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Partons and Deep Inelastic Processes at High Energies

Deep inelastic electron scattering is a process that is theoretically attractive. The electron is a *known* and *weak* probe. Moreover, if we choose the kinematics in the appropriate way, we can make direct use of the impulse approximation. In studying those other composite systems—the atom and the nucleus—the natural starting point is to analyze the bound state in terms of its constituents, viz electrons and nucleons, respectively. Then, if we consider a scattering process in which we specify the kinematics, so that these constituents can be treated as instantaneously free during the sudden pulse carrying a large energy transfer from the projectile, we can neglect the effects of their binding during the interaction and we can treat the kinematics of the collision as between two free particles—the constituent and the projectile. With these conditions the impulse approximation applies—and we learn from a nuclear target, for example, the momentum distribution of its nucleons and hence one important key to the structure of its ground state.

Turning to a nucleon, whose constituents or, as named by Feynman, "partons" have not yet been deciphered, whatever and however numerous they may be, it is in the Bjorken limiting region of deep inelastic scattering with large momentum transfer that we satisfy this condition for applying the impulse approximation. In contrast to the nucleus the "partons" are very strongly bound together by an energy at least comparable to and probably greater than their rest energies as viewed in the proton's rest frame. However, as suggested by Feynman, we may help our intuition and view them as long lived, almost real states, if we take advantage of the time dilation by viewing the proton from an infinite momentum frame. Then, if this bound state describing a proton in the rest system can be formed by momentum components that are limited in magnitude below some fixed maximum—i.e. if there exists a finite $k_{\max}$—then, as viewed in an infinite momentum frame $P \to \infty$, the partons will each share a finite fraction $0 < x_i < 1$ of P and move closely parallel to it as illustrated in Fig. 1. The lifetime of these parton states is characterized by

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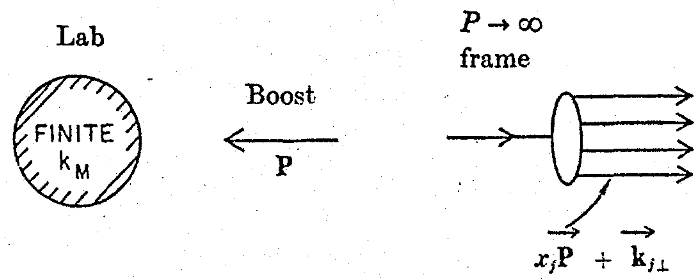

FIG. 1. Proton viewed in the rest frame and in an infinite momentum frame in terms of partons.

$$\tau_{\text{life}} \sim \frac{1}{\Delta E} \sim P/M_{\text{eff}}^2, \tag{1}$$

where for finite $k_{\max}$ and for a finite fraction x not too close to its end-point values of zero or unity, M_{eff} is measured typically in GeV units. Equation (1) exhibits the time-dilation effect.

In the deep inelastic-scattering region τ_{life} is long compared with the duration of the pulse, τ_{int}, from the inelastically scattered electron. In the electron–proton collision center-of-mass system and in the high-energy limit, so that in this system $P = \frac{1}{2}\sqrt{s} \to \infty$, τ_{int} is given by

$$\tau_{\text{int}} \sim \frac{4P}{2M\nu - Q^2}, \tag{2}$$

where $Q^2 > 0$ is the negative of the invariant squared mass transferred to the proton and $\nu \equiv (1/M)\text{P} \cdot \text{q}$ is the frequency transferred in the proton rest system, as illustrated in Fig. 2. We see then that

$$\tau_{\text{int}} \ll \tau_{\text{life}}, \tag{3}$$

provided

$$2M\nu - Q^2 \gg M_{\text{eff}}^2. \tag{4}$$

Equation (4) is the condition for applying the *impulse approximation.* The current interaction is sudden relative to the lifetime of the partons, which are essentially free, and the energy introduced across the current vertex in Fig. 2 is large enough so that energy conservation for the overall process can be approximated by energy conservation across the vertex.

In order to satisfy condition (4) for applying the impulse approxi-

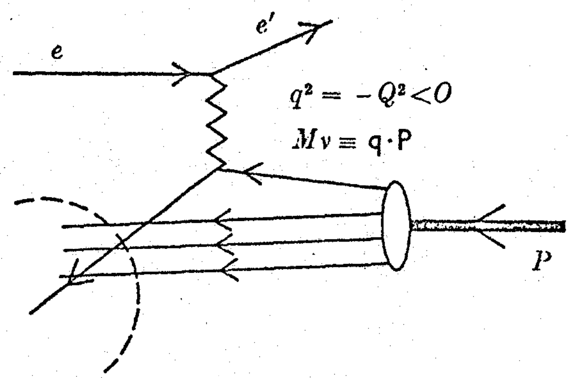

FIG. 2. Inelastic electron–proton scattering viewed in the $P \to \infty$ frame.

mation we require not only high energy ($\nu \gg M$) and inelasticity ($2M\nu - Q^2 \gg M^2$), but also we must fix the fraction x of longitudinal momentum on the parton which scatters the electron, so that it approaches neither extremity $x \sim 1$ or $x \sim 0$. Otherwise, as indicated in Fig. 3, $M^2_{\text{eff}} \sim M^2/x(1-x)$ and violates condition (4) for the impulse approximation. We are also assuming that there are no huge masses coupling to the physical proton state.

$$\Delta E = \sqrt{x^2P^2 + k_\perp{}^2 + M^2} + \sqrt{(1-x)^2P^2 + k_\perp{}^2 + \mu^2} - \sqrt{P^2 + M^2}$$

$$\approx \frac{1}{P}\left\{\frac{k_\perp{}^2 + M^2(1-x)^2 + \mu^2 x}{2x(1-x)}\right\} = \frac{1}{P} M^2_{\text{eff}} \qquad \text{if} \qquad 0 < x < 1$$

$$\approx P \qquad \text{if} \qquad x > 1 \quad \text{or} \quad x < 0$$

FIGURE 3.

As first discussed by Feynman, the fraction x of longitudinal momentum on the parton in the $P \to \infty$ frame, from which the electron scatters, is given by

$$x = Q^2/2M\nu. \tag{5}$$

Thus, the condition for applying the impulse approximation is satisfied if we work in the Bjorken region of finite x and at high inelasticities $\nu \gg M$. Equation (5) is just the condition for elastic scattering from the bare parton in a $P \to \infty$ frame.

To show this we refer to Fig. 2 and observe that the kinematics of the collision as viewed in the center-of-mass system,

$$\mathsf{p} = (P, 0, 0, -P), \qquad m_e \simeq 0,$$

$$\mathsf{P} = \left(P + \frac{M^2}{2P}, 0, 0, P\right),$$

and the relations $(\mathsf{p} - \mathsf{q})^2 \cong 0$, $(\mathsf{P} + \mathsf{q})^2 = 2M\nu - Q^2$, lead to the momentum components

$$q^\mu: \qquad \left(\frac{2M\nu - Q^2}{4P}, \sqrt{Q^2}\,\mathrm{i}_\perp, -\frac{2M\nu + Q^2}{4P}\right). \tag{6}$$

We have thus satisfied the condition for applying an impulse approximation and determining the longitudinal momentum distribution of a parton which, in terms of the structure functions of e–p inelastic scattering as usually defined, is given by

$$G(x) \equiv \frac{1}{x}[\nu W_2]_x \equiv \frac{1}{x}F_2(x). \tag{7}$$

The scaling behavior observed for the structure functions is experimental support for this simple description.

As we have emphasized, the ratio x must be definite for this simple result. Otherwise we will be forced to deal with very slow partons in the $P \to \infty$ system, or, as seen in the rest system of the proton, with the high-momentum extremities of the bound-state structure, and for these the impulse approximation breaks down. The beauty of the electron scattering is that it allows us to tune the mass of the virtual photon line as we choose—either space-like for the scattering or time-like for the deep inelastic annihilation process

$$e + \bar{e} \to H + \cdots$$

and in this way probe the structure by an impulse treatment and with the aid of concrete models purporting to represent gross features of the proton's structure.

However, when we return to the world of only real external hadrons, we have no large mass since $Q^2 \to M^2$ while $2M\nu \to s$, the total collision energy, and thus the fraction of momentum on a parton becomes very small—or "wee". Our condition for applying the impulse approximation also fails and the value of the parton concept is less certain. Nevertheless, as suggested by Feynman, we may hope for clues to the behavior here by studying $F_2(x)$ in (7) as x decreases to very small values, and thus be led to insights into what is going on here in the "wee" region. To see how these purely hadronic processes can be described in parton language, let us view a p–p collision from the center-of-mass system in which there will be an assemblage of right-moving partons colliding with left-moving ones. How do they interact? In field theory the interactions are due to the exchange of "partons", or the constituents forming the physical state; for QED these are the bare photons and electrons. Without specifying what the partons are for hadrons, in order for there to exist an interaction between A and B in Fig. 4 there must be some "confused" partons that do not know right from left. These are the "wee" ones with $x \sim (1 \text{ GeV})/W$, for which relations (1) and

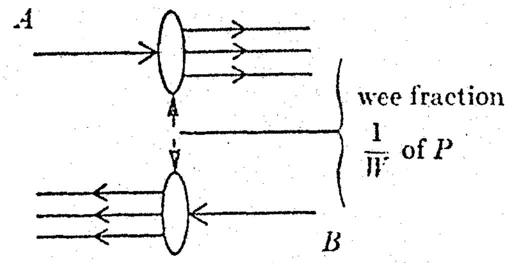

FIG. 4. Hadron–hadron interaction in the center-of-mass system at high energy via "wee" parton exchange.

(2) are replaced by $\tau_{\text{life}} \sim Px/M^2 \sim 1/\text{GeV} \sim \tau_{\text{int}}$. A normal parton in A with finite fraction x to the right cannot be inserted into the left-running state B without paying the penalty of a factor $1/s$, as computed directly from the energy denominators. This is the price to introduce a relative momentum of magnitude $2P$ into the wave function of a ground state built predominantly from finite momentum components, which we take to be a working hypothesis (see Fig. 3). We must therefore turn to these "wee" partons with $x \sim (1 \text{ GeV})/W$ as being responsible for the hadronic cross sections. In order to account for constant total cross sections (to within logarithmic factors) at very high energies, Feynman has suggested a dx/x or bremsstrahlung spectrum for slow partons with x much less than one. In this case we see, by squaring the amplitude to emit and absorb "wee" partons between two colliding hadrons, that

$$\sigma_{\text{tot}} \sim \left(\int^{C/E_a} \frac{dx_a}{x_a^\alpha}\right)\left(\int^{C/E_b} \frac{dx_b}{x_b^\alpha}\right) \sim (E_a E_b)^{2(\alpha-1)} \sim s^{2(\alpha-1)}, \tag{8}$$

which for $\alpha = 1$ is a constant up to logs.

Whether or not the parton idea proves to be successful, both quantitatively and conceptually for high-energy hadronic processes, it should not be forgotten that we are here dealing with "wee" partons as in (8). In contrast, it is the hard partons that satisfy the criteria for the impulse approximation that are important in the application to deep inelastic electron scattering in the Bjorken limit. The parton ideas may meet with different successes or failures in these separate regions.

If we want to find other processes which satisfy the same kinematical constraints as in (4) and (5) and allow application of the impulse picture of partons in an infinite momentum frame, we need look for interactions at high energies s which absorb or produce a lepton system of high mass Q^2 such that the ratio Q^2/s is finite. We confine our attention here to massive lepton systems which can be treated safely by perturbation theory in the electromagnetic or weak couplings, although by further extending the assumptions for the theoretical framework, massive hadron systems could be included in the same kinematical framework just as well. Two examples meeting these constraints are the deep inelastic neutrino processes and electron–positron annihilation cross sections:

$$\nu + p \to e + \cdots \qquad \text{and} \qquad e + \bar{e} \to \text{hadron} + \cdots .$$

In particular, the hard-parton region and scaling behavior will also stand critical challenge from electron clashing rings now under construction through the study of

$$e + \bar{e} \to H + \cdots \tag{9}$$

in the deep inelastic region. This reaction is related by crossing to the deep inelastic-scattering region and predictions of the magnitude of its cross section, as well as of its scaling behavior and of its ratios for different hadrons according to the unitary-symmetry scheme, have been derived for experimental test. One general result is that the cross section has a dependence on the colliding-ring energy that is the same as for point particles for fixed momentum fraction x, i.e.

$$\frac{d\sigma}{dx} = \frac{4\pi\alpha^2}{3Q^2} f(x), \tag{10}$$

where Q^2 is the square of the total colliding-ring energy and $f(x)$ is a dimensionless function of the scale variable $x = Q^2/2M\nu > 1$ for colliding rings, where ν denotes the total energy of all the hadrons as measured in the rest system of the one detected in (9). This is the colliding-ring analogue of ν in (2) for scattering. Values for $f(x)$ near $x = 1^+$ have been inferred from scattering data near $x = 1^-$ and the magnitude of (10) is characteristically 4 orders of magnitude larger than predicted two-body "elastic" events, such as $e\bar{e} \to p\bar{p}$, at total colliding-ring energies of 6 BeV as presently in construction and/or planning.

Finally, there is another measurable cross section that meets the conditions for applying an impulse analysis, and that is

$$\left.\begin{matrix} p + p \\ \bar{p} + p \\ \pi + p \\ \gamma + p \end{matrix}\right. \to (\mu\bar{\mu}) + \cdots; \quad \text{or} \quad \to (\mu\nu) + \cdots \tag{11}$$

where dots denote all other hadron channels open so that this is an "inclusive" process and Q^2, the mass of the lepton pair, is large so that Q^2/s is finite. We will discuss this class of processes in a later Comment.

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