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DUALITY AND INELASTIC ELECTRON-NUCLEON SCATTERING*

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I. Introduction

The large cross sections and regularities observed in experiments on inelastic electron-proton scattering have led to intensive theoretical efforts attempting to understand the origin of the observed behavior of the scattering, particularly the deep inelastic scattering at high energies and large momentum transfers. In this talk we will also be discussing deep inelastic electron-nucleon scattering, with particular attention to the ways in which the behavior of elastic scattering and nucleon resonance electroproduction are related to the behavior of deep inelastic scattering. As we will see, there are close and striking connections between the resonance and deep inelastic behaviors, connections between low and high energies which provide some striking examples of the idea of duality — an idea which had its origins in considerations of purely strong interaction processes.[1]

Before discussing duality, however, we will first review in the next section the present state of the available experimental information on inelastic electron-nucleon scattering. That will put us in a position to discuss the behavior of the resonances, show the relations to deep inelastic behavior, and generally discuss duality as applied to inelastic electron-nucleon scattering. Finally, in the last part of this talk we will discuss some additional aspects of some theoretical models proposed so far to explain deep inelastic scattering, as well as some very interesting open questions on the nature of the hadronic final state.

II. The Experimental Situation and Scaling

Presumably everyone here knows about the famous structure functions W_1 and W_2, which summarize the results of inelastic scattering of electrons on nucleons.[1] They are determined from the double differential cross section,

$$\frac{d^2\sigma}{d\Omega' dE'} = \frac{4\alpha^2 E'^2}{q^4}\left[2W_1(\nu, q^2)\sin^2\frac{\theta}{2} + W_2(\nu, q^2)\cos^2\frac{\theta}{2}\right], \qquad (1)$$

for scattering of the incident electron of energy E through an angle θ to a smaller energy E' due to the exchange of a single virtual photon, which has laboratory energy

$$\nu = E - E' \tag{2}$$

and four-momentum squared

$$q^2 = 4EE' \sin^2 \frac{\theta}{2} \quad . \tag{3}$$

Through knowledge of ν and q^2 (from measuring the incident and scattered electron), the invariant mass W of the final hadrons is fixed by

$$s = W^2 = 2M_N\nu + M_N^2 - q^2 \quad . \tag{4}$$

We can also consider inelastic electron scattering as a collision between the exchanged virtual photon and the target nucleon. One is then simply studying the total cross section for the process "γ" + nucleon → hadrons, where the hadrons have an invariant mass W and we are able to vary the mass of the incident photon beam (i.e., q^2) and its longitudinal and transverse polarization by changing the electron's angle of scattering and energy. Through the optical theorem, we are just studying the imaginary part of the forward amplitude for virtual photon-nucleon scattering, an object which lends itself easily to application of our accumulated wisdom on forward hadron-hadron scattering amplitudes. The amplitudes $W_1(\nu, q^2)$ and $W_2(\nu, q^2)$ are then just the imaginary parts of the two invariant amplitudes for forward virtual photon-nucleon scattering (averaged over nucleon spins) and are related by

$$W_1 = \frac{(\nu - q^2/2M_N)}{4\pi^2\alpha} \sigma_T$$

and (5)

$$W_2 = \frac{(\nu - q^2/2M_N)}{4\pi^2\alpha} \frac{q^2}{q^2 + \nu^2} (\sigma_T + \sigma_S)$$

to the total virtual photon-nucleon cross sections σ_T and σ_S for transversely and longitudinally polarized photons. The longitudinal total cross section σ_S is kinematically forced to vanish at $q^2=0$, while σ_T at $q^2=0$ is simply the total photoabsorption cross section into hadrons for real photons.

The first deep inelastic scattering experiments[2] not only showed the large, point-like magnitude of the cross section referred to in the Introduction but also provided evidence for the scaling behavior proposed by Bjorken.[3] We take "scaling" as the statement that as ν and $q^2 \rightarrow \infty$, νW_2 and W_1 become nontrivial functions of the dimensionless ratio $\omega = 2M_N\nu/q^2$ only, rather than functions of both ν and q^2 separately as would be the case a priori. Since scaling is a statement of behavior as ν and $q^2 \rightarrow \infty$, any other dimensionless variable ω' such that $\omega' \rightarrow \omega$ as ν and $q^2 \rightarrow \infty$ is, in principle, just as suitable for defining scaling behavior. The experimental data, which only exists at finite values of ν and q^2, could scale in some other variable ω' sooner in the sense that νW_2 and W_1 would become independent of q^2 (and thus equal to their $q^2 \rightarrow \infty$ limiting values) if they are studied as functions of q^2 for fixed ω' rather than ω.

This is in fact the case for inelastic electron-proton scattering. If we consider data points with $W > 2$ GeV, so that for the moment we stay away from the prominent N* resonances, then there is a more rapid approach to scaling behavior if we use the variable

$$\omega' = 1 + \frac{W^2}{q^2} = 1 + \frac{s}{q^2} = \omega + \frac{M_N^2}{q^2} \quad . \tag{6}$$

Clearly $\omega' \rightarrow \omega$ in the limit $\nu, q^2 \rightarrow \infty$ considered by Bjorken.[3] While one could consider the more general form $\omega' = \omega + m^2/q^2$, a best fit[4] for m^2 gives a value of $m^2 = M_N^2 \pm 0.1$ GeV^2, and we use the value $m^2 = M_N^2$ because of the simple expression for ω' in terms of $W^2 = s$ and q^2. There is also evidence from a new analysis[5] of inelastic neutrino-nucleon scattering data that $\omega' = 1 + W^2/q^2$ results in scaling sooner than ω does. We shall therefore use ω' as the scaling variable in what follows.

In order to actually test for the presence of scaling behavior in the experimental observations of inelastic electron scattering one must separate the contributions of W_1 and W_2 to the double differential cross section in Eq. (1). This is accomplished experimentally by measuring the scattering at the same value of ν and q^2, but at different angles. Separation of W_1 and W_2 is equivalent, through Eq. (5), to a knowledge of $R = \sigma_S/\sigma_T$, and the value of R obtained by averaging over the present data[4] between ω' of 1 and 10 is 0.18 ± 0.05. In other words, the scattering is dominated by transverse photons.[6] There is no indication of any strong dependence on ν, q^2 or ω'. With R=0.18, Fig. 1 shows νW_2 and $2M_N W_1$ as functions of ω' for various q^2 intervals and W > 2 GeV. Both νW_2 and W_1 clearly scale, i.e., are functions of ω' to within the accuracy of the data for ω' in the range $1 < \omega' < 10$, as long as $q^2 \geq 1\ \mathrm{GeV}^2$ and $W \geq 2$ GeV.[7]

For inelastic neutrino-nucleon scattering one is not yet able to separate the (three) different form factors. There is, however, evidence for a consequence of scaling behavior of the structure functions, namely, the apparent linear rise of the total cross section with incident neutrino energy. Specifically, the data from propane give[5]

$$\sigma = (0.52 \pm 0.13)\ \frac{G^2 M_N E}{\pi} \qquad (7)$$

per nucleon.

An alternate way of looking at the inelastic electron-proton scattering data is shown in Fig. 2, where the experimentally measured combination[8] of total cross sections $\sigma_T + \epsilon\sigma_S$ is plotted against q^2/W^2 for various hadron masses W. Also shown is $G_E^2(q^2) + (q^2/4M_N^2)\ G_M^2(q^2)$, the analogue of $\sigma_T + \sigma_S$ for W=0.94 GeV, i.e., elastic scattering. Notice in particular the slow (like $1/q^2$) falloff of $\sigma_T + \epsilon\sigma_S$ when $1/9 < q^2/W^2 < 1/3$ corresponding to the relatively flat part of νW_2 between ω' of 4 and 10 in Fig. 1. But when q^2/W^2 becomes large we come below the knee in νW_2 and $\sigma_T + \epsilon\sigma_S$ falls rapidly, roughly like $1/q^6$ for fixed W. From Eq. (5), a $1/q^6$ behavior for $\sigma_T+\sigma_S$ as $q^2 \to \infty$ implies that $\nu W_2 \propto (W^2/q^2)^3 = (\omega' - 1)^3$ as $q^2/W^2 \to \infty$ or $\omega' \to 1$. The behavior $\sigma_T+\sigma_S \propto 1/q^6$ as $q^2 \to \infty$ at fixed W is of course just the behavior of the elastic analogue of $\sigma_T+\sigma_S$, $G_E^2(q^2) + (q^2/4M_N^2)\, G_M^2(q^2)$, at large q^2 if we take dipole forms for $G_{Ep}(q^2)$ and $G_{Mp}(q^2)$. We will return to this point later. For now, we would just emphasize the close similarity in behavior of the cross section for different values of W when considered as a function of q^2/W^2. As noted many times previously, the deep inelastic (W > 2 GeV) cross section does fall with increasing q^2 more slowly than elastic scattering at the <u>same value of q^2</u>, particularly for values of q^2 in the few GeV^2 range for which ω' is in the range where νW_2 is approximately constant. But for sufficiently large values of q^2 the cross section for any fixed W falls rapidly, very much as elastic scattering does already at values of q^2 of a few GeV^2. This would have been realized much sooner if it were not that, because of scaling, the behavior of elastic scattering for q^2 values of a few GeV^2 should properly be compared with inelastic scattering at, for example, W=2 GeV and q^2 values in the 10 to 20 GeV^2 range!

In contrast to the region near $\omega'=1$, for $\omega' > 10$ there is presently no data over a large range of q^2, nor is there a separation of W_1 and W_2 to permit a conclusive investigation of possible scaling behavior. If we use the same (small) value of $R=\sigma_S/\sigma_T$ found for $\omega' < 10$, then the available data with $q^2 \geq 1$ GeV^2 are consistent with scaling and show[4] νW_2 decreasing with ω' for large ω'. From the relation of W_1 and W_2 to the total cross sections σ_T and σ_S, or from a

Regge pole analysis, one expects νW_2 and W_1/ω' to behave at large ω' as $(\omega')^{\alpha-1}$, where α is the Regge intercept (at $t=0$) of the relevant exchange in forward virtual photon-nucleon scattering. If this is the Pomeron, corresponding to diffractive virtual photon-nucleon scattering, then $\alpha=1$ and νW_2 and W_1/ω' should be constant at large ω', while they should decrease if "ordinary" exchanges and a corresponding nondiffractive component of virtual photon-nucleon scattering are present in the scaling region. From the available data,[4] and assuming scaling for all ω' as long as $q^2 \geq 1$ GeV^2, either or both νW_2 and W_1/ω' appear to decrease by ~20% between their maxima at $\omega' \simeq 6$ and $\omega' \simeq 25$. This is because both νW_2 and W_1/ω' appear to decrease by ~20% over the above ω' range if we assume R=0 for $\omega' > 10$, and, as the assumed value of R for $\omega' > 10$ is increased, the values of νW_2 obtained from the measured differential cross sections increase, but those of W_1/ω' decrease. Alternately one can consider directly the values of σ_T at points where a separation of σ_T and σ_S has been made.[4] In particular, at $q^2=1.5$ GeV^2, σ_T is a maximum near $\omega'=4$ and falls with increasing energy at least as much as the total photoabsorption cross section does over the same ν or W^2 range at $q^2=0$. Thus there is evidence from the large ω' dependence of νW_2 and W_1 or σ_T for some nondiffractive component or "ordinary" exchanges in virtual photon-nucleon scattering in the scaling region. Its exact magnitude, however, is beyond the ability of the energy dependence of the present data at large ω' to reveal. In particular, even assuming scaling, one cannot show if νW_2 and W_1/ω' have already reached their asymptotic values or continue to fall as $\omega' \to \infty$.

A more direct piece of evidence for the presence of an isospin dependent, and therefore nondiffractive component of the amplitude is the observation of a difference between inelastic scattering from protons and neutrons. Neglecting possible deuterium corrections,[9] the data indicate[4] neutron inelastic cross sections that are definitely smaller than the proton cross sections over a large kinematic range. In particular, assuming the same value of $R=\sigma_S/\sigma_T$ for the neutron and proton, then $\nu W_{2n}/\nu W_{2p}$ is smaller than unity at least for $\omega' < 6$, and νW_{2n} scales within the accuracy of the data. The quantity $\nu W_{2p} - \nu W_{2n}$ is a maximum near $\omega' = 4$, at which point $\nu W_{2p} - \nu W_{2n} \simeq 0.1$ and $\nu W_{2n}/\nu W_{2p} \simeq 2/3$. While the neutrino data may also suggest that the scattering of neutrinos on neutrons and protons is also different,[5] the electron data is the most direct and conclusive evidence for an isospin dependent, nondiffractive component of the virtual photon-nucleon amplitude in the scaling region.

From the above discussion of the results of the present inelastic electron scattering experiments it is clear that the existing single arm experiments are rather complete. Aside from refinements on existing measurements and filling in some accessible but unexplored regions between existing measurements at presently available beam energies, the next step experimentally is to investigate the nature of the hadronic final state. In fact we already have preliminary results from Cornell[10] on rho electroproduction, which indicate that the rho plus nucleon final state forms a decreased percentage of the total hadrons produced as q^2 increases from zero to $\sim 1\ \mathrm{GeV}^2$. Experiments have also been performed at CEA and DESY[11] on forward π^+ and proton electroproduction, and several other electron or muon experiments with observation of the final hadrons are planned at DESY, Cornell, SLAC, and NAL. Up to now our only view of the entire (charged) hadronic final state comes from a few hundred neutrino produced events in a heavy liquid bubble chamber.[5] This very interesting subject is in its infancy both experimentally and theoretically, and we shall return to it again in the last section of this talk.

III. Resonance Behavior and Duality in Deep Inelastic Scattering

We are all familiar with the correlation in purely hadronic amplitudes between the presence and properties of low energy s-channel resonances and the presence and properties of "ordinary" (non-Pomeron) t-channel exchanges at high energies. This correlation of resonances and "ordinary" exchanges is part of the "two component" picture[12] of duality for two-body amplitudes in which Pomeron exchange or diffraction scattering at high energies is connected to (or, is "dual" to) the low energy "background", while the exchange of "ordinary" trajectories is connected to the low energy resonances. This connection is made quantitative through the use of finite energy sum rules[13] which relate integrals over the imaginary part of the amplitude at low energies to the properties of the high energy amplitudes.[14]

Since we have seen that experimental evidence for a nondiffractive component of the forward virtual photon-nucleon amplitudes exists, we expect that low energy resonance electroproduction will have a behavior which is correlated with that of the deep inelastic scattering.[15] To examine this question, we have plotted in Figs. 3-7 the function νW_2 at various values of q^2 between 1 and 3 GeV^2, obtained[16] by interpolating the published 6^o and 10^o data[17] up to a hadron mass, W, of 3 GeV. The solid line, which is the same in all cases, is a fit[18] to all data with $W \geq 2$ GeV and $q^2 \geq 1$ GeV^2, where scaling is observed to hold in ω'. We shall use the term "scaling-limit-curve" to refer to this smooth fit to the data in the region of scaling behavior. Above W = 2 GeV, where there are no prominent resonances, the interpolated values of νW_2 at fixed q^2 agree with the scaling-limit-curve, $\nu W_2(\omega')$, as they should.

From the figures we can distinctly see the prominent N* resonances for values of q^2 where νW_2 scales for W > 2 GeV. A given resonance (including the elastic peak), occurs at $\omega'_R = 1 + M_R^2/q^2$ and moves toward $\omega' = 1$ as q^2 increases. The prominent resonances do not disappear with increasing q^2 relative to a "background" under them. (Note that for values of q^2 beyond about 3 GeV^2 the present data in the low W region is not of sufficient quality statistically to reveal whether the prominent resonances are still there.) Instead, the prominent resonance bumps roughly remain of the same magnitude relative to "background" and follow in magnitude the height of the scaling-limit-curve at the corresponding value of ω'. This can be seen more clearly in Figs. 8-10, where the heights[19] of the N*(1238), N*(1520), and N*(1688) nucleon resonance bumps in νW_2 divided by $\nu W_2(\omega'_R)$ (where $\omega'_R = 1 + M_R^2/q^2$ corresponds to the given resonance and value of q^2 measured in the 6^o and 10^o inelastic scattering experiments) are plotted versus q^2. Clearly, the ratio of height of the resonance bump to the magnitude of the scaling-limit-curve at the same point in ω' remains roughly constant as q^2 changes from 1 to 3 GeV^2.

Thus at least the prominent resonances have a behavior which is strongly correlated with the behavior of νW_2 in the region of scaling behavior. In addition a recent analysis[20] of $R = \sigma_S/\sigma_T$ for $W \leq 2$ GeV shows the same small value (consistent with zero) that is found in the scaling region of deep inelastic scattering. Although we cannot determine what the many broad, low spin, N* resonances that we know exist from πN phase shifts are doing without a detailed partial wave analysis of the hadronic final state, the behavior of the prominent N* resonances that we can see and separate give us the clue as to what is happening.[15] In the duality framework we say that the nucleon and other N* resonances at low energy correspond to the presence of non-Pomeron exchanges at high energy, which will result in a falling σ_T or $\nu W_2(\omega')$ curve at high energies and a difference between neutron and proton inelastic scattering.

What is unique to studying duality in electroproduction is the experimentally observed scaling behavior. This allows us to consider data at fixed values of ω' but different values of q^2 and W^2, both within and outside the region where prominent resonance bumps exist. Thus we can compare data where there are prominent resonances directly with data for $\nu W_2(\omega')$ at large q^2 and W^2 where nature has accomplished the appropriate averaging of the many broad resonances and background or t-channel exchanges present there. Thus, without any interpolation from high to low energies using a theory or model, one can see in Figs. 3-7 the clear oscillations of νW_2 in the low W region about the scaling-limit-curve, which represents either the average of many resonances and background or t-channel exchanges at large W.

In spite of warnings to the contrary, some have been confused by the use of ω' rather than ω in the analysis given above and in Ref. 15 into thinking that the conclusions depend on using ω' rather than ω. But neither the decrease of νW_2 or σ_T at high energies, nor the difference between neutron and proton elastic scattering or N* electroproduction and the similar difference between neutron and proton inelastic scattering, nor the small value of R measured in both the resonance region and deep inelastic scattering, nor the presence of prominent resonance bumps in νW_2 for values of q^2 where scaling holds above W = 2 GeV, nor even the semi-quantitative correlation between the height of the prominent resonance bumps and the magnitude of the scaling-limit-curve depend on using ω'. All of these important aspects of the physics which are basic to our arguments can be seen when we look at the data when plotted with respect to other variables like ω. In some ways the particular choice of variable is similar to the situation in the choice of ν_{lab} or s in extrapolating high energy fits or models of pion-nucleon charge exchange into the low energy region. While extrapolation with some variables results in better averaging of the resonance region, the essential physics, which was the impetus for much of the original thinking about duality, does not change, e.g., the correlation between zeroes in the angular distributions of the prominent resonances and the zeroes at fixed t in the high energy spin flip and spin nonflip amplitudes.[13] Similarly in electroproduction, the essential physics does not depend on ω'.

That is not to say that ω' does not have advantages. First, as we saw in the last section, scaling occurs earlier in ω'. Second, if νW_2 is considered as a function of ω, the nucleon pole term in νW_2, corresponding to elastic scattering, always occurs at ω=1. All the other resonances are at values of $\omega > 1$ and move toward ω=1 as q^2 increases. Using $\omega' = 1 + W^2/q^2$, however, the nucleon and all other resonances occur at values of $\omega' > 1$. The nucleon is then not treated in a special way. As we will see shortly, this allows one to understand in an alternate way the connection found previously between the behavior of the elastic form factors and of νW_2 as $\omega' \to 1$. Third, the use of ω' allows a much more local averaging of the region below W=2 GeV where there are prominent resonances. In short, while the use of ω' does not affect the basic arguments, it does direct one's attention to certain aspects of the physics and make what is happening rather clear even to a cursory viewer.

As mentioned previously, our qualitative discussion of duality can be made quantitative in terms of finite energy sum rules. If q^2 and ν_{max} are in a region where νW_2 scales, then by a derivation closely paralleling that of the usual finite energy sum rules,[13] we find[15, 16]

$$\frac{2M_N}{q^2} \int_0^{\nu_{max}} d\nu \; \nu W_2(\nu, q^2) = \int_1^{1+W^2_{max}/q^2} d\omega' \; \nu W_2(\omega') \qquad (8)$$

where $W^2_{max} = 2M_N \nu_{max} + M_N^2 - q^2$ is chosen above the region of prominent resonances. Equation (8) states that for $\nu < \nu_{max}$, $\nu W_2(\omega')$ acts as a smooth average function for $\nu W_2(\nu, q^2)$ in the sense of finite energy sum rules.

The validity of the sum rule in Eq. (8) can be tested by using interpolations to fixed q^2, such as in Figs. 3-7 on the left-hand side and the scaling-limit-curve, $\nu W_2(\omega')$, on the right-hand side. Using $W_{max} = 2.0$ and 2.5 GeV, and values of q^2 between 1 and 4.5 GeV^2, one finds that the agreement of the two sides is to 10% or better over the whole range of q^2, while each side is separately changing by over an order of magnitude.[16] Considering the statistical as well as systematic errors present in both the data and the interpolation to fixed q^2, the agreement is extremely good. Furthermore, the prominent resonances and elastic peak are a significant part of the agreement of the two sides of Eq. (8); their removal from the left-hand side would destroy it.

The success of the sum rule, the apparently almost resonance by resonance averaging seen in Figs. 3-7, and the behavior of the prominent resonances in "following" $\nu W_2(\omega')$, leads us to ask what must be the behavior in q^2 of a given hadronic final state of mass W, if it is to participate in the scaling behavior of νW_2. It is rather simple to show[15] that if $G(q^2)$ is the excitation form factor of the hadronic final state of mass W and

$$G(q^2) \xrightarrow[q^2 \to \infty]{} (1/q^2)^{n/2} \tag{9}$$

while νW_2 can be parameterized by

$$\nu W_2 \xrightarrow[\omega' \to 1]{} (\omega' - 1)^p \ , \tag{10}$$

then the coexistence of the two behaviors demands that

$$n = p + 1. \tag{11}$$

If we apply this in the low energy region to a given resonance of mass M_R, then all resonances which follow $\nu W_2(\omega')$ in magnitude (as we have seen the prominent N^* resonances do) must have the same power of falloff in q^2 as $q^2 \to \infty$, and this power is related to the power with which νW_2 rises at threshold. Presumably this includes the elastic peak, or zeroeth resonance, which has $n \simeq 4$. That the resonance excitation form factors all have a behavior at large q^2 which is similar to the behavior of the elastic form factors is well known.[2,4] Further, we saw in Section II that $p \simeq 3$, so that Eq. (11) is also at least approximately satisfied. Equation (11) for the case of the elastic peak is just the relation of Drell and Yan[21] first found in an entirely different way in the parton model.

All these successes of attempts to relate the resonance region to scaling and deep inelastic scattering suggest that the finite energy sum rule average in Eq. (8) can actually be made over a much more local region of W. In particular, the possibility that the scaling behavior is reflected in the behavior of the resonances on an almost resonance-by-resonance basis leads us to try using Eq. (8) over very small regions of W of order a few hundred MeV. While we generally have no detailed partial wave decomposition of inelastic scattering so that we could study each of the resonances present in a given W range, there is one place where we know exactly what resonances are present — namely, at the elastic peak in νW_2. It is then interesting to carry the assumption of local duality to an extreme and assume that the area (in the sense of the left-hand side of Eq. (8)) under the elastic peak in νW_2 for large q^2 is also the same as the area (in the sense of the right-hand side of Eq. (8)) under the scaling-limit-curve between $\omega'=1$ and a value of ω' corresponding to a hadron mass W_t around physical pion threshold. Taking the derivative with respect to q^2 of both sides of the resulting equation gives

$$\nu W_2 \left(\omega' = 1 + \frac{W_t^2}{q^2}\right) = \left(\frac{1}{\omega' - 1}\right) \left(-q^2 \frac{d}{dq^2} G^2(q^2)\right) , \qquad (12)$$

where

$$G^2(q^2) = \frac{G_E^2(q^2) + \frac{q^2}{4M_n^2} G_M^2(q^2)}{1 + \frac{q^2}{4M_N^2}} \qquad (13)$$

Thus, given some very strong assumptions, Eq. (12) allows us to calculate $\nu W_2(\omega')$ in terms of the elastic form factors once we have chosen W_t. Given W_t, we would presumably only expect such strong assumptions to work when the elastic peak is pushed into the threshold region of $\nu W_2(\omega')$, i.e., when $q^2 \gg 1\ \text{GeV}^2$ and $\omega' - 1 = W_t^2/q^2 \ll 1$.

However, independent of what value we choose for W_t, a number of semi-quantitative results follow. First of all we again obtain the Drell-Yan relation[21] between the behavior of $G(q^2)$ at large q^2 and νW_2 near $\omega' = 1$, as one can easily convince oneself from Eq. (12). Next, by comparing Eq. (12) for neutrons and protons that near $\omega' = 1$ we obtain

$$\frac{\nu W_{2n}}{\nu W_{2p}} = \frac{-q^2 \frac{d}{dq^2} \left[G_n(q^2)\right]^2}{-q^2 \frac{d}{dq^2} \left[G_p(q^2)\right]^2} \xrightarrow[\omega' \to 1]{} (\mu_n/\mu_p)^2 = 0.47 \quad . \qquad (14)$$

This is in agreement with at least the trend of the available data[4] (which extends down to $\omega' \simeq 1.7$), but only experiments at smaller values of ω' can verify the truth of Eq. (14). In addition, if we apply the same assumptions of elastic dominance in a finite energy sum rule for W_1, then we obtain

$$R = \sigma_S/\sigma_T \xrightarrow[\omega' \to 1]{} 0 \quad , \qquad (15)$$

which is again quite consistent with our experimental knowledge.[4]

The actual result[16] of computing νW_{2p} from the measured proton elastic form factors and Eq. (12) is shown in Fig. 11 for $W_t = 1.08$ and 1.23 GeV, together with the large angle data points[4] near ω'=1. While we obtain the correct shape for $\omega' \lesssim 1.5$ in either case, one must go to a value of $W_t \simeq 1.23$ GeV to obtain a calculated curve which passes through the data in this region. Stated another way, the elastic contribution to the left-hand side of Eq. (12) equals the area under $\nu W_2(\omega')$ from ω'=1 all the way up to an ω' which corresponds to a hadron mass just below the first resonance. We are actually hampered here by our lack of knowledge of $G_{Ep}(q^2)$ at large q^2. While $G_{Mp}(q^2)$ dominates Eq. (12) as $q^2 \to \infty$ or $\omega' \to 1$, changing the form of $G_{Ep}(q^2)$ for values of q^2 beyond the presently explored region can make nonnegligible changes in the calculated $\nu W_2(\omega')$. This is even more true of $G_{En}(q^2)$, for which we have no knowledge at large q^2. So, strangely enough, the calculation of the inelastic structure functions through Eq. (12) runs into difficulties because of lack of knowledge of elastic scattering! It will clearly be quite interesting to have data on neutron elastic and resonance excitation form factors for $q^2 \geq 1$ GeV^2 to compare with the deep inelastic electron-neutron data near ω'=1. We may yet find ourselves in the embarrassing position of having the predictions of Eqs. (14) and (15) found to be true experimentally at values of ω' away from 1 where the elastic form factors and Eq. (12) predict deviations from the asymptotic ($\omega' \to 1$) behavior.

In summary, the correlations between elastic scattering, resonance electroproduction and the scaling behavior of deep inelastic scattering are very close. Besides the striking qualitative connections, some of which can be seen in Figs. 3-10, there are quantitative connections through finite energy sum rules which are satisfied to within the errors in the existing data. Even the very extreme assumption of averaging over only the elastic delta function leads to at least qualitatively the correct behavior and shape of the scaling-limit-curve near $\omega'=1$. Taken all together, this represents one of the most striking examples of the many connections that exist between low and high energy behavior and of the general ideas of duality.

IV. Some Open Questions and Comments

The arguments in the last sections and experiment show that there is a substantial nondiffractive component present in virtual photon-nucleon scattering in the scaling region. Our arguments or experiment, however, do not rule out the presence of some diffractive (or Pomeron exchange) component, particularly at large ω'. From the success of the relations between the elastic and resonance form factors and $\nu W_2(\omega')$ near $\omega'=1$, as well as from the measurements of inelastic electron-nucleon scattering in the same region, it seems that there is little or no diffractive component near $\omega'=1$. Further, from the size of the prominent resonance bumps which we can see at $q^2 \simeq 1\ \text{GeV}^2$ we estimate using the connection of resonances to non-Pomeron exchanges and that the resonances are to be added to the "background", that at least a quarter of νW_2 at $\omega'=5$ or 6 (where $\nu W_2 \simeq 0.33$) is due to the nondiffractive component. We thus expect νW_2 to decrease at least to ~ 0.25 at very large ω', but cannot demonstrate that the drop will be more than this.

There are, however, a number of theoretical reasons to expect an even larger nondiffractive component and a correspondingly greater drop with increasing ω'. First, if there is an isospin one exchange in the t-channel of virtual photon-nucleon scattering, as the observed difference in neutron and proton scattering implies exists, then one expects a corresponding isospin zero, non-Pomeron, exchange also. In Regge pole language, if A_2 exchange is present, then its SU(3) partner the P' or f also should be there if allowed by quantum numbers. In fact, from forward meson-baryon scattering[22] or from a quark-parton model of inelastic scattering one expects the P' to make a contribution to forward virtual photon-proton scattering which is roughly twice that of the A_2. Second, if we demand that certain sum rules be valid, then much more than 25% of νW_2 must be associated with non-Pomeron exchanges.[23] The relevant sum rules here include the Gottfried sum rule[24] and that of Cornwall, Corrigan and Norton,[25] as well as combinations of these with other sum rules.[26] Third, since there are presumably many other resonances present that cannot be seen as bumps in νW_2 at $q^2 \simeq 1$ GeV^2, our estimate of 25% for the nondiffractive part is certainly an underestimate, perhaps by a factor of two or better at $\omega'=5$ or 6.

All these theoretical "arguments" point toward νW_2 falling to ~ 0.15 or less as $\omega' \rightarrow \infty$, and they imply that at $\omega'=5$ or 6, half or more of the magnitude of νW_2 is to be associated with the nondiffractive, non-Pomeron exchange part of the amplitude. If this is true, and I emphasize that it has not been proven, then deep inelastic scattering is qualitatively different[26] at the maximum in νW_2 near $\omega' \simeq 6$ from elastic hadron-hadron scattering at say $\nu_{lab} = 6$ GeV where non-Pomeron exchanges are only 20 to 30% of the Pomeron exchange part of the forward amplitude. If for example the non-Pomeron and Pomeron exchange parts of νW_2 are equal at $\omega' \simeq 6$ then one may have to go to values of ω' between 50 and 100 to find oneself in the same situation as in elastic hadron-hadron scattering at a few GeV!

Among other things this could well mean that trying to fit $\nu W_2(\omega')$ to one or two Regge terms starting at $\omega' \sim 5$ may be a very dangerous business. Asymptotia may be far away and Regge behavior, in the sense of a description in terms of a few trajectories, may start at much larger values of ω'. This might also imply difficulties for attempts to relate the structure functions in the scaling region to the real ($q^2=0$) photon-nucleon total cross section.[27] While I do not feel that these attempts have been entirely successful quantitatively up to now in any case, a large difference between the ratio of non-Pomeron to Pomeron parts of the amplitude at $q^2=0$ and in the deep inelastic region could provide an insurmountable obstacle to their success.

Even in the absence of a definitive settlement of the question of how big the diffractive part of the amplitude for deep inelastic scattering actually is, a number of authors have tried to construct models of electroproduction where the hadronic final state is entirely a sum of resonances.[28] Naturally, many of these attempts have been within the framework of the Veneziano model. Up to now, all such attempts have been affected by at least one of two diseases[29]: either they have bad asymptotic behavior in ν or q^2 (e.g., Gaussian form factors, no scaling behavior, violation of current algebra constraints) or they lack factorization of the resonance couplings, which should be a basic property of any model based on resonances. In addition most of the models which agree with some aspect of experiment involve at least as many ad hoc assumptions or parameters as there are results obtained. Even so, such models often provide theoretical laboratories for examining certain questions and for showing the consistency of one kind of behavior with another. In particular, such models do show very explicitly the compatibility of a model world built entirely of resonances with the existence of scaling behavior and with the observed shape of $\nu W_2(\omega')$.

The beginnings of experimental observations of the hadronic final state in deep inelastic scattering make it an opportune time to discuss briefly where experiment and theory stand with respect to the quantum numbers and distributions of the produced hadrons.[30] There are already some restrictions from existing experiments on possible theories. For example, up to $q^2 \simeq 1\ \text{GeV}^2$ the completed experiments at Cornell,[10] CEA[11] and DESY,[11] as well as the inelastic neutrino events in the CERN heavy liquid bubble chamber,[5] already make it fairly clear that the final state is made up largely of pions and nucleons.[31] For $W < 2$ GeV, this is also implied by the prominent N* resonance bumps (whose decay modes are known) that can be seen in the single arm experiments. Also if the trend of the CEA data[11] continues to higher q^2 and ν, then the nucleon is not a "leading" particle along the direction of the incident virtual photon, i.e., as in photoproduction, the nucleon in the final state does not tend to go with large momentum along the direction of the photon's three-momentum. The "leading" particles along the photon's direction will then have to be pions, as in photoproduction.

In short, the little bits of experimental evidence we now have point toward the final state hadrons in electroproduction at $q^2 \simeq 1\ \text{GeV}^2$, where scaling behavior has already set in, not being wildly different from the final hadrons in photoproduction. If this statement holds up under the much more detailed experimental tests at larger q^2 and W^2 to which it will soon be subjected, we can already rule out a number of theories — examples are those theories which consider the nucleon or other baryons to be "partons" which would then be found in the final state moving along the incident photon's direction, those which predict large numbers of baryon-anti-baryon pairs in the final state, those which predict a large percentage of strange particles, etc.

More generally it would seem that the most naive versions of the parton model as applied to electroproduction also have serious difficulties if the "leading" final hadrons in electroproduction are similar to those in photoproduction, i.e., are mostly pions. This is because the measured value of $R=\sigma_S/\sigma_T$ suggests spin 1/2 partons, and because the sum rules for the sum of the parton charges squared and for the mean squared charge give small values for these quantities.[4] Both these facts would argue against the spin zero, integrally charged pion being a parton. In other words, what is needed in the naive parton models for interacting inside the nucleon with the incident virtual photon is not the same as what it seems is likely to be observed outside in the final state travelling along the direction of the incident photon. While one can always claim final state interactions change the naive partons to the final state hadrons, this makes it difficult to make unambiguous predictions for the final state hadrons.

The real question still remaining is how are the pions and nucleons distributed in the final state and what, if any, are the differences with photoproduction. Does, for example, the cross section for the diffractively produced rho and nucleon final state disappear relative to the overall total cross section at large q^2, as the Cornell experiments[10] suggest it might? Is the multiplicity changing with q^2 (fixed W)? Is the charged multiplicity anything like in photoproduction where it is quite consistent with a logarithmic increase with incident momentum up to 10 GeV at which point the average charged multiplicity in γp interactions is nearly four?

Without answering these questions, let us at least divide the proposed theories into two classes.[32] In the first class are those theories which predict a finite average multiplicity, $\bar{n}$, in the Bjorken limit of ν and $q^2 \to \infty$, but ω or (ω') fixed. In this case scaling is directly reflected in the hadron multiplicity and at least one particular multiplicity must also be finite in the Bjorken limit (barring undamped oscillations). Included in this class are the multiperipheral models[33] which exhibit scaling and where $\bar{n} \propto \ln\omega$, as well as some models where particular channels exhibit scaling.[32, 34] The second class is where the average multiplicity of final hadrons is infinite in the Bjorken limit. Included here are models as different as those which say the multiplicity is some increasing function of ν or W^2, e.g., $\bar{n} \propto \ln W^2$, but does not depend on q^2, and some recent work[35] on the predictions of the idea of limiting fragmentation which predicts that $\bar{n}$ increases with q^2 at fixed W.

While I would guess that the models in the first class will be shown to be wrong experimentally (Will for example, a final state with W = 3 GeV and $q^2 = 1\ \text{GeV}^2$ really have the same multiplicity as one with W = 9 GeV and $q^2 = 9\ \text{GeV}^2$?), I suspect that those mentioned above in the second class are also wrong. In particular, the increase in the photon's three-momentum as q^2 increases at fixed W leads one to expect excitation of different partial waves as q^2 changes at fixed W. Everything may not be as in photoproduction. Hopefully there are still some surprises waiting for us.

REFERENCES

1. For notation, kinematics, metric, and a review of the experimental and theoretical situation as of a year and a half ago, see the invited talks of R. E. Taylor and F. J. Gilman in Proceedings of the Fourth International Symposium on Electron and Photon Interactions at High Energies, Liverpool, 1969 (Daresbury Nuclear Physics Laboratory, Daresbury, Near Warrington, Lancashire, England, 1969). In this talk we shall not treat the parton or light cone commutator approaches to deep inelastic scattering, as these subjects are discussed in the talks of J. D. Bjorken and M. Gell-Mann at this conference. We also will not discuss the history or other aspects of duality ideas as applied to inelastic electron-nucleon scattering, such as that through the sum rules of H. Leutwyler and J. Stern (Phys. Letters 31B, 458 (1970)), or that of V. Rittenberg and H. Rubinstein (Ref. 27), which we leave to the discussion session.
2. See the rapporteur's talk of W.K.H. Panofsky in Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, 1969 (CERN Scientific Information Service, Geneva, Switzerland, 1969).
3. J. D. Bjorken, Phys. Rev. 179, 1547 (1969).
4. E. D. Bloom et al., "Recent results in inelastic electron scattering," Report No. SLAC-PUB-796, report presented to the XVth International Conference on High Energy Physics, Kiev, USSR (1970).
5. G. Myatt and D. H. Perkins, Oxford University preprint (1970) (unpublished); I. Budagov et al., Phys. Letters 30B, 364 (1969).
6. The value of R is small and compatible with zero within two standard deviations at each point where it is measured. Given the possible systematic errors it is possible, although not likely, that R=0; see Ref. 4. In any case,

the small uncertainty in R does not change any conclusion about scaling behavior in what follows.

7. While scaling in ω' may even extend to somewhat smaller values of q^2 than 1 GeV^2, we will not examine this in detail here.

8. The quantity ϵ varies between 0 and 1, depending on kinematics, and gives the proportion of longitudinal photons; see Ref. 1. Since $R=\sigma_S/\sigma_T$ is small, the variable value of ϵ makes little difference in Fig. 2.

9. While there may exist the possibility of nonnegligible deuterium corrections (see G. West, Stanford University preprint, 1971), we very much doubt that there will be any qualitative change in the results presented in Ref. 4.

10. D. E. Andrews et al., Cornell University preprint, CLNS-136 (1971) (unpublished).

11. A. M. Eisner et al., Bull. Am. Phys. Soc. 15, 1634 (1970); G. J. Feldman et al., ibid, 1634 (1970); C. Driver et al., DESY preprint (1971) (unpublished).

12. H. Harari, Phys. Rev. Letters 20, 1395 (1968); P. Freund, Phys. Rev. Letters 20, 235 (1968).

13. R. Dolen, D. Horn and C. Schmid, Phys. Rev. 166, 1768 (1968).

14. For the case of πN scattering, see Ref. 13 and F. J. Gilman, H. Harari and Y. Zarmi, Phys. Rev. Letters 21, 323 (1968).

15. E. D. Bloom and F. J. Gilman, Phys. Rev. Letters 25, 1140 (1970).

16. E. D. Bloom and F. J. Gilman, to be published.

17. E. D. Bloom et al., Phys. Rev. Letters 23, 930 (1969). M. Breidenbach, MIT thesis (1970) (unpublished).

18. G. Miller, Stanford thesis and Report No. SLAC-129 (1970) (unpublished).

19. We use fits of M. Breidenbach, Ref. 17, to the 6^0 and 10^0 spectra in terms of Breit-Wigner forms plus a polynomial background to obtain the magnitude of the resonance bumps. The quoted errors are those given by Breidenbach.

20. F. W. Brasse et al., DESY preprint (1970) (unpublished).

21. S. D. Drell and T. M. Yan, Phys. Rev. Letters 24, 181 (1970). See also G. West, Phys. Rev. Letters 24, 1206 (1970).

22. See, for example, the recent Regge analysis of electroproduction by H. R. Pagels, Phys. Rev., to be published.

23. See F. J. Gilman, Invited talk presented at the Symposium on Hadron Spectroscopy, Balatonfured, Hungary, September 1970, Report No. SLAC-PUB-842 (unpublished).

24. K. Gottfried, Phys. Rev. Letters 18, 1174 (1967).

25. J. M. Cornwall, D. Corrigan, and R. E. Norton, Phys. Rev. Letters 24, 1171 (1970).

26. I thank F. Close and J. Gunion for discussions of their work on Regge fits and sum rules in electroproduction and C. H. Llewellyn Smith and H. Harari for discussions on fixed poles and Regge behavior in hadronic processes and electroproduction.

27. See, for example, the recent work by V. Rittenberg and H. Rubinstein, Weizmann Institute preprint (1971) (unpublished).

28. Out of many works on this subject, see for example the recent models of P. V. Landshoff and J. C. Polkinghorne, Nucl. Phys. B19, 432 (1970) and that of G. Domokos and S. Kovesi-Domokos, Johns Hopkins University preprints (1970) (unpublished).

29. See the review by M. Ademollo, talk presented at the Symposium on Meson Photo- and Electroproduction at Low and Intermediate Energies, Bonn, September 1970 (unpublished).

30. Many aspects of this and related subjects are discussed in the talk at this conference by J. D. Bjorken.

31. This is not to imply that various strong interaction resonances are not intermediate states; just that the final decay products are ultimately pions and nucleons.

32. This division is close to one discussed by T. D. Lee, Columbia University preprint (1970), to be published in Ann. Phys. (N.Y.).

33. A very recent example is the work of S. S. Shei and D. M. Tow, Phys. Rev. Letters 26, 470 (1971).

34. See also in this connection the light cone expansions for exclusive processes of Y. Frishman *et al.*, Weizmann Institute preprint (1971) (unpublished).

35. H. T. Nieh and J. M. Wang, Stony Brook preprint (1971) (unpublished).

FIGURE CAPTIONS

1. The functions W_1 and νW_2 plotted versus $\omega' = 1+W^2/q^2$ for $W \geq 2$ GeV and various ranges of q^2 assuming $R = \sigma_S/\sigma_T = 0.18$.
2. Measured values[4] of $\sigma_T + \epsilon\sigma_S$ for various final hadron masses, W, plotted versus q^2/W^2. The solid line is $G^2_{Ep}(q^2) + (q^2/4M^2_N)\, G^2_{Mp}(q^2)$, the elastic analogue of $\sigma_T+\sigma_S$, under the assumption of dipole form factors. Values for the N*(1520) cross section are taken from the thesis of M. Breidenbach.[19]
3. The function $\nu W_{2p}(\nu, q^2)$ plotted versus $\omega' = 1 + W^2/q^2$ from an interpolation of data to a fixed q^2 value of 1.0 GeV^2. The solid line is the scaling-limit-curve, $\nu W_2(\omega')$, a smooth fit[18] to the data in the scaling region where $q^2 \geq 1$ GeV^2, $W \geq 2$ GeV. The arrow indicates the position of the elastic peak.
4. Same as Fig. 3, but for $q^2 = 1.5$ GeV^2.
5. Same as Fig. 4, but for $q^2 = 2.0$ GeV^2.
6. Same as Fig. 5, but for $q^2 = 2.5$ GeV^2.
7. Same as Fig. 6, but for $q^2 = 3.0$ GeV^2.
8. The ratio of the height of the N*(1238) bump in νW_2 to the value of the scaling-limit-curve, $\nu W_2(\omega')$, at the corresponding value of $\omega' = 1 + M^2_{N*}/q^2$ for values of q^2 between 1.0 and 3.0 GeV^2. Values for the height of the resonance bump are taken from fits to the 6° and 10° inelastic spectra by M. Breidenbach.[19] The values of $\nu W_2(\omega')$ are from G. Miller.[18]
9. Same as Fig. 8, but for the N*(1520).
10. Same as Fig. 9, but for the N*(1688).
11. Computed values of $\nu W_{2p}(\omega')$ for $W_t = 1.08$ and 1.23 GeV using Eq. (12) and the measured proton elastic form factors. The data points are from the large angle experiments,[4] plotted assuming $R = 0.18$.

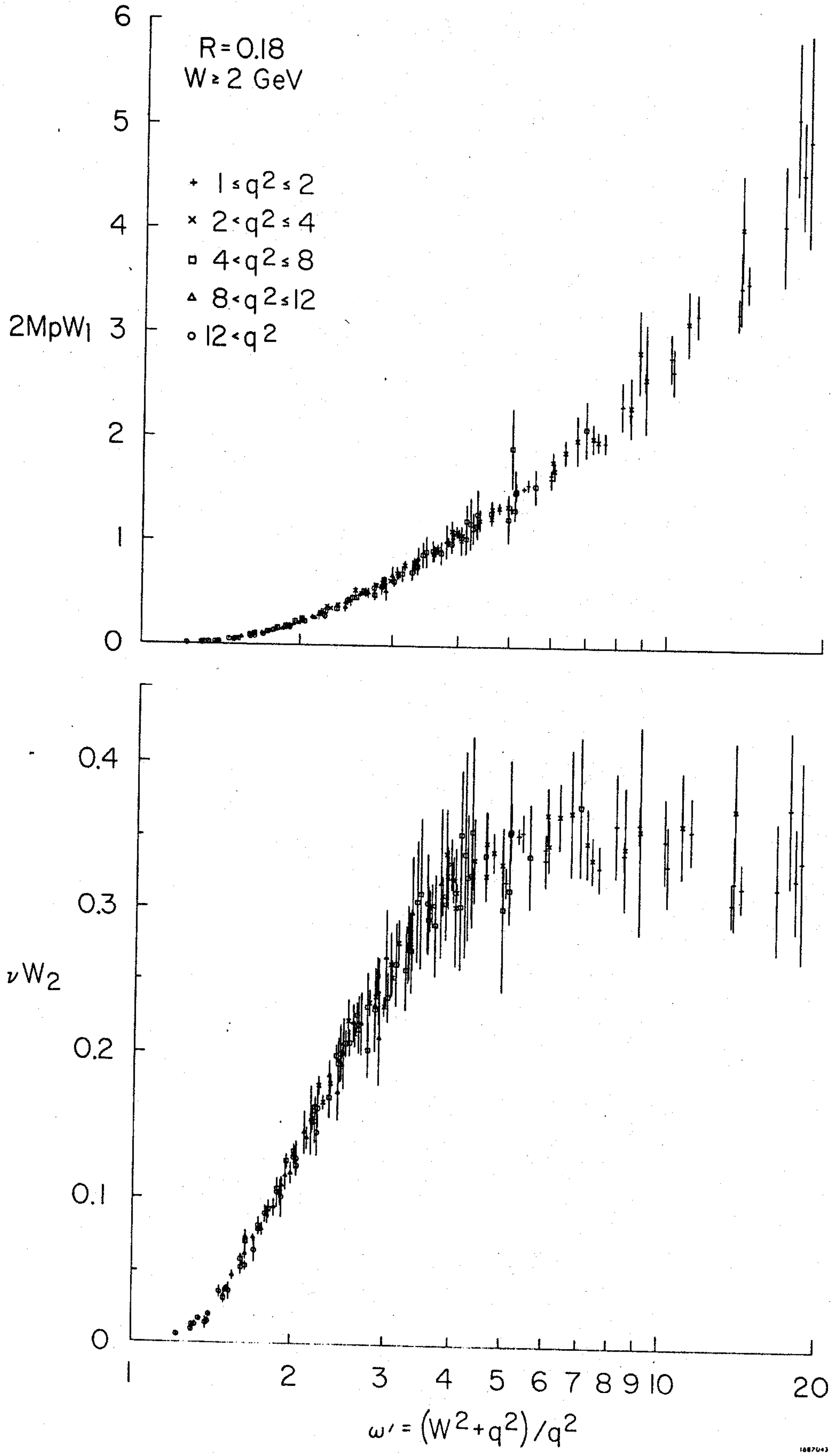
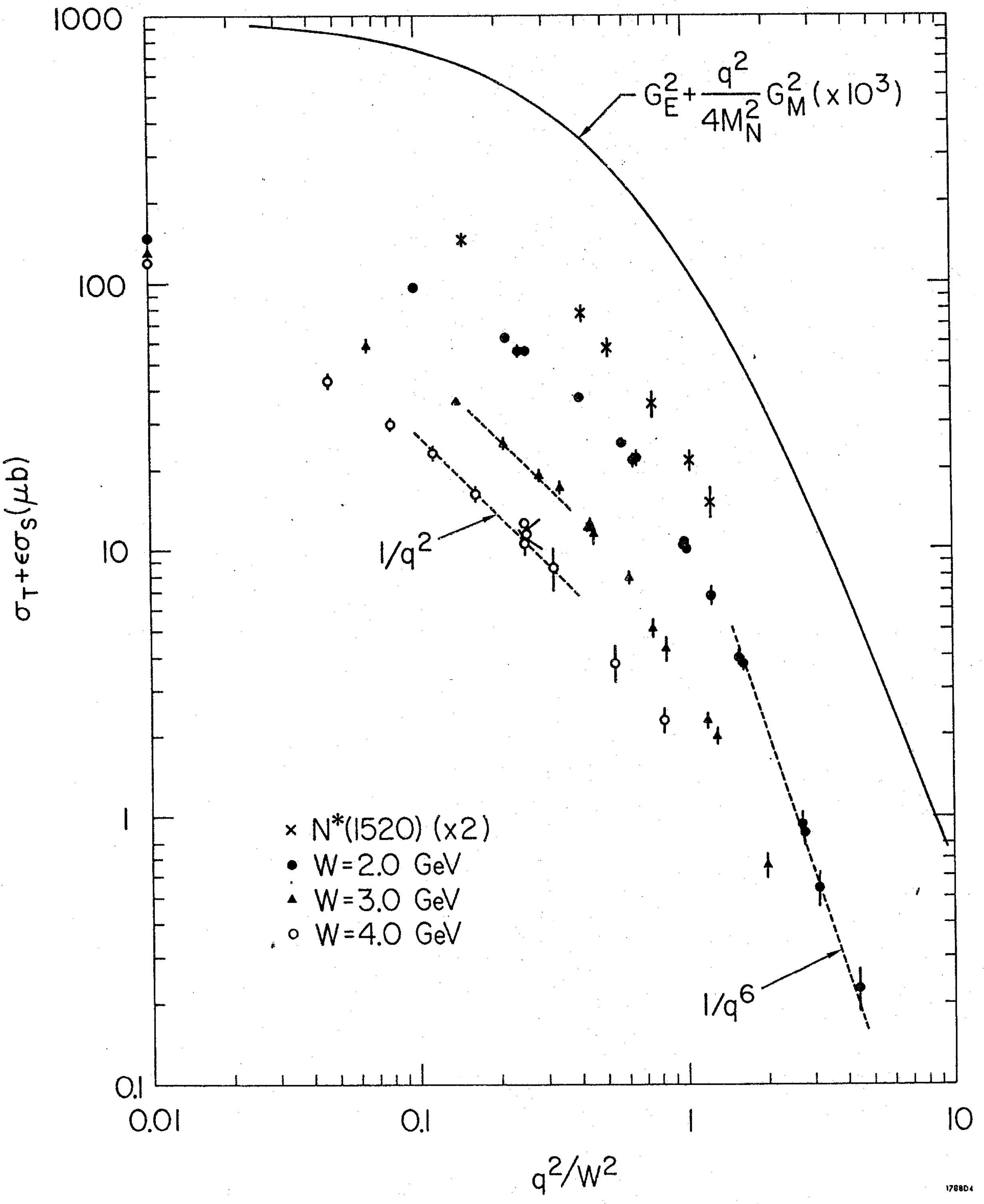
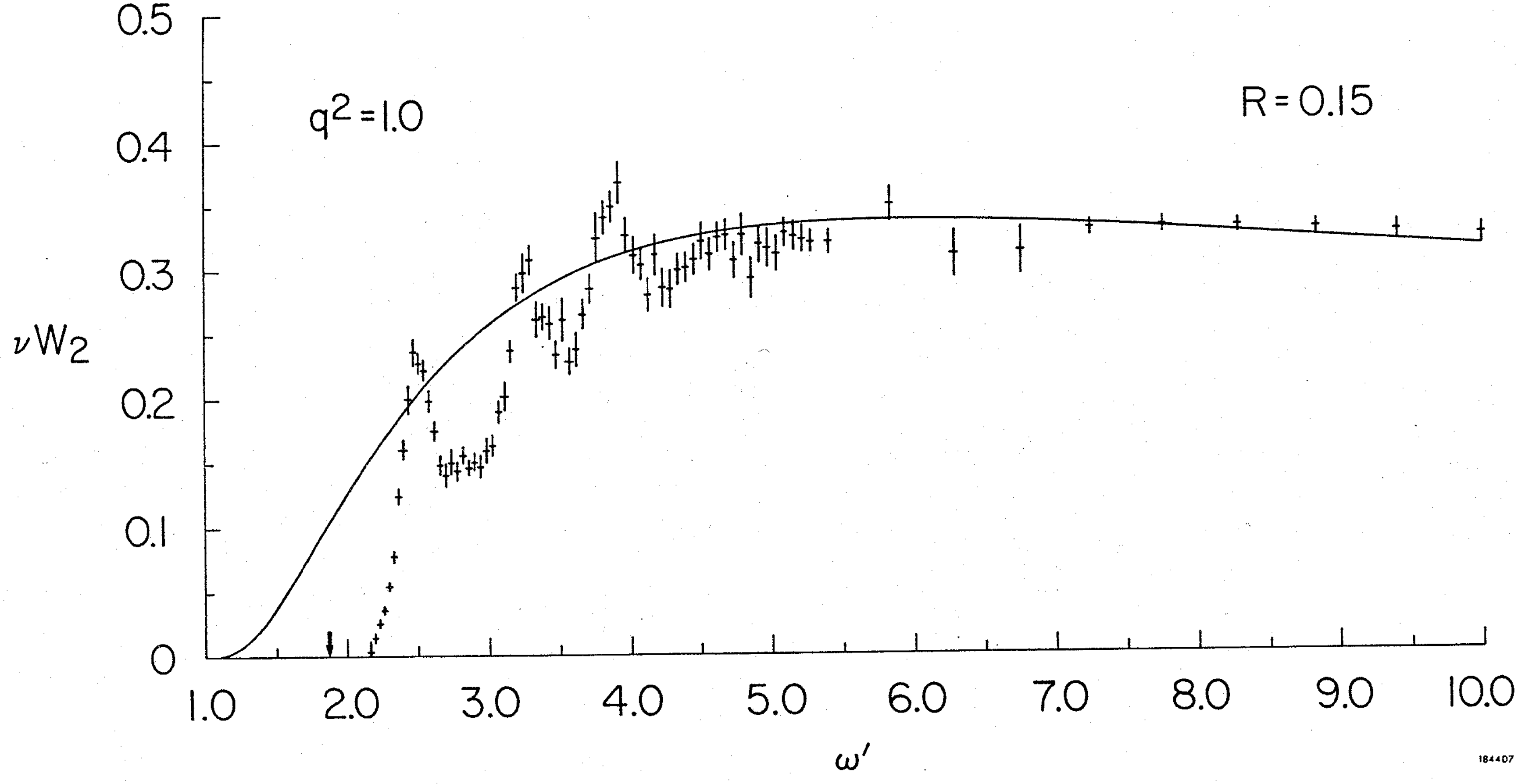

R=0.18
W ≥ 2 GeV
+ 1 ≤ q2 ≤ 2
× 2 < q2 ≤ 4
□ 4 < q2 ≤ 8
△ 8 < q2 ≤ 12
○ 12 < q2
2MpW1
νW2
ω' = (W2+q2)/q2
0
1
2
3
4
5
6
0.1
0.2
0.3
0.4
7
8
9
10
20

Fig. 1



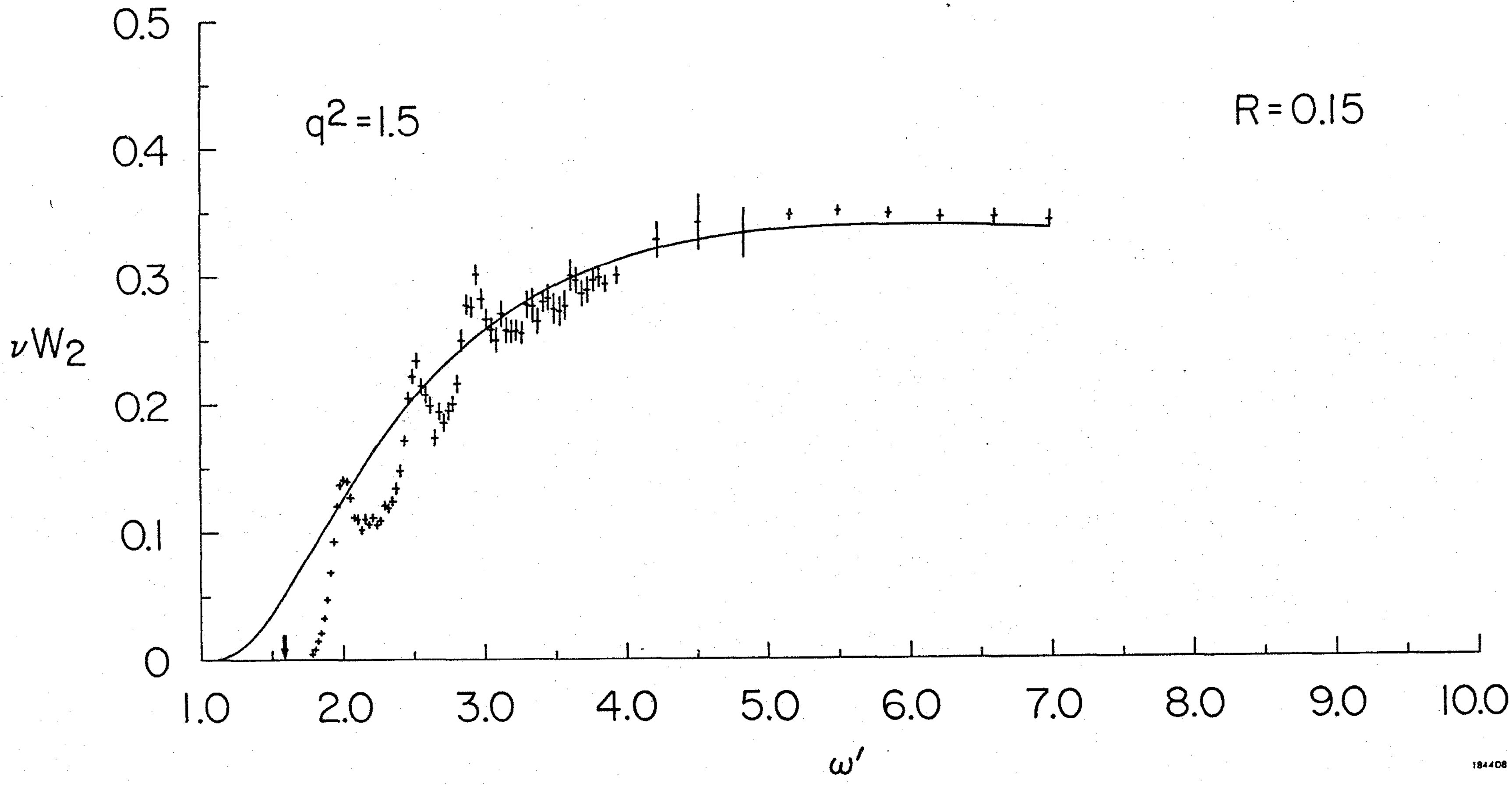
1000
100
10
1
0.1
0.01
0.1
1
10
$\sigma_T + \epsilon\sigma_S (\mu b)$
q^2/W^2
$G_E^2 + \frac{q^2}{4M_N^2} G_M^2 (\times 10^3)$
$1/q^2$
$1/q^6$
× N*(1520) (x2)
● W=2.0 GeV
▲ W=3.0 GeV
○ W=4.0 GeV
1788D4

Fig. 2



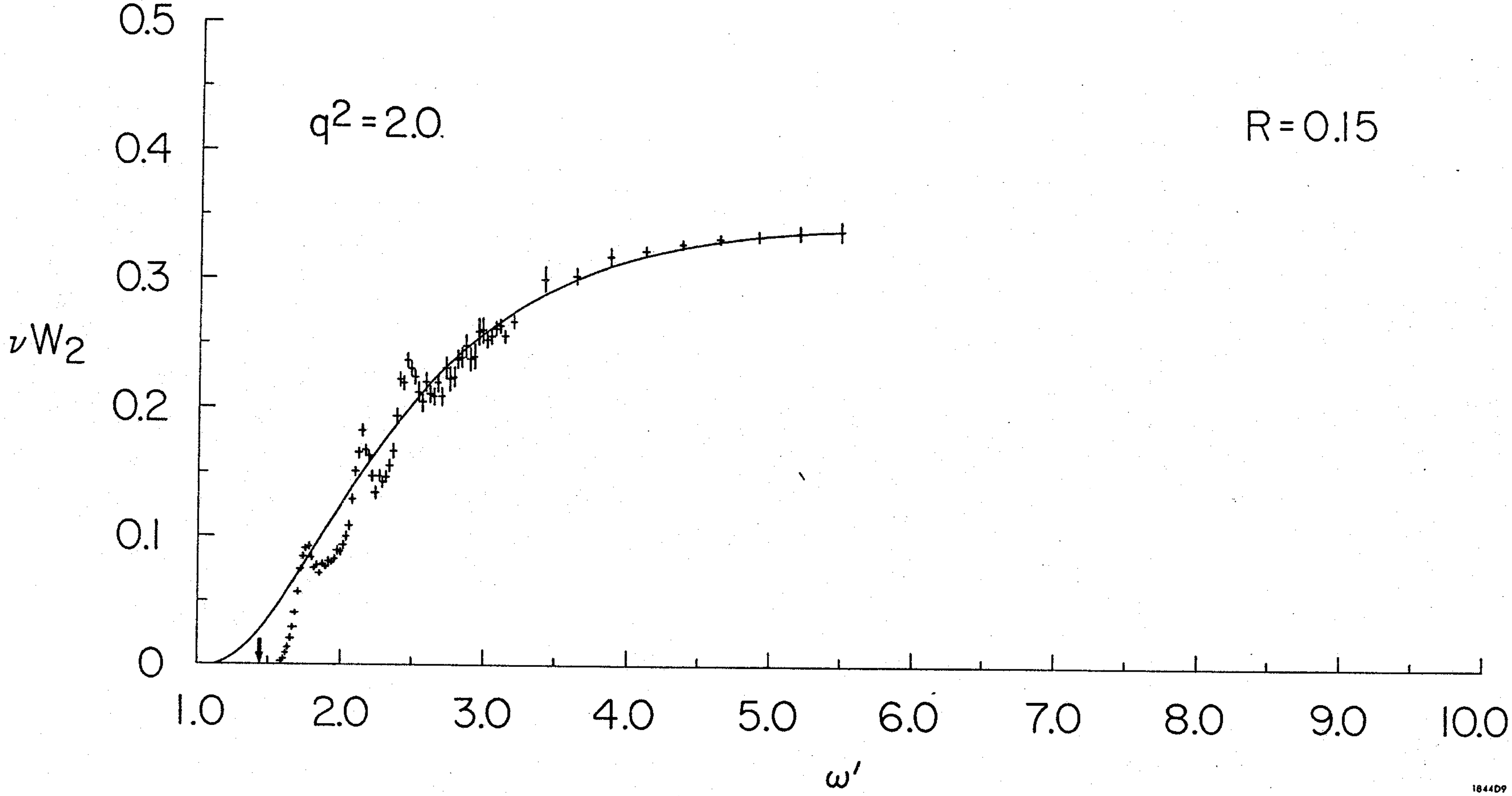
0.5
0.4
0.3
0.2
0.1
0
νW_2
$q^2 = 1.0$
R = 0.15
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
ω'
184407

Fig. 3



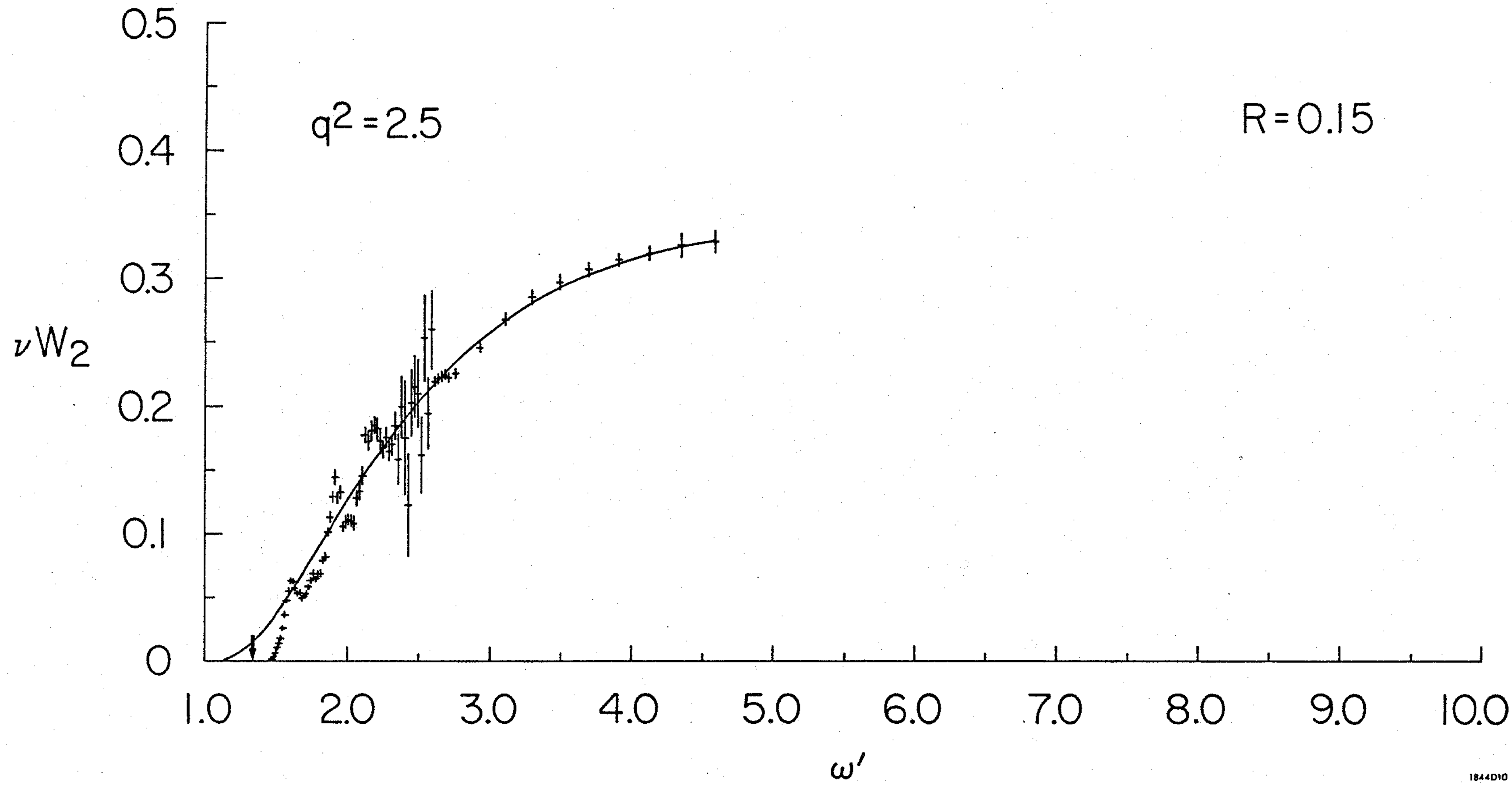
0.5
0.4
0.3
0.2
0.1
0
νW_2
$q^2 = 1.5$
R = 0.15
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
ω'
1844D8

Fig. 4



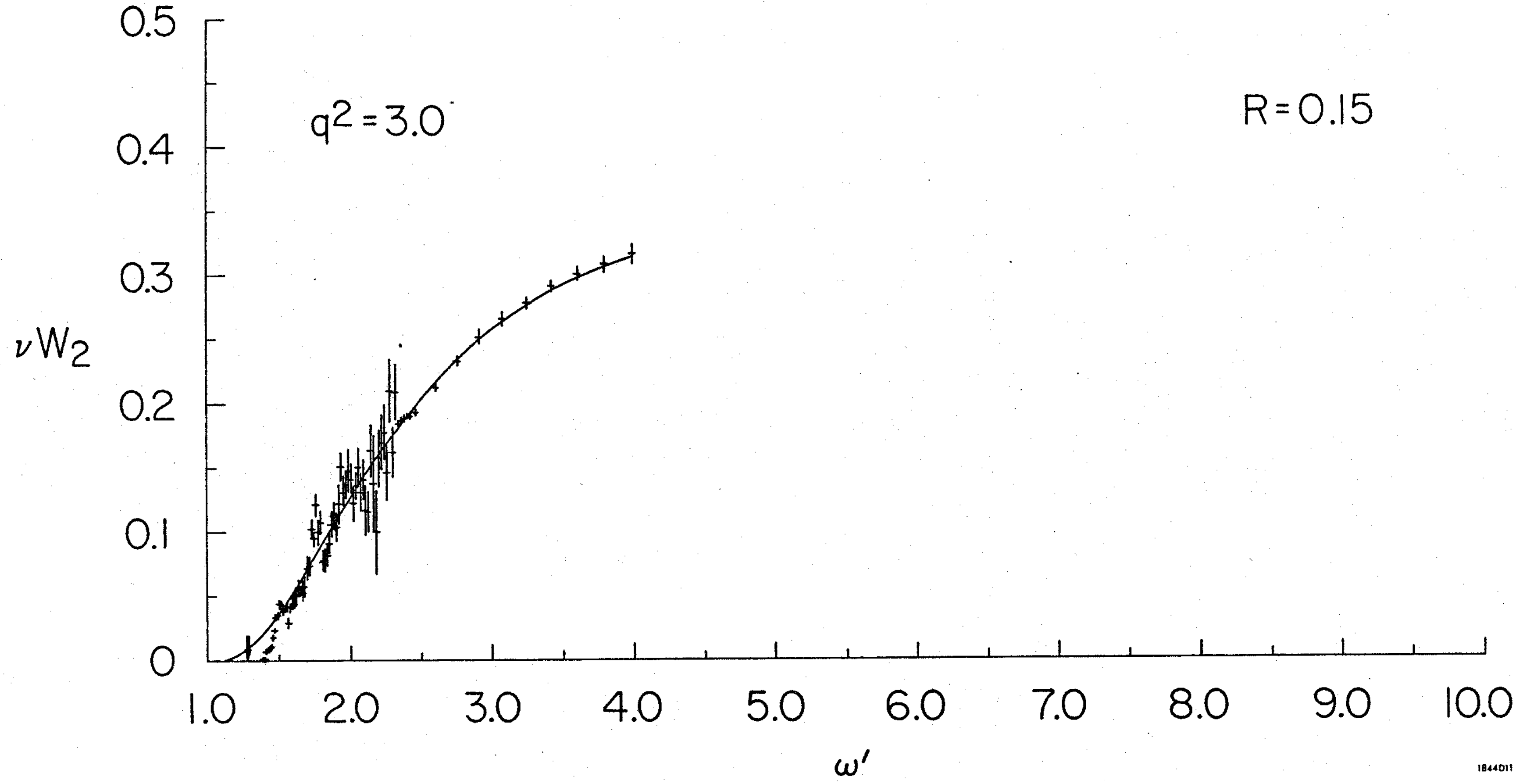
0.5
0.4
0.3
0.2
0.1
0
$q^2 = 2.0$
R = 0.15
νW_2
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
ω'
1844D9

Fig. 5



0.5
0.4
0.3
0.2
0.1
0
νW_2
$q^2 = 2.5$
R = 0.15
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
ω'
1844D10

Fig. 6



0.5
0.4
0.3
0.2
0.1
0
νW_2
$q^2 = 3.0$
R = 0.15
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
ω'
1844D11

Fig. 7

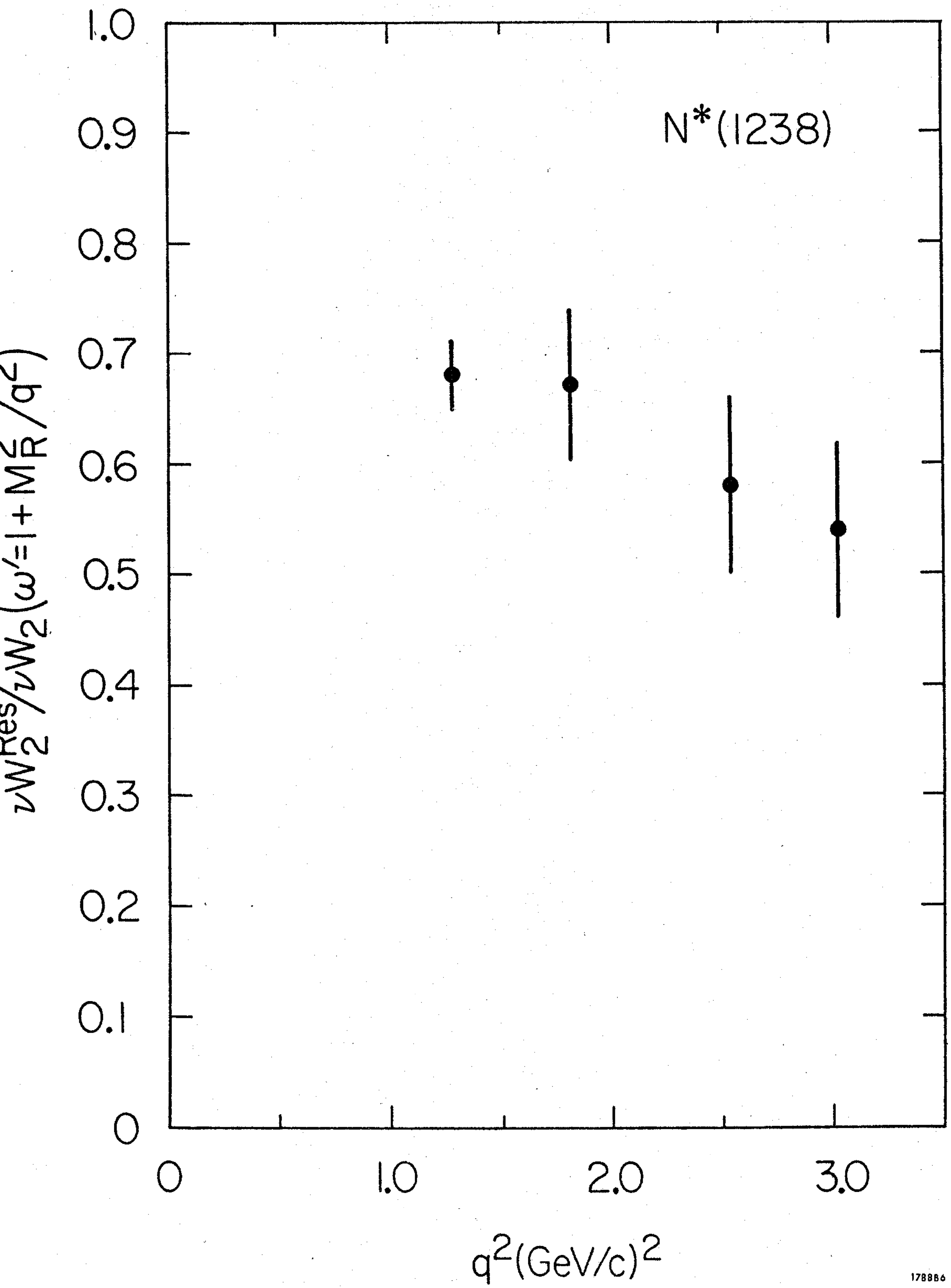

N*(1238)
$\nu W_2^{Res}/\nu W_2(\omega' = 1 + M_R^2/q^2)$
$q^2 (GeV/c)^2$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1.0
2.0
3.0
178886

Fig. 8

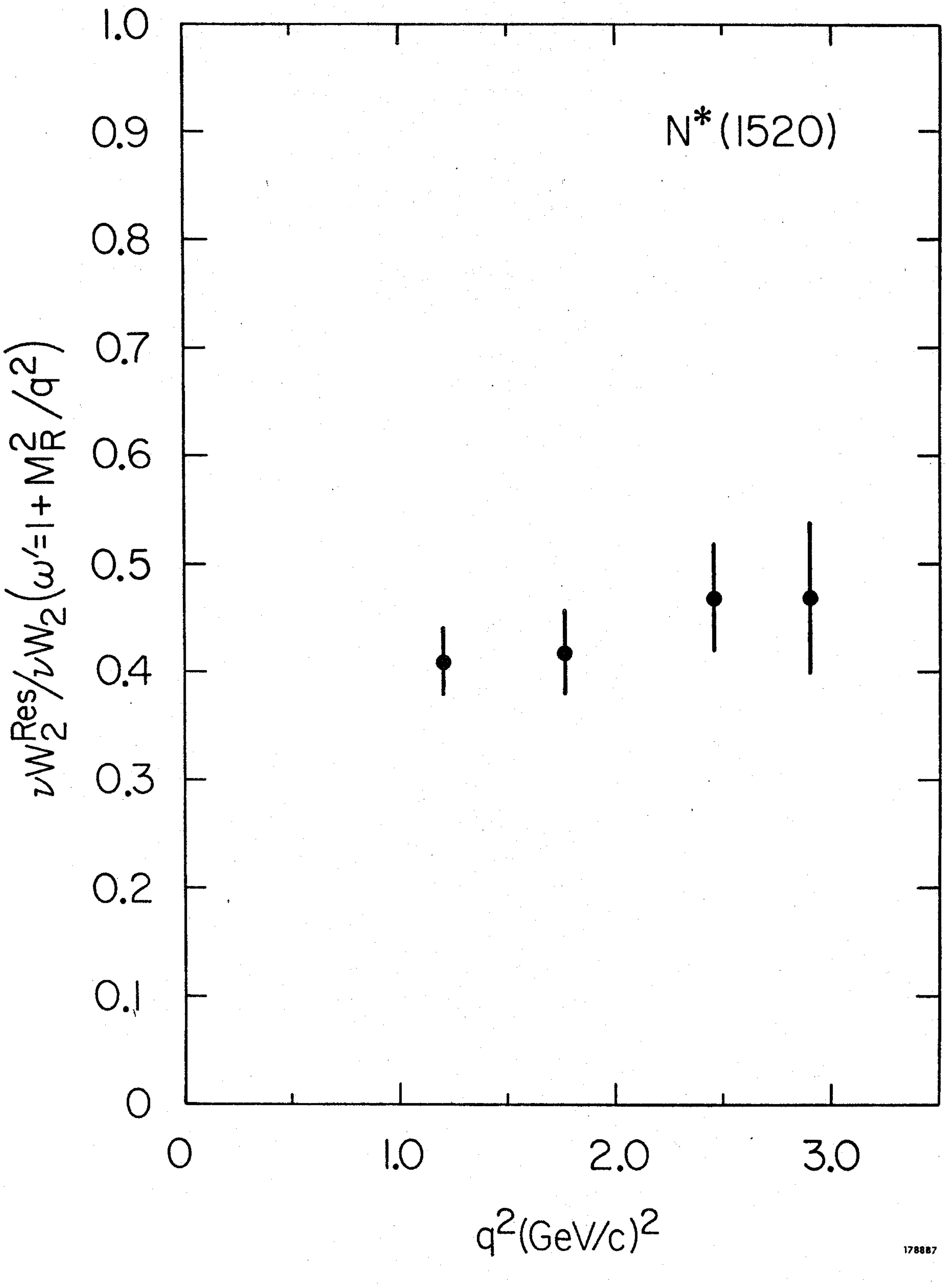
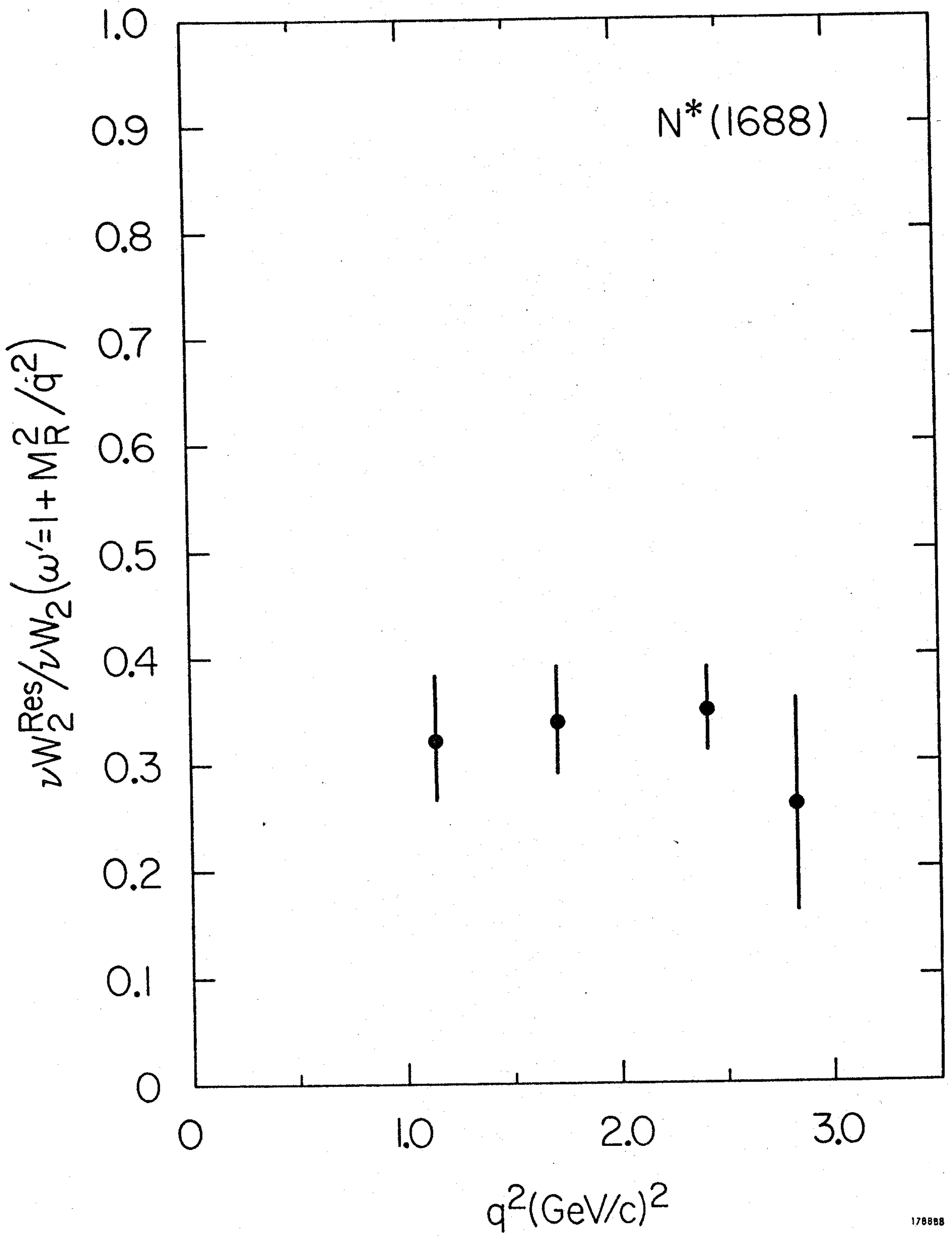
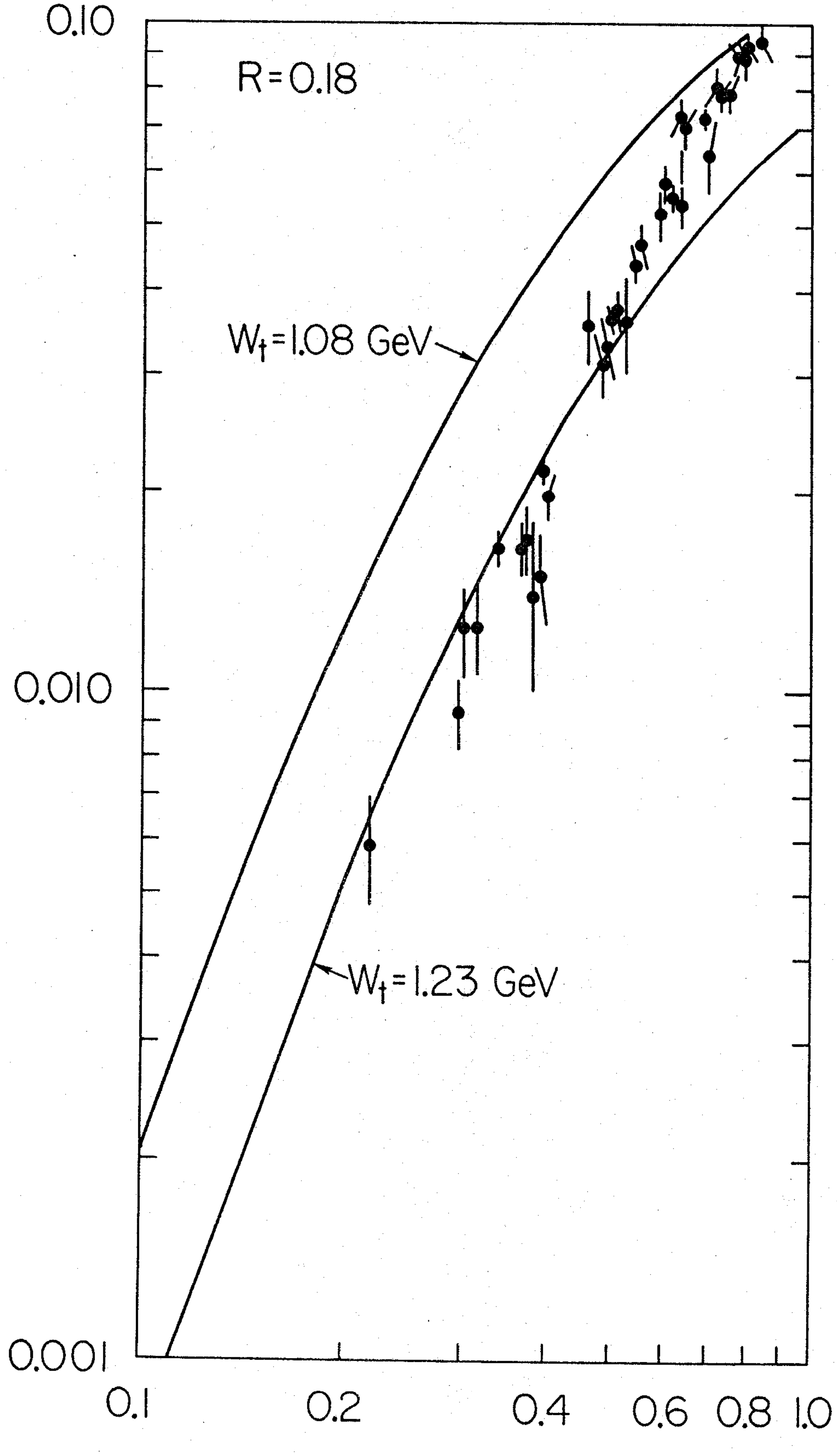

N*(1520)
$\nu W_2^{Res}/\nu W_2(\omega'=1+M_R^2/q^2)$
$q^2 (GeV/c)^2$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1.0
2.0
3.0
178887

Fig. 9



N*(1688)
$\nu W_2^{Res}/\nu W_2(\omega' = 1 + M_R^2/q^2)$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1.0
2.0
3.0
$q^2 (GeV/c)^2$
178888

Fig. 10



0.10
R=0.18
W_t=1.08 GeV
0.010
W_t=1.23 GeV
0.001
0.1
0.2
0.4
0.6
0.8
1.0
ω'-1
1788C15

Fig. 11