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A DUAL ABSORPTIVE MODEL FOR DIPS IN INELASTIC HADRON PROCESSES*

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ABSTRACT

We assume that the imaginary part of any inelastic hadronic amplitude is dominated by the peripheral ($\ell \sim qr$) resonances, and that the same imaginary part can also be described by a combination of t-channel poles and cuts. The strength of the required cut-term is determined by whether or not the pole-term itself is already peripheral. The real part has no reason to be peripheral and can be easily determined from the peripheral imaginary part only when the cuts happen to be relatively weak. These assumptions lead to a successful qualitative description of all $t \sim 0.6$ dip effects in vector and tensor exchange inelastic and elastic reactions.

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In the absence of a theory of hadronic interactions, many phenomenological models have been proposed [1-4] for the observed behavior of inelastic hadronic reactions. The rise and fall of these models was often related to their ability or inability to explain the apparent erratic behavior of dips in inelastic differential cross sections. The presence of $|t| \sim 0.6$ BeV^2 dips in $\pi^- p \rightarrow \pi^0 n$ and $\gamma p \rightarrow \pi^0 p$ or their absence in $\pi^+ n \rightarrow \omega p$ and $\pi^- p \rightarrow \eta n$ are just a few examples of this puzzling behavior. Every one of these effects has been properly explained in some of the models, but everyone of the models has failed to explain some of the effects.[5]

In this paper we present a simple dual absorptive scheme which accounts for the systematic pattern of these dips. Our model, which has been already applied to elastic scattering,[6] is still qualitative, but we feel that its overall success is sufficient for encouraging the pursuit of a detailed quantitative analysis. We hope to report on such an analysis in the near future.

The starting point of our model is the recognition that the t-channel description of an inelastic hadronic amplitude f(s,t) must involve Regge poles as well as cuts and that the combination of these poles and cuts is dual to the s-channel resonances. These resonances dominate Im f(s,t) in a local way — namely, at a given value of s, Im f(s,t) is dominated by resonances of mass $m \sim s^{1/2}$. On the other hand, Re f(s,t) is not locally controlled by the nearby resonances.[7] It is actually fed by distant resonances, including those with $s < 0$ (u-channel resonances).

Any t-channel description would tend to predict that structures in the angular distribution will occur (if at all) at approximately fixed values of t at all energies. This is supported by the data. How can the s-channel description of f(s,t) reproduce such an effect? This can happen only if strong correlations exist between the different s-channel resonances. The simplest (but not the only[8]) way to guarantee a fixed, energy independent, t-value for a given structure (dip, bump, etc.) is to demand that every single prominent resonance will possess this structure.[9] The sum of all resonance contributions will then automatically exhibit the same structure in t at any given energy. The condition that has to be obeyed by all prominent resonances in order to insure this behavior is $\ell \propto s^{1/2}$, where ℓ is the spin of a resonance and $m = s^{1/2}$ is its mass.[7] Since Im f(s, t) is locally dominated by the resonances, we conclude that at any value of s, the important partial waves in Im f(s, t) will have $\ell \propto s^{1/2}$. We cannot draw a similar conclusion for Re f(s, t) and there will be no simple correlation between s and the ℓ-values of the dominant partial waves of Re f(s, t).

Most versions of the absorption model[1-4] assume that the low ($\ell \ll qr$) partial waves of an inelastic amplitude are largely absorbed by the many open channels and that the full amplitude is dominated by the largest impact parameter within the range of interaction or, equivalently, by the $\ell \sim qr$ partial waves (q-c.m. momentum; $r \sim 1f$-the interaction radius). This assumption coincides with our $\ell \propto s^{1/2}$ duality relation, since $q \propto s^{1/2}$. However, from the duality point of view, it is evident that <u>only Im f(s, t) should be dominated by the $\ell \sim qr$ waves, while Re f(s, t) need not obey such a behavior</u>. Such a departure from the conventional ideas of the absorption model is actually desirable from another point of view. As $s \to \infty$, at fixed t, a definite relation must exist[10] between the s-dependence and the phase of f(s, t). In most versions of the absorption model this relation is ignored.[3,4] However it is easy to see that, in general, the asymptotic phase for an $s^{\alpha(t)}$ energy dependence does not allow both Im f(s, t) and Re f(s, t) to be dominated by the $\ell \sim qr$ partial waves.[11] It is therefore rather satisfactory that our duality argument leads us to accept the conventional absorption picture for the imaginary part but not necessarily for the real part.

We are now ready to state our model:

(i) Im f(s, t) is dominated by s-channel resonances. The prominent resonances have $\ell \sim qr$. Consequently, Im f(s, t) is dominated by the most peripheral s-channel partial waves. For total s-channel helicity flip $\Delta\lambda$, this gives[3, 6] — $\mathrm{Im}\, f^s_{\Delta\lambda}(s,t) \propto$ "J"$_{\Delta\lambda}(r\sqrt{-t})$ where "J"$_{\Delta\lambda}$ has the same general structure (zeroes, maxima, minima) as the Bessel function $J_{\Delta\lambda}(r\sqrt{-t})$ and $r \sim 1f$. A realistic candidate[6] for "J"$_{\Delta\lambda}$ is $A\, e^{Bt}\, J_{\Delta\lambda}(r\sqrt{-t})$. For exotic s-channel processes, $\mathrm{Im}\, f(s,t) \sim 0$.

(ii) The t-channel description of Im f(s, t) is given by a combination of Regge poles and cuts.[12] This combination is always required to be dominated by the s-channel $\ell \sim qr$ waves. In some cases the pole term has large contributions from $\ell \ll qr$ partial waves. In such cases the absorption by a cut is necessary and substantial. In other cases, the pole term itself is strongly dominated by the peripheral partial waves and already includes much of the required absorption. In such cases the cut term is small or even absent, since there is very little for it to absorb in the $\ell \ll qr$ waves. An easy way to decide whether a strong cut term is needed, is to transform the imaginary part of the single pole term to its impact parameter representation and to observe whether or not it is dominated by the peripheral waves.[11] In the case of the exchange degenerate vector and tensor trajectories $\alpha(t) \sim \frac{1}{2} + t$ and the imaginary parts of the pole terms in both $f^s_{\Delta\lambda=0}$ and $f^s_{\Delta\lambda=1}$ have a single zero[13] at $\alpha=0$. The impact parameter representation of $\mathrm{Im}\, f^s_{\Delta\lambda=0}$ has large $\ell \ll qr$ contributions while that of $\mathrm{Im}\, f^s_{\Delta\lambda=1}$ is clearly dominated[14] by $\ell \sim qr$. It is therefore evident that in this case a strong cut term is needed for $\Delta\lambda=0$ and a very weak or no cut term — for $\Delta\lambda=1$. When the cut influence is weak, $\mathrm{Im}\, f(s,t) \propto s^{\alpha(t)}$. When the cut influence is strong, log s terms as well as a modified "effective" $\alpha(t)$ function will appear.

(iii) The s-channel description of Re f(s, t) is obscured, in the absence of a simple resonance description. From the t-channel point of view, Re f(s, t) is described by the same poles and cuts which control Im f(s, t). When the pole description of Im f(s, t) is peripheral and the cut term is therefore small, the phase of f(s, t) is correctly given by the usual signature factor. When the cut term is strong, the phase must approach the signature factor as $s \to \infty$ but it may do so very slowly.[11] In this case we can say very little about Re f(s, t).

In the case of processes dominated by the exchange of the vector and tensor trajectories and their associated cuts, the t-dependence of f(s, t) will therefore be given by:

$$\mathrm{Im}\, f^s_{\Delta\lambda=0}(s,t) = "J_0"(r\sqrt{-t}) \quad ; \qquad \mathrm{Im}\, f^s_{\Delta\lambda=1}(s,t) = "J_1"(r\sqrt{-t})$$

$$\mathrm{Re}\, f^s_{\Delta\lambda=0}(s,t) = \,? \quad ; \qquad \mathrm{Re}\, f^s_{\Delta\lambda=1}(s,t) = "J_1"(r\sqrt{-t}) \begin{cases} \tan \frac{\pi\alpha(t)}{2} & \text{(vector)} \\ -\cot \frac{\pi\alpha(t)}{2} & \text{(tensor)} \end{cases}$$

The first zeroes of "J_0" are at $|t| \sim 0.2,\ 1.2\ \mathrm{BeV}^2$. For "$J_1$" they are at 0, 0.6 BeV^2. Notice that the $\Delta\lambda=0$ amplitude will not exhibit a $|t| \sim 0.6$ dip while the dip structure of the $\Delta\lambda=1$ contribution depends on whether we have a vector exchange or a tensor exchange.

Before we can discuss specific processes we have to make an assumption concerning the relative strengths of the $\Delta\lambda=0$ and $\Delta\lambda=1$ terms for ω, ρ, f^o and A_2 exchange, where these symbols represent the combined pole + cut contribution with the appropriate t-channel quantum numbers. There is good evidence from elastic scattering on nucleons that the f^o and ω contribute almost purely to $\Delta\lambda=0$, while ρ and A_2 exchange are dominated (but not so decisively) by the $\Delta\lambda=1$ amplitude.[15] This agrees with vector dominance estimates which indicate that the (magnetic) $\Delta\lambda=1$ vector nucleon coupling is almost pure isovector while the (electric) $\Delta\lambda=0$ coupling is dominated by the isoscalar term.[3]

We now discuss several concrete examples:

(a) The processes $\pi^- p \to \pi^o n$, $\pi^- p \to \eta n$, $K^- p \to \bar{K}^o n$ and $K^+ n \to K^o p$ are dominated by ρ and A_2 exchange. In all of these cases the $\Delta\lambda=1$ amplitude is dominant, as suggested above. This is confirmed by the $t \sim 0$ dips observed in these processes. If we assume $d\sigma/dt \sim |f_{\Delta\lambda=1}(s,t)|^2$ we find:

$$\frac{d\sigma}{dt}(\pi^- p \to \pi^o n) \propto ("J_1")^2 \left(1 + \tan^2 \frac{\pi\alpha}{2}\right) = ("J_1")^2 / \cos^2 \frac{\pi\alpha}{2}$$

$$\frac{d\sigma}{dt}(\pi^- p \rightarrow \eta n) \propto (``J_1'')^2 \left(1+\cot^2 \frac{\pi\alpha}{2}\right) = (``J_1'')^2/\sin^2 \frac{\pi\alpha}{2}$$

$$\frac{d\sigma}{dt}(K^- p \rightarrow \bar{K}^o n) \propto (``J_1'')^2 \left[4 + \left(\tan \frac{\pi\alpha}{2} - \cot \frac{\pi\alpha}{2}\right)^2\right] = 4\,(``J_1'')^2/\sin^2 \pi\alpha$$

$$\frac{d\sigma}{dt}(K^+ n \rightarrow K^o p) \propto (``J_1'')^2 \left(\tan \frac{\pi\alpha}{2} + \cot \frac{\pi\alpha}{2}\right)^2 = 4\,(``J_1'')^2/\sin^2 \pi\alpha$$

$(``J_1'')^2$ has a double zero around $|t| \sim 0.6$. In $\pi^- p \rightarrow \pi^o n$ this will not be cancelled and we expect a dip. In the three other cases the double zero is cancelled by the double zero of $\sin^2 \frac{\pi\alpha}{2}$ or $\sin^2 \pi\alpha$. No dip is therefore expected. All four predictions agree with experiment.

(b) A similar situation occurs for $\pi N \rightarrow \pi\Delta$, $\pi N \rightarrow \eta\Delta$, $KN \rightarrow K\Delta$ and $\bar{K}N \rightarrow \bar{K}\Delta$. A dip is expected and observed for $\pi N \rightarrow \pi\Delta$. It is not predicted and not observed in the three other processes.[16] The only modification needed here is the assumption that the $\rho N\Delta$ and $A_2 N\Delta$ vertices are dominated by the $\Delta\lambda=1$ term. This is, again, consistent with vector dominance as well as with the $t \sim 0$ behavior of these processes.

(c) The processes $\gamma p \rightarrow \pi^+ n$ and $\pi^+ n \rightarrow \omega p$ involve I=1 exchange. The helicity flip term presumably dominates the nucleon vertex. The $\gamma\pi\rho$ vertex obviously involves a single helicity flip and the $\pi\omega\rho$ vertex is probably similar. The total $\Delta\lambda$ is thus predominantly 0 or 2, although the $\Delta\lambda=1$ amplitude probably does not vanish. Since $\Delta\lambda=1$ does not dominate, we have no reason to expect a $|t| \sim 0.6$ dip. In both processes such dips are not observed.[17]

(d) A similar conclusion, using a similar argument, applies to $\gamma p \rightarrow \pi^- \Delta^{++}$ and $\pi^+ p \rightarrow \omega \Delta^{++}$. Here, again, we have to repeat our assumption on the $N\Delta$ vertex. No dips are predicted or observed, at $|t| \sim 0.6$.[17]

(e) In $\gamma p \rightarrow \pi^o p$ and $\pi^+ p \rightarrow \rho^+ p$, ω exchange is dominant and the nucleon vertex is dominated by the nonflip term. The $\omega\pi\gamma$ vertex involves a helicity flip and the $\omega\pi\rho$ vertex is similar. $\Delta\lambda=1$ is dominant. Since the exchanged ω has negative signature we expect:

$$\frac{d\sigma}{dt} \propto ("J_1")^2 \left(1+\tan^2 \frac{\pi\alpha}{2}\right) = ("J_1")^2/\cos^2 \frac{\pi\alpha}{2}$$

and a $|t| \sim 0.6$ dip is predicted in both cases. The dips are observed.

(f) In $\gamma p \rightarrow \eta p$, ρ exchange is dominant. The nucleon and meson vertices both involve a single helicity flip and the dominant term is, again, $\Delta\lambda=0,2$. A dip is not predicted and not observed.[17]

(g) In elastic $\pi^{\pm}p$, $K^{\pm}p$, pp and $\bar{p}p$ scattering, Im $f_{\Delta\lambda=0}$ is projected out by the differences between particle and antiparticle cross sections, while Re $f_{\Delta\lambda=1}$ is projected out by the polarizations. In all cases the data agree with our predictions[6] and the entire dip systematics in the elastic differential cross sections and polarizations is explained.

As stated above, the $|t| \sim 0.6$ structure of every one of the 15 inelastic reactions discussed here was correctly described by several models, but every model has failed to account for some of the observations. We shall group the existing models into the two usual families — the weak cut model[2] as well as the Regge pole model or the Veneziano amplitude will be referred to as class I models. The strong cut model[4] as well as the Dar-Weisskopf model[3] will be referred to as class II models.

Class I models fail in the reactions $\gamma p \rightarrow \pi^+ n$, $\pi^+ n \rightarrow \omega p$, $\gamma p \rightarrow \pi^- \Delta^{++}$, $\pi^+ p \rightarrow \omega \Delta^{++}$, $\gamma p \rightarrow \eta p$ and the elastic differential cross sections.

Class II models fail in $\pi^- p \rightarrow \eta n$, $K^- p \rightarrow \bar{K}^o n$, $K^+ n \rightarrow K^o p$, $\pi p \rightarrow \eta \Delta$, $KN \rightarrow K\Delta$, $\bar{K}N \rightarrow \bar{K}\Delta$ and the elastic polarization.

Both classes are successful in $\pi^- p \rightarrow \pi^o n$, $\pi N \rightarrow \pi \Delta$, $\gamma p \rightarrow \pi^o p$ and $\pi^+ p \rightarrow \rho^+ p$.

A quick glance at these lists immediately reveals that all failures of class I models stem from an inadequate description of Im $f^S_{\Delta\lambda=0}$ (namely — instead of $|t| \sim 0.2$ zero, it has a $|t| \sim 0.5$ zero which can be moved slightly, but not enough, by the weak cut). In these models Im $f^s_{\Delta\lambda=0}$ is not dominated by the $\ell \sim qr$ partial waves, contrary to our assumptions. All failures of class II models stem from an inadequate description of Re $f_{\Delta\lambda=1}$ (namely — instead of "J_1" $\tan \frac{\pi\alpha}{2}$ or "J_1" $\cot \frac{\pi\alpha}{2}$ it behaves like "J_1"). In these models Re $f^s_{\Delta\lambda=1}$ is required to be dominated by the $\ell \sim qr$ partial waves, contrary to our assumptions.

We believe that our description represents correctly the gross features of the relevant amplitudes and that it provides a successful solution to the puzzle of $|t| \sim 0.6$ BeV^2 dips. A more quantitative study would be extremely interesting.

Many problems are left open, however. We mention only a few:

(i) In our 15 inelastic processes as well as in elastic scattering Re $f^s_{\Delta\lambda=0}$ did not play a crucial role. We therefore succeeded in explaining many pieces of data without making explicit assumptions on this amplitude.[11] Strangeness exchange reactions as well as π-exchange processes may enable us to determine the characteristics of Re $f^s_{\Delta\lambda=0}$.

(ii) We showed that Im f is dominated by the ℓ qr partial waves. What remains to be determined is the s-dependence of the radius r (constant ?$(\log s)^{1/2}$? log s ?) as well as the details of the impact parameter or partial wave description[11] (what is the "width" of the peripheral peak of Im f as a function of ℓ? How does it depend on energy?).

(iii) Finally, we assumed that $r \sim 1$f. Does the radius depend on the nature of the colliding hadrons? Is it very different for, say, $\pi\pi$ scattering and NN scattering?

Numerous helpful discussions with Adam Schwimmer are gratefully acknowledged.

FOOTNOTES AND REFERENCES

1. For a review of pole and cut models see J. D. Jackson, Proc. of the Lund Conference (1969) and references therein.
2. Examples of "weak cut" models are e.g., R. C. Arnold, Phys. Rev. 153, 1523 (1967); A. Capella and J. Tranh Thanh Van, Nuovo Cimento Lett. 1, 321 (1969).
3. A. Dar, T. Watts and V. F. Weisskopf, Nucl. Phys. B13, 477 (1969).
4. F. S. Henyey, G. L. Kane, J. Pumplin and M. Ross, Phys. Rev. 182, 1579 (1969).
5. Previous partial explanations of dip systematics include H. Harari, Proc. of the Liverpool Conference (1969); A. Dar, Proc. of the Columbia Conference (1969); M. Bander and E. Gotsman, Phys. Rev. D2, 224 (1970); C. B. Chiu and S. Matsuda, Phys. Lett. 31B, 455 (1970); R. Carlitz and M. Kislinger, Phys. Rev. D2, 336 (1970); J. Tranh Thanh Van, Orsay preprint (1971).
6. H. Harari, SLAC-PUB-821, Ann. Phys., in print; SLAC-PUB-837, to be published in the Proc. of the 1970 Erice School; M. Davier and H. Harari, SLAC-PUB-893, to be published.
7. For a detailed discussion see e.g., H. Harari, Proc. of the Brookhaven Summer School (1969).
8. A more complicated way is offered by the Veneziano formula in which the many s-channel resonances at any given energy produce dips at fixed t-values through delicate cancellation effects. This amplitude is, however, not dominated by the peripheral partial waves and we shall see below that it provides an inadequate description of several $|t| \sim 0.6$ effects.
9. This was shown by R. Dolen, D. Horn and C. Schmid, Phys. Rev. 166, 1768 (1968), to be true for πN scattering. See also Refs. 6 and 7.
10. N. N. Khuri and T. Kinoshita, Phys. Rev. 137B, 720 (1965).
11. A detailed discussion of this problem will be given in a forthcoming paper by H. Harari and A. Schwimmer.

12. In this paper we shall ignore the double-particle-exchange cuts and consider only the pole-Pomeron cuts. This is really justified only at energies of the order of, say, 8 - 10 GeV or more (see e.g., H. Harari, SLAC-PUB-887, to be published).

13. In the t-channel helicity nonflip amplitude, Im $f^t_{\Delta\lambda=0}$ has a ghost-killing factor of α for the tensor exchange. Exchange degeneracy requires a similar term for vector exchange. The t-channel amplitude Im $f^t_{\Delta\lambda=1}$ must also have such a factor. Since both t-channel amplitudes have these factors, the imaginary parts of both s-channel helicity amplitudes will also possess them.

14. Im $f^{pole}_{\Delta\lambda=0}$ will have a zero at $\alpha=0$, i.e., $|t|\sim 0.5$. It does not resemble the "J_0" function (which would have a $|t|\sim 0.2$ zero for r=1f) and is therefore not peripheral. Im $f^{pole}_{\Delta\lambda=1}$ has a kinematic zero at t=0 and a zero at $\alpha=0$. These imitate the "J_1" structure, and the amplitude is therefore peripheral.

15. It seems that for vector and tensor exchange in $\Delta\lambda=0$, D/F $\sim$ 0 while for $\Delta\lambda=1$, D/F $\sim$ 3. The latter ratio completely decouples ω and f from the nucleon for $\Delta\lambda=1$. The first ratio predicts a 3:1 ratio between the $\Delta\lambda=0$ couplings of f or ω and ρ or A_2 to the nucleon. For a discussion, see, e.g., R. Odorico et al., Phys. Lett. 32B, 375 (1970); C. Michael and R. Odorico, CERN preprint (1971).

16. This discussion is consistent with the usual pole-model description of these amplitudes. See e.g., M. Krammer and U. Maor, Nucl. Phys. B13, 651 (1969).

17. Since the ρNN coupling involves some nonflip contribution, our qualitative statements in cases (c), (d) and (f) are actually weaker than our other predictions. Only a detailed s-channel helicity analysis of these processes can give a complete picture.