is nicely compatible with the basic view of electrons scattering incoherently from the pointlike partons.

2. $F(\infty) > 0$ implies a logarithmically infinite number of partons in an infinite-momentum nucleon.

3. The smallness of σ_S/σ_T (0.2 ± 0.2) suggests spin $\frac{1}{2}$ for the partons.

4. The smallness of νW_2 $(\lesssim 0.35)$ suggest either a small mean-square charge per parton $(\lesssim 1/6)$ or that the picture fails.

All this points to quarks as the most favorite parton, but that is far from conclusive. The kindergarten calculations leading to these conclusions[12, 13] need not be elaborated here. I will only mention a different way of looking at them. Instead of tossing partons into ordinary phase-space d^3p, one may try to put them into covariant phase space. In infinite-momentum variables, this means $d^3p/E = d^2p_\perp \frac{d\eta}{\eta}$. Suppose there is one kind of parton, and the differential probability for finding n partons is (ignoring for the moment the dependence on transverse variables)

$$dP_n \sim Z\, g^n \frac{d\eta_1}{\eta_1} \cdots \frac{d\eta_n}{\eta_n}\, \delta\left(\sum_{i=1}^{n} \eta_i - \eta\right) \tag{2.1}$$

For normalization purposes the η_i are restricted to be larger than η_{min}. Evidently Z depends on η_{min} (and η). But as $\eta_{min} \to 0$, a pleasant calculation gives a sensible result .

$$\nu W_2 = g Q^2 (1-x)^{g-1} \qquad x = 1/\omega \tag{2.2}$$

where Q is the parton charge. With $g \sim 2$ and $Q^2 \sim 1/6$ νW_2 is very roughly fit. If one puts, in addition to these "cloud" partons, k valence partons $\{j\}$ with momentum-spectrum $\eta_j^{\beta_j - 1}\, d\eta_j$ and charge Q_j, νW_2 becomes, after another pleasant calculation

$$\nu W_2 = g Q^2 (1-x)^{g+\beta-1} + \sum_{j=1}^{k} Q_j^2\, x^{\beta_j} (1-x)^{g+\beta-\beta_j-1} \left[\frac{\Gamma(\beta_j)\Gamma(g+\beta-\beta_j)}{\Gamma(g+\beta)}\right]^{-1} \tag{2.3}$$

$$\text{where} \quad \beta = \sum_{j=1}^{k} \beta_j$$

I write this down because the answer has a beta-function in it, and this is a conference on duality. Kuti and Weisskopf[14] have found nice fits to the data with three valence quarks + a cloud of $q\bar{q}$ pairs and an equal number of neutral gluons mixed in.

But in either case, the fact that $\nu W_2 \to$ const as $\omega \to \infty$ seems to be correlated with a $d\eta/\eta$ distribution of the partons. Can we construct wave functions with this property? We give two examples:

1. The multiperipheral model provides such a wave function. It is most easily viewed in transverse configuration-space and longitudinal momentum space. Then the diagram in Fig. 3 implies (in old-fashioned perturbation theory as $\eta \to \infty$) that $\eta > \eta_1 > \eta_2 \cdots > \eta_n$

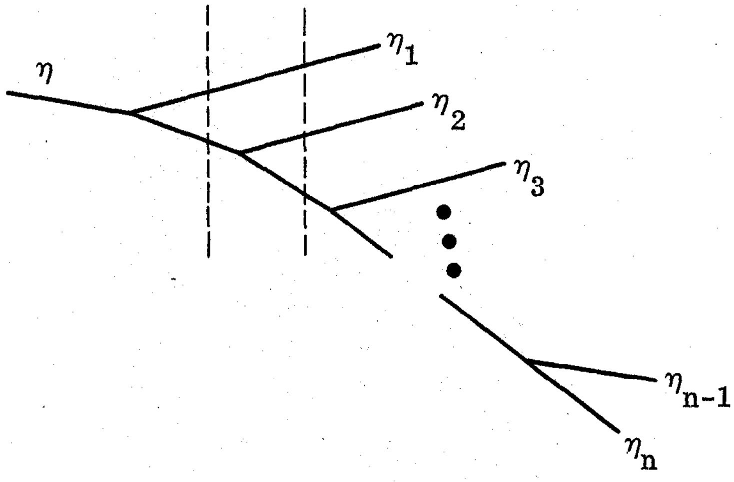


Figure 3: Diagram for multiperipheral wave function.

If $\eta_1 \gg \eta_2 \gg \eta_3 \ldots$, etc., the situation is especially simple. The parton 1 is the source for parton 2, which is the source for parton 3, etc. Neglecting spin, the dependence on η_i in Ψ factors out and the wave function is just proportional to

$$\Psi \sim g^n K_0 \left(\mu \left|x_1 - x_2'\right)\right. K_0 \left(\mu \left|x_2 - x_3\right|\right) \ldots K_0 \left(\mu \left|x_{n-1} - x_n\right|\right) \qquad (2.4)$$

where μ is the mass of the exchanged parton. The K_0 factors again are just the Fourier transforms of the energy-denominators shown in Figure 3 by the dotted lines. Introducing an external field and allowing only the wee parton n to interact with it gives, of course, the Regge trajectory of the multiperipheral model, but the point we want to make here is the physical picture which emerges from the non-relativistic analogy. Recalling that $\eta \longleftrightarrow$ mass, we have an ordering of the partons in η-space. The wave function requires only that neighbors in η be within a distance $|\underset{\sim}{x}_\perp| \lesssim \mu^{-1}$ of each other. It is something like a planetary system: the "lightest" parton n orbits about the next lightest parton n-1, the pair orbit about n-2, all three of which orbit about (n-3), etc. Or one may think of the system as a two-dimensional chain (Figure 4)

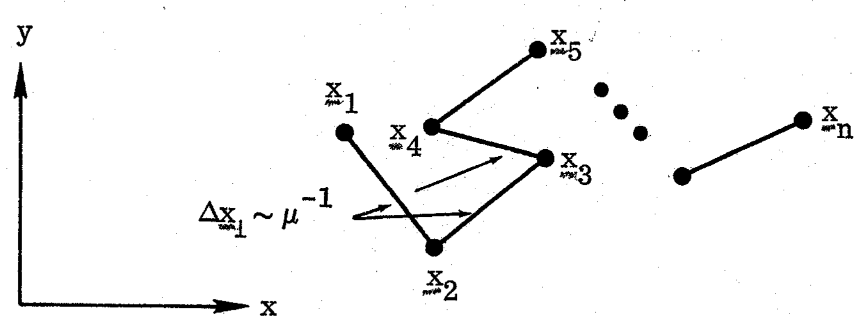


Figure 4: Multiperipheral wave function.

whose average length is the mean multiplicity of partons ($\bar{n} \cong g \log \eta$) for the multiperipheral wave-function. The radius of the hadron is then the distance between parton 1 and parton n, which by random walk is proportional to $\sqrt{\bar{n}} \sim \sqrt{\log \eta}$. Thus the diffraction peak would be expected to shrink logarithmically with energy.

Now the multiperipheral wave function has what would appear to be some unrealistic properties. It presumes the source of parton 2 is a point parton. In the physical nucleon we would expect the situation to be more complex and the effective source-function to be nonlocal. Furthermore neighboring partons with comparable η have low subenergies; such partons would be expected to interact most strongly with each other.

All this suggests constructing a different kind of wave function for the nucleon, for which we suppose:

a) The energy associated with fluctuation in parton number may be neglected compared to the interaction-energy between neighboring partons.

b) Only nearest neighbors in η-space are correlated by a potential $V(\underset{\sim}{x}_i - \underset{\sim}{x}_{i\pm1})$, which is attractive and which may be approximated by a harmonic well.

So for the "energy" H of a hadron we write*

$$H = \frac{M^2}{2\eta} = \sum_{i=1}^{n} \frac{\underset{\sim}{p}_i^2 + m_i^2 + \omega^4 (\underset{\sim}{x}_i - \underset{\sim}{x}_{i+1})^2}{2\eta_i} \tag{2.5}$$

and in order to solve simply for the spectrum we go to the continuum limit and introduce a field

$$\underset{\sim}{x}(\eta) = \underset{\sim}{x}_i \quad \text{and} \quad \underset{\sim}{\pi}(\eta) = \rho(\eta) \underset{\sim}{p}_i$$

* This form guarantees invariance under "Galilean" boosts and z-boosts, but is not fully Lorentz covariant.

where

$$[x(\eta),\ \pi(\eta')] = i\delta(\eta-\eta') \tag{2.6}$$

and $\rho(\eta)$ is the density of partons $dN/d\eta$. Then H becomes

$$H = \frac{M^2}{2\eta} = \int_0^{\eta} d\eta' \rho(\eta') \frac{\left\{\pi^2(\eta') + \omega^4 \left(\frac{\partial x}{\partial \eta'}\right)^2\right\}}{2\eta\rho^2(\eta')} + \text{constant} \tag{2.7}$$

Thus if and only if $\rho(\eta) = g\eta^{-1}$, the hamiltonian reduces to that of a uniform violin-string, where the string-coordinate is η. With introduction of creation and destruction operators a_ℓ in the standard way, we get linearly rising trajectories.

$$M^2 = \frac{4\pi\omega^2}{g} \sum_\ell \ell a_\ell^+ a_\ell + \text{constant} \tag{2.8}$$

Now suppose we let the string in the ground state absorb and emit a small amount of momentum from some external agent. We are then invited to calculate the scattering amplitude

$$T(q_1, q_2) = \frac{1}{2\eta} \sum_n \langle 0 | e^{iq'\cdot x(0)} | n \rangle \frac{1}{E_n - E_0 - H(q_0)} \langle n | e^{-iq\cdot x(0)} | 0 \rangle \tag{2.9}$$

Here we have assumed the momentum is absorbed by the "wee" partons of low η. We also do not allow $\underset{\sim}{q}$ to become large because it is clear that the physical picture is that of a collective excitation where the partons are moving coherently relative to each other. The energy denominator ΔH is

$$\Delta H = \sum_\ell \frac{\mu^2 \ell a_\ell^+ a_\ell - 2\eta H(q_0)}{2\eta} = \frac{\mu^2 \Sigma \ell a_\ell^+ a_\ell - s + M^2}{2\eta} \tag{2.10}$$

where to get the last form we use

$$s = (P + q)^2 \approx 2P\cdot q + M^2 \qquad \mu^2 = \frac{4\pi\omega^2}{g} \tag{2.11}$$

and

$$P\cdot q = P_0 q_0 - P_3 q_3 \approx \eta\left(\frac{q_0 - q_3}{\sqrt{2}}\right) = \eta H(q_0) \tag{2.12}$$

The rest of the calculation is then isomorphic to the operator formalism of the Veneziano model,[15,16,17] and the Veneziano scattering amplitude emerges in the same way.

$$T = \int_0^1 dx\,(1-x)^{-gt/2\mu^2}\, x^{-s/\mu^2 + \text{const.}} \tag{2.13}$$

where we put $t = q_\perp^2 + q_\perp'^2 - 2\mathbf{q}_{1\perp}\cdot\mathbf{q}_{2\perp} \approx -2\mathbf{q}_{1\perp}\cdot\mathbf{q}_{2\perp}$.

The only difference is that the formalism is of two-dimensional oscillators with positive metric rather than four-dimensional oscillators of indefinite metric.

I am not convinced that this is just an infinite momentum reformulation of the fishnet-diagram methods.[18,19] For example, it is not clear the above method is covariant. [Infinite-momentum techniques must ultimately cope with the infamous "angular condition".[20] This we have not done.] Secondly, hard partons are at one end of the string and wee at the other. I know of no fishnet enthusiast who likes that. I am, in fact, not sure that when quantum numbers are put in sensibly, the model is dual.

III. Dynamics in the Deep-Inelastic Region

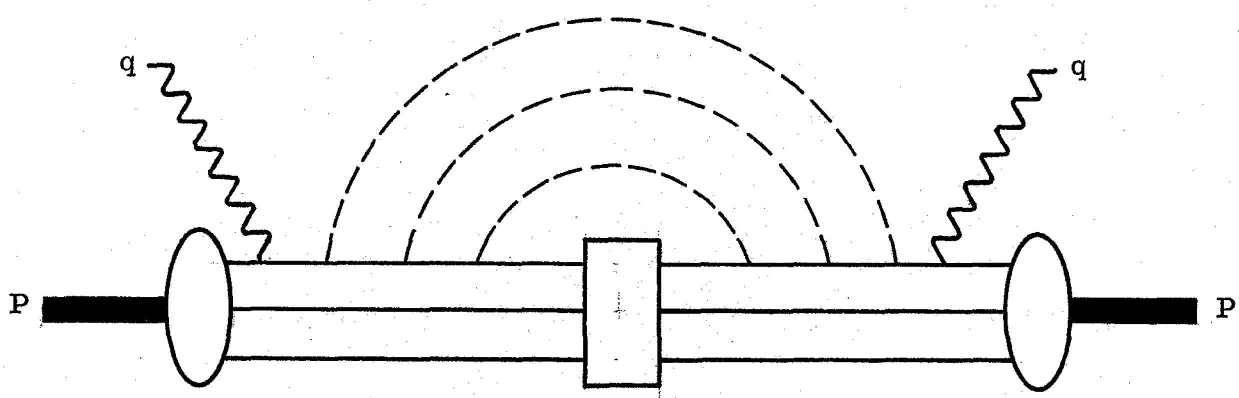
When measurement of the secondary hadrons are made for deep-inelastic phenomena, what is it we will want to know? Any given exclusive process most likely vanishes in the scale limit. After all the only channels surviving for $s \to \infty$, Q^2 small are elastic scattering and diffraction dissociation. So only such channels would seem to have a chance in the limit of large s and Q^2, unless some other channel cross-sections grow with increasing spacelike Q^2 as a power of Q^2 at fixed s. The best candidate in electroproduction for an exclusive process that scales would therefore be the $\rho^o p$ channel. However, the failure of ρ-dominance to account for the Q^2-dependence of the transverse photoabsorption cross-section $\sigma_T(Q^2)$ for large ω is not encouraging. In the colliding beam process, it has been proposed[21] that the pion is a parton and the channel $e^+ + e^- \to \pi^+\pi^-$ scales. This hypothesis faces the problem of why the pion doesn't make a large contribution to $\sigma_S(Q^2)$ in electroproduction. But, aside from the possibilities mentioned, probably the measurement of inclusive variables is most relevant theoretically as well as most accessible experimentally. These include single particle distributions of π, K, p, and the Q^2 and ν dependence of multiplicity and $< p_\perp >$. Any striking differences from what occurs in hadron phenomena might give a clue as to the nature of the underlying dynamics. We consider the various processes in turn.

A. Deep-Inelastic Electroproduction and Neutrino Processes; ω not large.

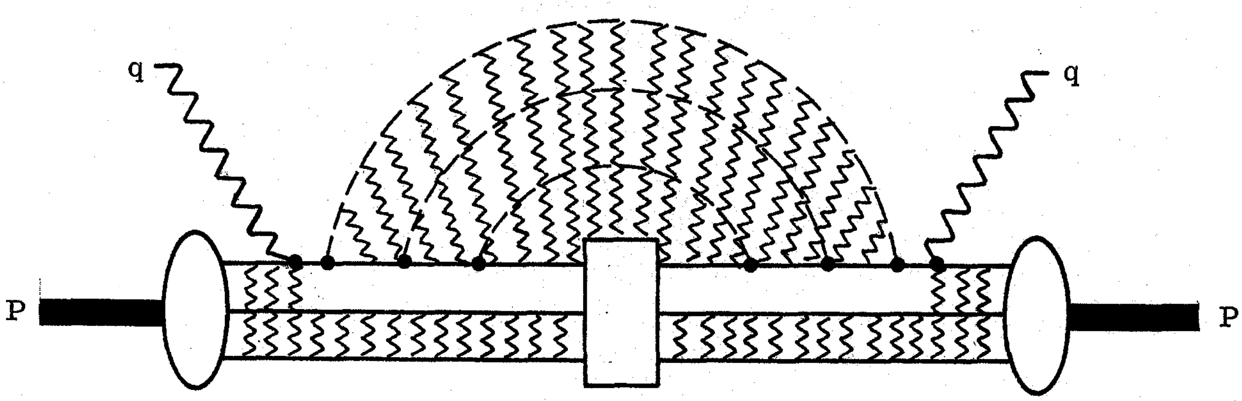
When viewed in the laboratory frame, the virtual hadron state immediately after the lepton has hit the parton is composed of a parton of momentum q_μ and small mass, plus a residual nucleon composed of wee partons. The struck parton then has large subenergy relative to any or all of the residual partons. What happens to this state? Let us assume the parton is a quark. Then it certainly doesn't get free of the nucleon. But it must behave as a free particle for those times and distances appropriate to the kinematics in order that the impulse-approximation not be violated. In terms of distances this probably means that the longitudinal distance is $\nu/Q^2 = \omega/2M = \omega \times 10^{-14}$cm. and the transverse distance is $\sim 1/\sqrt{Q^2}$ characteristic of the virtual Compton process.[22] Where does the quark go? We propose that the quark loses its initial momentum ($\approx \nu$) by successive emission of gluons or $q\bar{q}$ pairs of low invariant mass. In each such emission, the quark loses a finite fraction (say $\sim\frac{1}{2}$) of its initial 4-momentum. This means that the distribution of emitted glue is distributed with a $d\eta/\eta$ longitudinal momentum spectrum. When the quark becomes wee (in infinite momentum language very "light") it rapidly diffuses back to the target partons and pairs off with its residual antiquark.

In order for this process to make sense, the time (and distance) scale for emission of the first gluon is large: $\ell \sim (\text{const})\nu$. Emission of the subsequent gluons does not appreciably change this estimate (one sums a geometrical series: ℓ for the first emission, $\sim \ell/2$ for the second, $\sim \ell/4$ for the third, etc.). One sees that the multiplicity of gluons will go as $\log\nu$ (or better $\log s$, independently of Q^2). We conjecture that the observed hadron multiplicity would likewise follow the gluon distribution and be $\sim \log s$, much like in ordinary processes.

An important question is the distribution of transverse momentum in the emitted gluons and subsequent hadrons. If no cutoff is put in, the picture above is very similar to that of the renormalizable theories with the "rainbow" diagrams summed (Figure 5a). The calculations which have been made[23] destroy scaling behavior and are not a promising interpretation of the real world. When a transverse-momentum cutoff is imposed, Drell, Levy, and Yan find[24] a more palatable behavior. Where should this cutoff originate? After all we are discussing coupling of partons to partons and it seems wrong to make such a coupling nonlocal. A possible origin lies in the picture adopted for the "Veneziano" wave function in Section 2. There we assumed two time scales, the first, relatively



(a)



(b)

Figure 5: Two pictures of parton de-excitation for virtual Compton scattering.

long, being that associated with fluctuation of parton number (such as gluon emission) and the second, relatively short, associated with the elastic force between partons with comparable longitudinal momentum η. Therefore, there should in this picture be a large amount of parton-gluon and gluon-gluon interaction between successive gluon emissions. These interactions may well enhance the low subenergies (i.e., low $p_\perp$) and result in a secondary distribution of hadrons from the parton much like the distributions found (or expected) in strong interactions. Diagrammatically the situation is as in Figure 5b.

B. e^+e^- Annihilation into Hadrons; Decay of W into Hadrons

Given the hypothesis of partons, it is folklore[21] that the annihilation first produces a parton-antiparton pair with the point cross-section

$$\frac{\sigma_{tot}(e^+e^- \to \text{hadrons})}{\sigma_{tot}(e^+e^- \to \mu^+\mu^-)} = \sum_{\substack{\text{parton}\\ \text{species } i}} Q_i^2 \tag{3.1}$$

The parton-antiparton pair initially has large subenergy and independently emit their gluons. Two "jets" of hadrons are formed with probably low $p_\perp$ relative to the direction of the initial parton-antiparton pair. The longitudinal momentum distribution obeys Feynman scaling,[25] and the multiplicity of hadrons $\bar{n}$ is $\sim \log E_{CM}$.

C. Deep-Inelastic Electroproduction and Neutrino Processes; ω Large.

When ω is large it is possible the vector-dominant mechanism may be important. In the parton picture we visualize this as analogous to the μ-pair electroproduction; the incident virtual photon materializes into a hard parton-antiparton pair each of which then diffraction scatters from the essentially passive target. Indeed, coherent production from a nucleus separates out this mechanism the best. The parton-antiparton pair at production has a large transverse momentum; on the average $p_\perp \sim \frac{1}{\Delta x_\perp} \sim \sqrt{Q^2}$ owing to the "smallness" of the virtual photon, as discussed in Section 1. Then the parton materializes into hadrons with small $< p_\perp >$ relative to the parton direction which is not the virtual photon direction. In the laboratory frame of reference, the distribution of hadrons at sufficiently high energy is that of two jets with mean angle $< \theta > \sim \sqrt{Q^2}/\nu$ relative to the virtual photon direction (Figure 6). In terms of coordinates taken parallel and perpendicular to the virtual photon direction, this leads to large $< p_\perp >$ of the

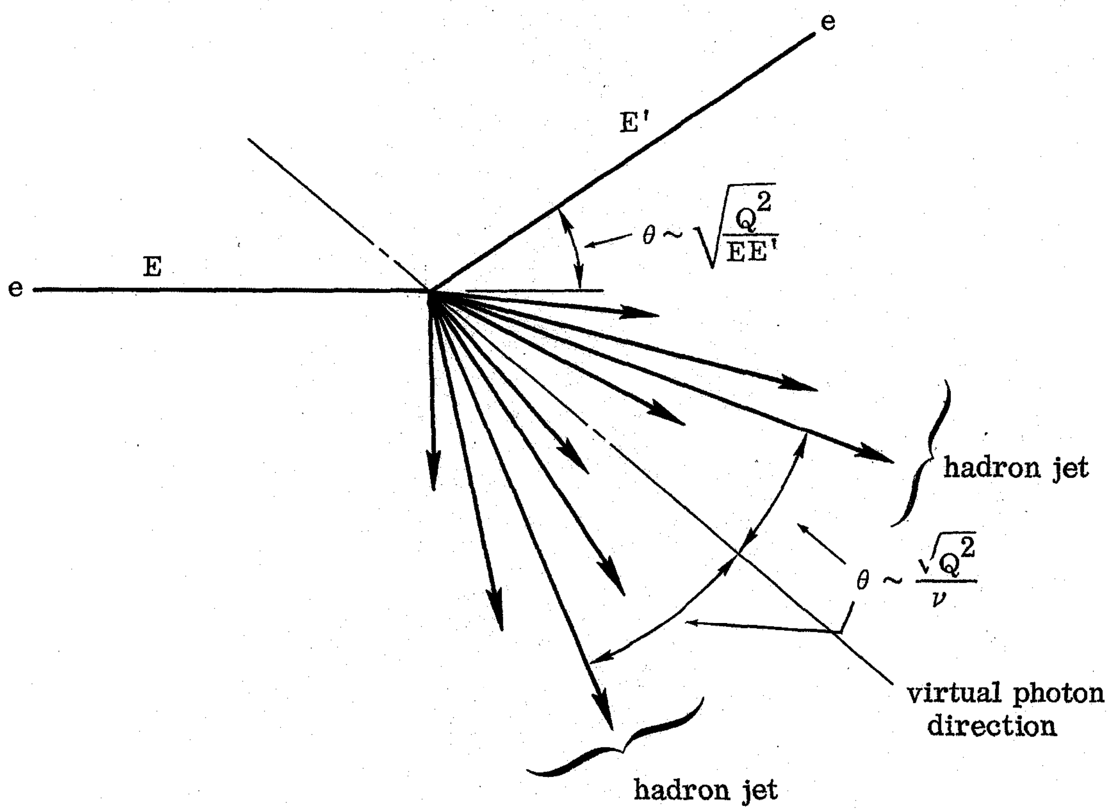


Figure 6: Configuration of possible "double-jet" deep-inelastic process.

outgoing hadrons:

$$<p_\perp^2> \approx <p_\perp^2>_{Q^2=0} + \lambda \left(\frac{p_\parallel}{\nu}\right)^2 Q^2 \cos^2 \phi \tag{3.2}$$

where the parameter λ is approximately constant and of order unity, and ϕ is the angle between the plane of the leptons and the plane defined by the momenta of the observed hadron and the virtual photon.

IV. Conclusions

While most of the above words are probably nonsense, it may be that some of the general consequences are still true. In particular, the hypothesis that the distribution of leading hadrons depends only on the internal quantum numbers and four-momentum of the parent parton is applicable to all deep inelastic processes in which the struck or created parton is isolated from all neighbors in phase space. Under such circumstances, we find the corresponding hadron distributions if we know

(1) The production cross-section for a parton of type i and momentum p, which is supposed to be given by the customary free-particle kindergarten calculations.

(2) The probability $\frac{dx}{x} f_{ij}(x)$ for finding a hadron of type j and of momentum xp in dx which is created from the parton i.

The cross section, in any frame where the hadron has large momentum, is then

$$\frac{d\sigma}{dp_j d\Omega} \approx \sum_i \int_{p_j}^{\infty} \left(\frac{d\sigma_i}{dp_i d\Omega}\right) f_{ij} \left(\frac{p_j}{p_i}\right) \frac{dp_i}{p_j} \tag{4.1}$$

The other conjectures for the various processes are summarized in the table. One sees that the diffractive, vector-dominant consequences look the most bizarre, and it may be that if such a mechanism exists, electroproduction or neutrino production from nuclei (which enhances such effects) may be of special interest.

Table 1

Process	Multiplicity	P_L Distribution	$p_\perp$ Distribution	Other
$e+N \rightarrow e+\text{hadrons}$ $\mu+N \rightarrow \mu+\text{hadrons}$ $\nu+N \rightarrow \mu+\text{hadrons}$ $\gamma+N \rightarrow \gamma+\text{hadrons}$ $\omega < 4$	$\bar{n} \sim \log s$	much like hadron induced processes $\frac{d\sigma}{dp_L} \sim \frac{1}{p_L} f\left(\frac{p_L}{\nu}, \omega\right)$	$\langle p_\perp \rangle$ small (relative to $\underset{\sim}{q}$) is perhaps preferred.	
$e^+e^- \rightarrow \text{hadrons}$ (via 1γ) $W \rightarrow \text{hadrons}$	$\bar{n} \sim \log Q^2$	jets; along the jet axis, much like hadron distributions. For large p, $\frac{d\sigma}{dp_L} \cong \frac{d\sigma}{dp} \sim \frac{1}{p} f\left(\frac{p}{\sqrt{Q^2}}\right)$	$\langle p_\perp \rangle$ small relative to jet axis again is perhaps preferred.	The f's for these processes are closely related by the hypothesis that they depend only on the distribution of partons ejected formula (4.1)].
$e+N \rightarrow e+\text{hadrons}$ $\mu+N \rightarrow \mu+\text{hadrons}$ $\nu+N \rightarrow \mu+\text{hadrons}$ $\gamma+N \rightarrow \gamma+\text{hadrons}$ $\omega \gg 4$	$\bar{n} \sim \log Q^2$	Because $n \ll \log s$, this looks like diffraction dissociation $\frac{d\sigma}{dp_L} \sim \frac{1}{p_L} f\left(\frac{p_L}{\nu}, \omega\right)$ $f(0,\omega) \rightarrow 0$ for large ω	Double-jets: relative to $\underset{\sim}{q}$ large $p_\perp$ for leading hadrons $\langle p_\perp^2 \rangle \approx \langle p_\perp^2 \rangle_{Q^2=0} + \lambda \frac{p_\parallel^2}{\nu^2} Q^2$ Low $\langle p_\perp \rangle$ relative to axes of the two jets.	Broadening of diffraction-peak for $ep \rightarrow ep\rho^0$ $\nu p \rightarrow \mu p \begin{cases} \rho^+ \\ A1^+ \\ K^{*+} \\ Q^+ \end{cases}$ Enhancement of coherent-production of ρ, A1, K^*, Q from nuclei relative to nucleon production [Eq. (1.11)].

V. Acknowledgements

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26. Right or wrong, this hypothesis has considerable predictive power, most notably in providing mechanisms for populating the high-$p_\perp$ regions of phase-space for all high-energy processes. Sam Berman, John Kogut, and I are now making a systematic survey of the consequences for γ, lepton and hadron-initiated processes.