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# ELASTIC $K^{\pm}p$ SCATTERING AND A DUAL ABSORPTIVE MODEL*

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## ABSTRACT

A recent measurement of the differential cross sections for elastic $K^+p$ and $K^-p$ scattering is discussed within the framework of a dual absorptive model which was proposed earlier. The non-Pomeron part of the elastic scattering amplitude is shown to be strongly dominated by the most peripheral partial waves within the interaction radius, namely — by the $\ell \sim qr$ partial waves.

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A simple qualitative model of elastic hadron scattering has recently been proposed[1]. The model incorporates the basic ideas of duality as well as the physical intuition of the absorption model, and it successfully explains the systematics of the $|t| \sim 0.6\ \mathrm{GeV}^2$ dips, the crossover phenomena, the relative strengths of particle and antiparticle elastic cross sections and certain features of the elastic polarizations.

A meaningful quantitative test of the model necessitates an analysis of the differences between $\frac{d\sigma}{dt}(xp \to xp)$ and $\frac{d\sigma}{dt}(\bar{x}p \to \bar{x}p)$, $x = K^+, \pi^+, p$, etc. This, in turn, requires accurate data for elastic $xp$ and $\bar{x}p$ scattering at the same energy, preferably using the same experimental technique (or even the same apparatus).

A recent experiment performed at CERN[2] has yielded an accurate determination of the $K^+p$ and $K^-p$ elastic differential cross sections at $p_{lab} = 5$ GeV/c, $|t| > 0.2\ \mathrm{GeV}^2$. In the present note we study the results of this experiment from the viewpoint of the dual absorptive model mentioned above. We find that the model is strongly supported by the new data. Our most significant conclusion is that the ordinary (non-Pomeron) exchanges are strongly dominated by the peripheral partial waves, as predicted by the absorptive model, and as opposed to the non-peripheral behavior predicted by pure Regge pole models.

We first quote the basic assumptions of the model[1]:

(1) The imaginary part of the elastic scattering amplitude is given by a sum of two components[3] — a Pomeron exchange term $P(s,t)$ which is dual to the s-channel background and an ordinary (non-Pomeron) exchange term $R(s,t)$ which is dual to the s-channel resonances. The s-dependence of P and R is given by $s^{\alpha_P(t)}$ and $s^{\alpha_R(t)}$, respectively, and the ratio R/P decreases as the energy increases. We shall omit the energy dependence throughout this paper.

(2) The P-term approximately conserves the s-channel helicity[4] and it involves significant contributions from all partial waves $\ell \leq qr$ (q — c.m. momentum; r — the interaction radius). Experiment indicates that P(t) is structureless* (no dips, bumps, etc.) for $|t| < 1$ GeV$^2$.

(3) The R-term is assumed to be dominated[1] by the most peripheral partial waves $\ell \sim qr$. For a total s-channel helicity flip $\Delta\lambda$, R(t) will therefore exhibit zeroes, minima and maxima similar to those of the Bessel function $J_{\Delta\lambda}(r\sqrt{-t})$, where $r \sim 1$ fermi.[5]

(4) The real part of the Pomeron term is negligible. The real part of the non-Pomeron term is derived from its imaginary part[1] using the asymptotic relation between the energy dependence and the phase of the amplitude.[6] We will not use this assumption in the present paper.

(5) The elastic differential cross section at small t is approximated, at energies above a few GeV, by its two leading terms — $[P(t)]^2$ and P(t)R(t). We neglect terms of the form $[R(t)]^2$. Since the Pomeron term is predominantly imaginary and has $\Delta\lambda = 0$, we essentially have:

$$\frac{d\sigma}{dt} \sim |\operatorname{Im} f^s_{\Delta\lambda=0}|^2 \ .$$

For exotic s-channel processes such as $K^+p \to K^+p$, we have no s-channel resonances and $R(t) \sim 0$.

In the case of $K^+p$ and $K^-p$ elastic scattering at $p_{lab} = 5$ GeV/c we therefore write:

$$\frac{d\sigma}{dt}(K^+p) = [P(t)]^2$$

$$\frac{d\sigma}{dt}(K^-p) = [P(t)]^2 + 2P(t)R(t) \ .$$

R(t) can be easily extracted from the data:

$$R(t) = \frac{\frac{d\sigma}{dt}(K^-p) - \frac{d\sigma}{dt}(K^+p)}{2\sqrt{\frac{d\sigma}{dt}(K^+p)}}$$

For $|t| > 0.2$ we use the data of reference 2. For $|t| < 0.2$ we have interpolated the available bubble chamber data.[7] The amplitude R(t) as deduced from the data is shown in figure 1. R(t) is predicted[1] to have the features of $J_0(r\sqrt{-t})$. We therefore parametrize it by:

$$R(t) = A e^{Bt} J_0(r\sqrt{-t}) .$$

A good fit is obtained for $A = 1.6 \text{ mb}^{\frac{1}{2}} \text{ GeV}^{-1}$; $B = 1.3 \text{ GeV}^{-2}$; $r = 4.8 \text{ GeV}^{-1} = 0.95$ fermi. The lower quality of the low-t data does not justify the pursuit of a "best fit", but it is evident from the figure that our fit is perfectly acceptable and that R(t) does have the features of a $J_0$ Bessel function.

Our next step is to investigate the impact parameter representation or the partial wave expansion of R(t). We may evaluate the Fourier-Bessel transform of our fit for R(t) or we may numerically project the partial wave amplitudes, using the "experimental" R(t) of figure 1.

The impact parameter representation for R(t) is given by:

$$f(b) = \int_0^{t_{max}} R(t) J_0(b\sqrt{-t})\, dt$$

With our parametrization for R(t) we find

$$f(b) = \frac{A}{B}\, e^{-\frac{r^2+b^2}{4B}}\, I_0\left(\frac{rb}{2B}\right)$$

where $I_0$ is a Bessel function of an imaginary argument.

f(b) has a strong peak around b ~ r and most of its strength is given by the impact parameters around this value, as predicted by the absorptive picture. Alternatively, we may write:

$$R(t) = \frac{\sqrt{\pi}}{q^2} \sum_J (J + \tfrac{1}{2})\, d^J_{\frac{1}{2}\frac{1}{2}}(\theta)\, a_J$$

The partial wave amplitude $a_J$ is then given by:

$$a_J = \frac{1}{\sqrt{\pi}\,(2J+1)} \int R(t) \cos\frac{\theta}{2} \left(P'_{J+\frac{1}{2}} - P'_{J-\frac{1}{2}}\right) dt$$

Figure 2 shows the $a_J$ amplitudes corresponding to R(t). The peripheral nature of this contribution is very dramatic and it confirms that the imaginary part of the non-Pomeron amplitude is dominated by the $\ell \sim qr$ partial waves. Within the framework of duality this means that the important s-channel resonances appear in the peripheral partial waves.

We may ask whether the impact parameter (or partial wave) representation of P(t) indeed shows significant contributions from all $\ell \leq qr$ partial waves. Figure 3 shows the $a_J$ coefficients for P(t) (denoted by $K^+p$ in the figure) as well as for P(t) + R(t) (denoted by $K^-p$ in the figure). It is obvious that the Pomeron amplitude is dominated by $\ell < qr$ partial waves. Notice that R(t) appears in figure 3 as a relatively minor correction to the P(t)-term. This leads us to two important observations:

(i) The terms of order $[R(t)]^2$ that we have neglected are much smaller than P(t)R(t).

(ii) Significant information on R(t) such as the one displayed in figures 1 and 2 can be obtained only from very accurate measurements of the differential cross section for the particle and antiparticle elastic processes at the same energy.

A few remarks concerning our approximations:

(1) We have neglected all $\Delta\lambda = 1$ terms. This is presumably justified for the smaller t-values[4], but may be more dubious for $|t| \sim 1\ \mathrm{GeV}^2$.

(2) The real part of the Pomeron term is certainly negligible around $t \sim 0$. If, however, $\alpha_p(t) \sim 1 + 0.4\,t$, we may have a large real part at $t \sim 1\ \mathrm{BeV}^2$. This will not change the qualitative results but would affect our quantitative statements.

(3) The $[R(t)]^2$ term as well as possible contributions of double particle exchange are negligible at small t, but, again, could be important at larger t-values.

All of these items indicate that for larger and larger t-values our simple description becomes less and less reliable. However, in terms of the partial wave projections of figures 2 and 3, no <u>major</u> changes would result from large-t contributions. As long as the <u>main</u> features of R(t) for $|t| \leq 1$ remain unchanged, all of our conclusions survive. Notice also that many of the neglected terms contribute equally to $K^+p$ and $K^-p$ scattering and would therefore not affect our analysis of R(t).

As the energy increases the $[R(t)]^2$ terms as well as the double particle exchanges become less and less important and our approximation improves. At the same time, however, the ratio R/P decreases and the accuracy required for extracting R(t) becomes more difficult to achieve, experimentally. It seems to us that an appropriate energy range for studying the nature of the P(t) and R(t) amplitudes may be anywhere above a few GeV, with stronger accuracy requirement at higher energies. There should also be large differences between different reactions. For instance, the ratio R/P for pp and $\bar{p}p$ elastic scattering at any given energy is roughly twice as large as the corresponding ratio for $K^{\pm}p$ scattering at the same energy. Consequently, an analysis such as ours could become quantitatively meaningful for pp and $\bar{p}p$ scattering only above, say, 8 or 10 GeV/c (although the qualitative features of the dip, crossover, etc. are already observed at 2-3 GeV and are confirmed by the recent 5 GeV/c measurements of $\bar{p}p$ elastic scattering[8]).

Another interesting question which can be answered by accurate elastic scattering experiments at different energies is the energy dependence of the parameters r and B in our expression for R(t). These parameters measure the position and width of the peak in the impact parameter description of R(t) (figure 2). Their energy dependence should be extremely interesting.

We believe that the evidence presented here strongly supports the dual absorptive model of reference 1. More important is, however, our general result, which does not depend on the details of the model. We have shown that the non-Pomeron exchange term in the elastic amplitude is definitely peripheral. This confirms the idea that such exchanges are subject to strong absorption corrections in one way or the other[5,1] and that models involving only Regge poles are qualitatively inadequate. This conclusion cannot be ignored in phenomenological or theoretical work related to hadronic reactions.

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## FOOTNOTE

* This was concluded in reference 1 on the basis of the absence of $|t| < 1$ GeV$^2$ dips in $K^+p$ and pp elastic scattering, where the non-Pomeron exchanges are absent in the imaginary part. The new $K^+p$ data of reference 2 confirms this with greater accuracy.

## FIGURE CAPTIONS

### Figure 1

The experimental results for

$$\frac{\frac{d\sigma}{dt}(K^-p) - \frac{d\sigma}{dt}(K^+p)}{2\sqrt{\frac{d\sigma}{dt}(K^+p)}}$$

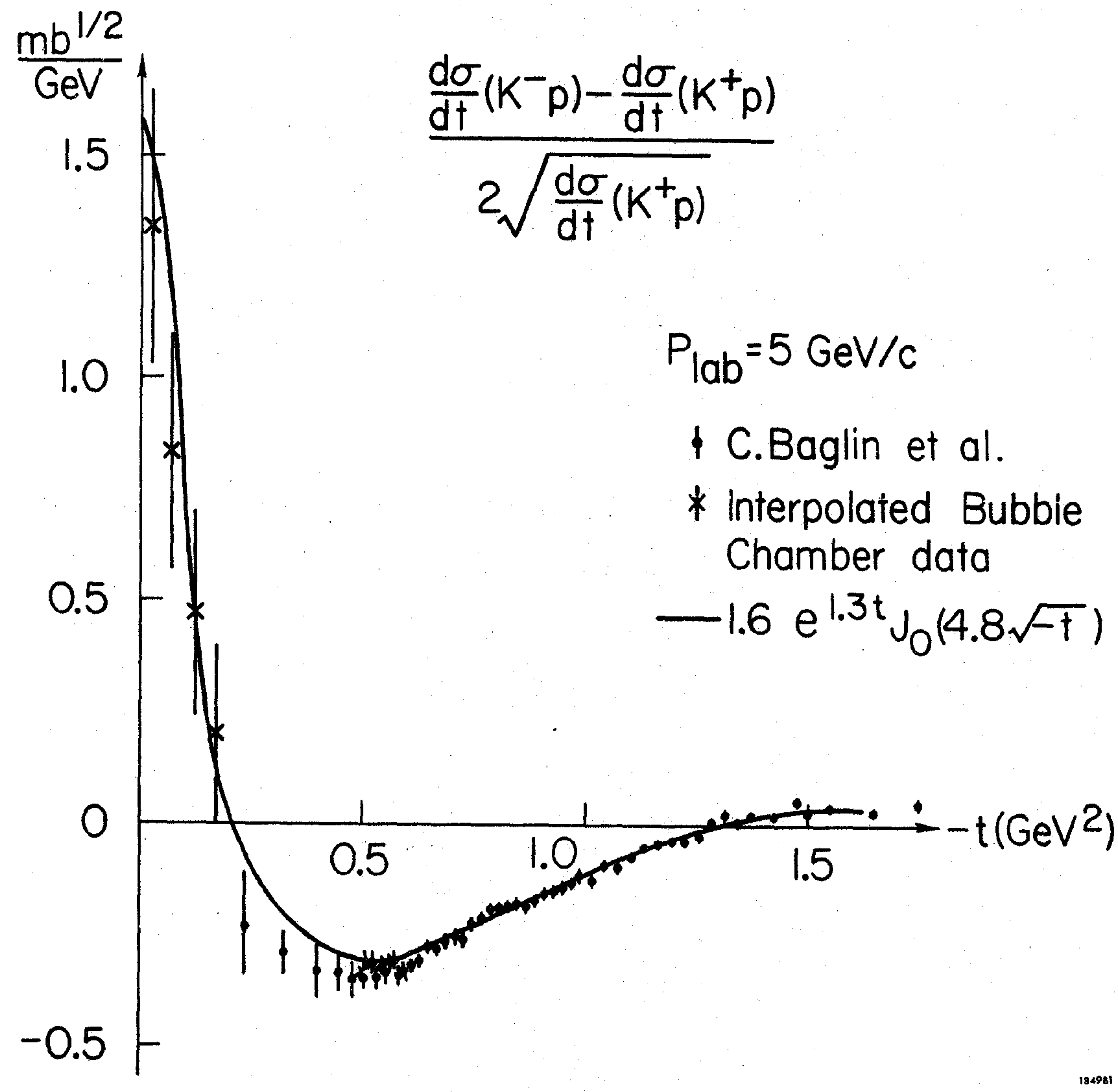
as a function of t and $p_{lab} = 5$ GeV/c. For $|t| < 0.2$ GeV$^2$ we have interpolated the 5 GeV/c $K^+p$ bubble chamber data of de Baere et al.[7] and the 5.5 GeV/c $K^-p$ data of Mott et al.[7], using the parametrization $A e^{Bt}$. Above $|t| = 0.2$ GeV$^2$ the data are those of reference 2. Only statistical errors are shown.

### Figure 2

Legendre coefficients $a_J$ (as defined in the text) for the amplitude R(t) shown in Figure 1. The dashed area between the two curves represents the uncertainty introduced by both statistical and systematic errors.

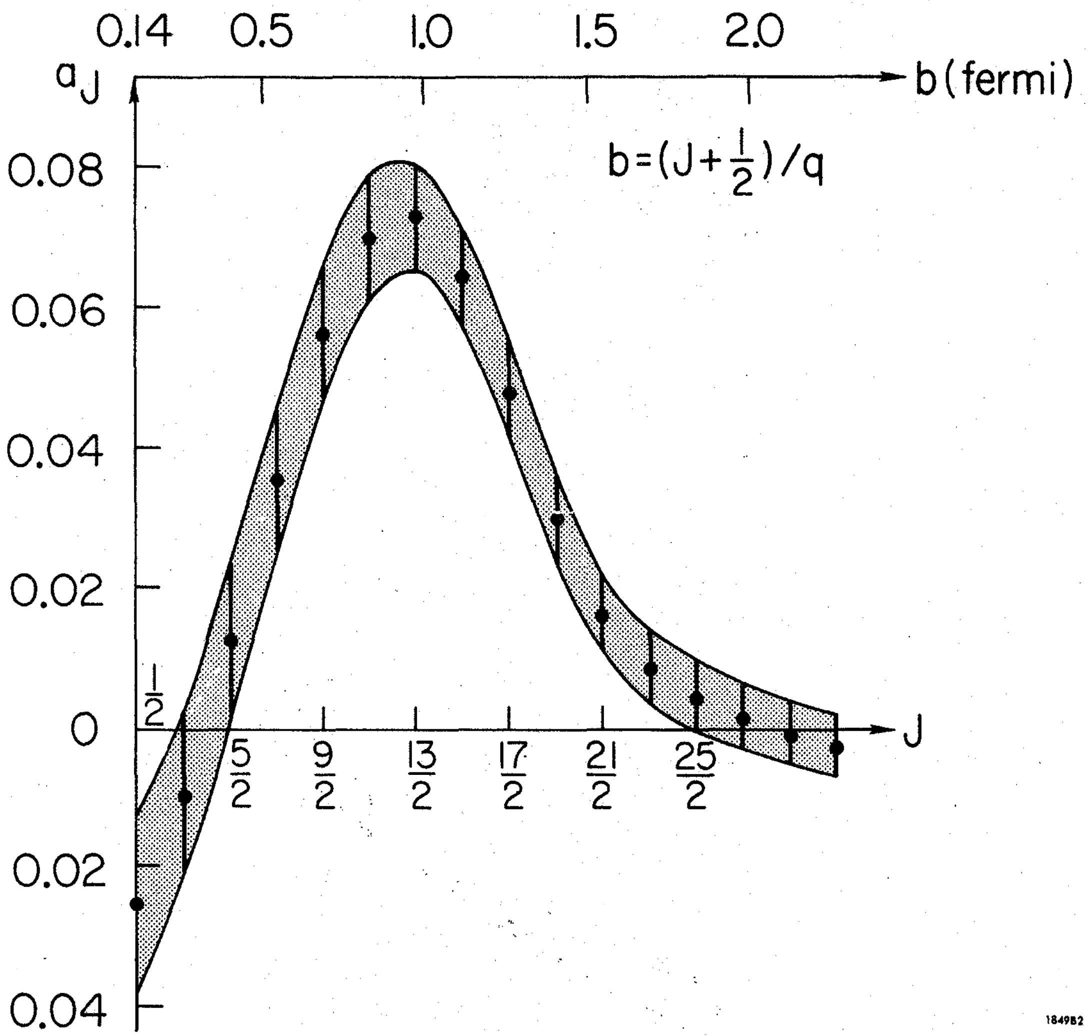
### Figure 3

Legendre coefficients $a_J$ for $P(t) = \sqrt{\frac{d\sigma}{dt}(K^+p)}$ (labeled $K^+p$) and for $P(t) + R(t) = \sqrt{\frac{d\sigma}{dt}(K^-p)}$ (labeled $K^-p$). The difference between the $K^+p$ and $K^-p$ coefficients is equal, within errors, to the coefficients displayed in figure 2.



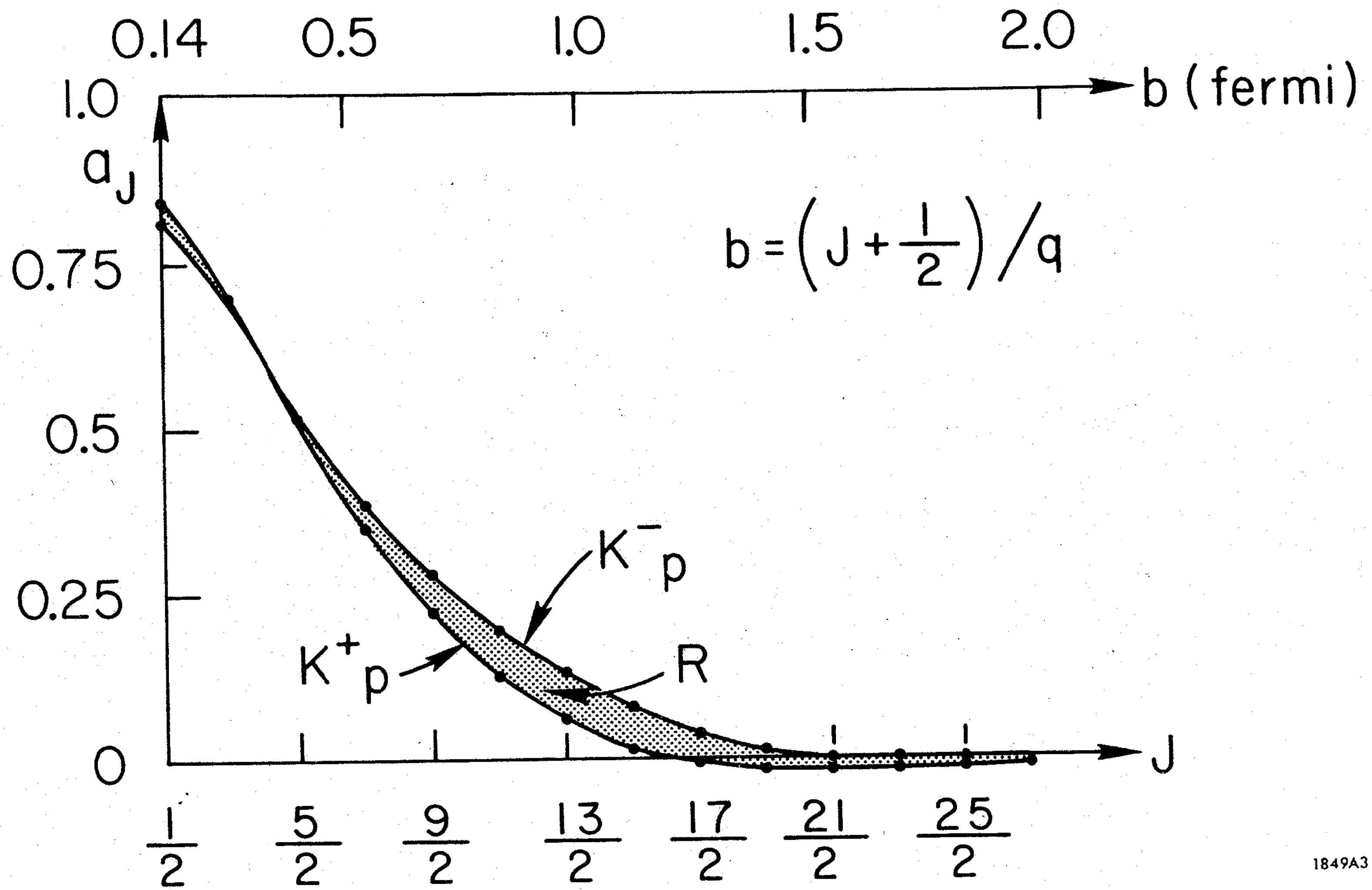
$\frac{mb^{1/2}}{GeV}$
1.5
1.0
0.5
0
-0.5
$\frac{\frac{d\sigma}{dt}(K^-p)-\frac{d\sigma}{dt}(K^+p)}{2\sqrt{\frac{d\sigma}{dt}(K^+p)}}$
$P_{lab}$=5 GeV/c
C.Baglin et al.
Interpolated Bubble Chamber data
— $1.6\ e^{1.3t}J_0(4.8\sqrt{-t})$
0.5
1.0
1.5
-t (GeV$^2$)
184981


Fig. 1



0.14
0.5
1.0
1.5
2.0
b(fermi)
$a_J$
0.08
0.06
0.04
0.02
0
0.02
0.04
$b=(J+\frac{1}{2})/q$
$\frac{1}{2}$
$\frac{5}{2}$
$\frac{9}{2}$
$\frac{13}{2}$
$\frac{17}{2}$
$\frac{21}{2}$
$\frac{25}{2}$
J
1849B2


Fig. 2



0.14
0.5
1.0
1.5
2.0
b (fermi)
1.0
$a_J$
$b = \left(J + \frac{1}{2}\right)/q$
0.75
0.5
0.25
0
$K^-p$
$K^+p$
R
J
$\frac{1}{2}$
$\frac{5}{2}$
$\frac{9}{2}$
$\frac{13}{2}$
$\frac{17}{2}$
$\frac{21}{2}$
$\frac{25}{2}$
1849A3


Fig. 3