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DOUBLE PARTICLE EXCHANGE IN HADRONIC REACTIONS*

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ABSTRACT

We show that, contrary to present practice and belief, double-particle-exchange (DPE) contributions to hadronic scattering amplitudes are not negligible. We introduce a hierarchy of exchange mechanisms: (i) Pomeron; (ii) single pole exchange (including pole-Pomeron cuts); (iii) DPE; etc. The relative strength of any pair of consecutive terms in this hierarchy of amplitudes is of order $\eta = \eta_0 \nu^{-\frac{1}{2}}$ where $\eta_0 \sim 0.6\ \mathrm{BeV}^{\frac{1}{2}}$. At $\nu = 5$ BeV, DPE amplitudes may contribute to 25%-50% of the cross section for inelastic processes, and should not be ignored.

Phenomenological studies of hadronic reactions at intermediate and high energies usually assume that the <u>single</u> particle exchange mechanism[1] is dominant at these energies. The absence of important "exotic" exchanges is usually considered as evidence for the smallness of the contributions of <u>double</u> particle exchange (or the Regge cuts generated by the exchange of two "ordinary" Regge poles). A typical parametrization of a hadronic amplitude at an energy of a few BeV's normally involves a few single-pole terms and perhaps a cut generated by the Pomeron and an "ordinary" pole (or some equivalent term such as an absorption correction). Double particle exchange is almost always ignored, even at 3 or 4 BeV.

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In this paper we argue that this practice is unjustified and that there is strong evidence for the importance of double-particle-exchange (DPE) at energies of a few BeV. Several clues indicate that the ratio between a DPE amplitude and a single-particle exchange[1] amplitude at any given energy is roughly the same as the ratio between the single particle exchange term and the Pomeron exchange amplitude at the same energy. This ratio is typically[2] 0.2 - 0.3 at $p_L \sim 5$ BeV. We may introduce a hierarchy of contributions for a given energy, with the Pomeron exchange amplitude as the leading term, followed by single-particle-exchange[1], double-particle-exchange, etc. The ratio η between the strengths of any two consecutive terms in this hierarchy is roughly given by[2] $\eta \sim \eta_0 \nu^{-\frac{1}{2}}$ where ν is the laboratory energy and $\eta_0 \sim 0.6\ \mathrm{BeV}^{\frac{1}{2}}$.

We have at least three reasons to believe, apriori, that DPE amplitudes are important.

(i) It has become clear that Regge pole models are totally inadequate for describing many aspects of hadron reactions. There is an overwhelming amount of evidence[3] that absorption corrections or pole-Pomeron cut contributions[1] are necessary and that these, in general, are of the same order of magnitude as the pole contributions. Suppose now that in a given process (say $\bar{p}p$ or Kp elastic scattering) the ω-exchange term and the $\omega \otimes P$ cut are comparable. How important would the $\omega \otimes \omega$ cut be? Any simple model would immediately predict that the relative strength of the $\omega \otimes \omega$ cut and the $\omega \otimes P$ cut is similar to the relative strength of the ω-pole and the Pomeron. Hence, in the same way that, in general, ω-exchange is not negligible with respect to the Pomeron, the DPE contribution ($\omega \otimes \omega$ cut) should not be negligible with respect to the single particle exchange term (ω-pole and $\omega \otimes P$ cut).

(ii) The strongest argument against important DPE terms is the absence of exotic exchanges. However, in at least half-a-dozen cases exotic exchanges have been observed[4] with cross-sections between 1% and 5% of the corresponding non-exotic ones at a few BeV's. The exotic amplitudes are, therefore, 10%-20% of the non-exotic ones. This could give substantial effects through exotic-nonexotic interference. However, a "typical" DPE term is actually expected to be even larger than the exotic exchanges. For instance — in pp reactions the amplitude of the $\rho \otimes \rho$ cut is probably an order of magnitude weaker than that of the $\omega \otimes \omega$ cut and in strangeness exchange reactions the $K^* \otimes \omega$ or $K^* \otimes f^0$ cuts are probably stronger than the $K^* \otimes \rho$ or $K^* \otimes A_2$ cuts. This follows from the stronger coupling of ω and f^0 to $N\overline{N}$, as compared with the ρ or A_2 couplings to the nucleon. As a result, nonexotic DPE terms should be in many cases substantially stronger than the corresponding exotic exchanges.

(iii) If we compare the cross sections for pairs of reactions such as $\overline{N}N \rightarrow \overline{N}\Delta$ and $NN \rightarrow N\Delta$ or $\overline{K}N \rightarrow \overline{K}\Delta$ and $KN \rightarrow K\Delta$ at 2-3 BeV, we find that the antiparticle cross section is always much smaller than the particle cross section. The usual explanation is that because of the larger total cross section for the antiparticle channel, there is much more absorption in an inelastic antiparticle reaction than in the corresponding particle reaction. This is presumably the main reason for the large difference between e.g. $\sigma(\overline{N}N \rightarrow \overline{N}\Delta)$ and $\sigma(NN \rightarrow N\Delta)$. This difference is therefore dominated by the difference between the two absorption corrections. The full absorption correction in $\overline{N}N \rightarrow \overline{N}\Delta$ or $NN \rightarrow N\Delta$ may be dominated by a pole-Pomeron cut, but the difference between the absorption in the $\overline{N}N$ and NN channels has nothing to do with the Pomeron. It must come from a DPE term[5] and it is certainly not negligible.

With these arguments in mind let us now postulate the following hierarchy of contributions to hadronic two-body scattering amplitudes:

(a) The leading term is the Pomeron exchange amplitude P. It is predominantly imaginary, has $I = S = 0$, $C = +1$ in the t-channel, approximately conserves the s-channel helicity, and grows like ν at $t \sim 0$.

(b) The second ranking term is the single-particle-exchange amplitude R, including a single t-channel pole as well as the pole-Pomeron cut.[1] Assuming that all poles are non-exotic, the amplitude R contributes only when the t-channel quantum numbers are nonexotic (since the pole-Pomeron cut is also nonexotic). We define the parameter $\eta = R/P$. Since the leading non-Pomeron pole and cut amplitudes grow like $\nu^{\frac{1}{2}}$ we expect $\eta = \eta_0 \nu^{-\frac{1}{2}}$, where η_0 is energy independent.

(c) The third term is the DPE amplitude denoted by $R \otimes R$. It can be non-exotic or exotic (but not too exotic; two mesons have $I \leq 2$, etc.). We claim that the ratio $(R \otimes R)/R$ is also approximately given by η. The leading DPE cut has $\alpha(0) \sim 0$. Hence — $(R \otimes R)/R \propto \nu^{-\frac{1}{2}}$.

(d) Additional terms such as triple particle exchange $R \otimes R \otimes R$, etc., exist; they are weaker than the DPE amplitude by another factor of η and fall faster with energy.

All differential cross-sections, polarizations, density matrix elements, etc. are bilinear forms in the amplitude. The hierarchy of contributions to such quantities will obviously be: (i) $P \cdot P$; (ii) $P \cdot R$ (order η); (iii) $R \cdot R$ and $P \cdot (R \otimes R)$ (order η^2); (iv) $R \cdot (R \otimes R)$ and $P \cdot (R \otimes R \otimes R)$ (order η^3); (v) $(R \otimes R) \cdot (R \otimes R)$, $R \cdot (R \otimes R \otimes R)$ and $P \cdot (R \otimes R \otimes R \otimes R)$ (order η^4). The sequence continues, of course, but only the first five terms can actually be isolated in various situations.

We shall now discuss examples of processes in which we can isolate, with some confidence, contributions of different orders in η. We show that similar η-values are obtained when we compare terms of different order in our hierarchy of amplitudes.

(1) R and P. A total cross-section is the only measurable quantity which actually represents an amplitude (via the optical theorem). At $\nu = 5$ GeV, the values of R/P are: $\eta \sim 0$ (for K^+p, pp); $\eta \sim 0.4$ (for K^-p); $\eta \sim 0.6$ (for $\bar{p}p$); $\eta \sim 0.3$ (for $\pi^\pm p$, γp). An "average" of $\eta \sim 0.3$ gives[2] $\eta_0 = \eta \nu^{\frac{1}{2}} \sim 0.7$.

(2) R$\otimes$R and P. The only way to isolate an R$\otimes$R term in the presence of Pomeron exchange is to consider cases in which the R-amplitude is absent because of some selection rule. This seems to happen in the K^+p and pp total cross-sections where the exchange degenerate vector meson exchange (V) and tensor meson exchange (T) cancel each other. The V$\otimes$V and T$\otimes$T cut contributions should be predominantly real but the V$\otimes$T cut contributes an imaginary amplitude leading to a positive term in $\sigma_{tot}(K^+p)$ and $\sigma_{tot}(pp)$. This term should have $\alpha \sim 0$. It is well known that $\sigma_{tot}(pp)$ has a small positive non-Pomeron part. This contribution actually falls like ν^{-1} (rather than the usual $\nu^{-\frac{1}{2}}$) corresponding to $\alpha \sim 0$ (see figure 1). For $\nu \geq 4$, $\sigma_{tot}(pp) \sim 38 + 15/\nu$ (σ in mb, ν in BeV). If we assume that the entire non-Pomeron part is, in this case, an R$\otimes$R term we find[2] $\eta_0^2 = 15/38$; $\eta_0 \simeq 0.6$. The K^+p data are not sufficiently accurate for such an analysis, but if there is an R $\times$ R term there, it seems to correspond to $\eta_0 \leq 0.3$. Most other total cross-sections, particularly $\sigma_{tot}(\bar{p}p)$, require $\alpha \leq 0$ terms in addition to the $\alpha \sim \frac{1}{2}$ contribution but their magnitudes cannot be easily determined.

(3) P·R and P·P. Elastic angular distributions are the obvious place for estimating the relative importance of P·R and P·P terms. Differences between, e.g. $\frac{d\sigma}{dt}(K^+p \to K^+p)$ and $\frac{d\sigma}{dt}(K^-p \to K^-p)$ are dominated by the P·R term. The obtained η-values are obviously similar to those obtained from the total cross-sections. It is important to remember that structures (such as the $|t| \sim 0.6$ dips in elastic πp, K^-p and $\bar{p}p$ scattering) which are due to the P·R term[7] disappear at high energies while those due to the P·P term (such as the $|t| \sim 1.2$ dip in elastic pp scattering) do not. The energy dependence of a given structure can identify its source in the hierarchy of contributions.

(4) P·(R$\otimes$R) and P·R. Elastic polarizations are dominated by P·R terms. A unique opportunity for isolating the P·(R$\otimes$R) term is offered by the $\pi^{\pm}p$ elastic polarizations. The P and f^0 exchange amplitudes seem to conserve the s-channel

helicity in πN scattering. Hence, the P·P, P·f^o and f^o·f^o terms do not contribute to the polarization. The only P·R term in the polarization is P·ρ which has C = -1 and contributes only to $\Delta = P_+ \frac{d\sigma}{dt}(\pi^+ p) - P_- \frac{d\sigma}{dt}(\pi^- p)$. The sum $\Sigma = P_+ \frac{d\sigma}{dt}(\pi^+ p) + P_- \frac{d\sigma}{dt}(\pi^- p)$ is therefore a P·(R⊗R) term, including the contributions of P·($f^o \otimes f^o$) and P·($\rho \otimes \rho$). A typical measure of the relative strength of P·(R⊗R) and P·R is given by the ratio[8] $\eta = \Sigma/\Delta \sim 1/3$ at $p_L = 5.15$, $t = -0.2$. This gives[2] $\eta \sim 0.7$.

(5) <u>R·(R⊗R) and R·R.</u> The most direct way of isolating an R·(R⊗R) term is to consider pairs of nonexotic reactions which would obey a specific ratio in the absence of exotic exchanges, and to study the experimental deviations from the predicted ratio. Two well known examples[9] are the ratios $\sigma(\gamma n \to \pi^+ \Delta^-)/\sigma(\gamma p \to \pi^+ \Delta^o)$ and $\sigma(\gamma n \to K^+ \Sigma^-)/\sigma(\gamma p \to K^+ \Sigma^o)$. In both cases deviations of 20%–30% from the ratios predicted by the absence of exotic exchanges are observed. The deviations are dominated by R·(R⊗R) terms, and we find[10] η = R·(R⊗R)/R·R ~ 0.2 at 11-16 BeV. This yields[2] <u>$\eta_0 \sim 0.7$.</u>

Another estimate of the R⊗R amplitude is offered by the remarkable systematic differences between $\overline{N}N \to \overline{N}\Delta$ and $NN \to N\Delta$; $\overline{K}N \to \overline{K}\Delta$ and $KN \to K\Delta$; $\overline{K}N \to \overline{K}^*N$ and $KN \to K^*N$; $\overline{K}N \to \overline{K}^*\Delta$ and $KN \to K^*\Delta$. In all of these cases the antiparticle reaction has a smaller cross-section than the particle reaction, presumably because of the larger absorption in the antiparticle channel. This difference in absorption between $\overline{N}N$ and NN reactions, $\overline{K}N$ and KN, etc., is due to DPE terms as discussed above. At $p_L = 2.8$ GeV/c, $\sigma(pp \to n\Delta^{++}) = 10.63 \pm 0.29$ mb.[11], $\sigma(\bar{p}n \to \overline{\Delta}^{--} p) = 4.25 {}^{+0.49}_{-0.21}$ mb[11], $\sigma(\overline{N}N \to \overline{N}\Delta)/\sigma(NN \to N\Delta) \sim 0.4$ yielding[10,2] <u>$\eta_0 \sim 0.7$.</u> At $p_L = 2.26$ BeV/c, $\sigma(K^+ p \to K^o \Delta^{++}) = 1.1 \pm 0.2$ mb[12]. At $p_L = 2.24$ BeV/c, $\sigma(K^- p \to \overline{K}^o \Delta^o) = 0.124 \pm 0.055$ mb [13]. Hence $\sigma(\overline{K}N \to \overline{K}\Delta)/\sigma(KN \to K\Delta) = 0.33 \pm 0.2$ giving[10,2] <u>$\eta_0 \sim 0.8$.</u> Similar results are obtained for $K^*\Delta$ and K^*N final states.

It is interesting to speculate that all experimental deviations from the predictions of exchange degeneracy for "line-reversed" reactions[14] are dominated by

$R\cdot(R\otimes R)$ terms. In that case, such deviations should decrease with energy, since the ratio $(R\otimes R)/R \propto \nu^{-\frac{1}{2}}$. Present data are inconclusive in that respect.

Another effect that could be "blamed" on the $R\cdot(R\otimes R)$ term is a dip in an inelastic cross section which disappears at high energies. While such disappearance may be due to other effects (shrinking?), its similarity to the disappearing $|t| \sim 0.6$ dips in elastic scattering[7] may hint that such dips are caused, again, by terms of a higher order in η. A specific process with such a behavior is $\pi^+ p \to K^+ \Sigma^+$ where a $|t| \sim 0.4$ dip disappears rapidly with energy.[15] The "depth" of this dip seems to be consistent with our values of η.

(6) $\underline{(R\otimes R)\cdot(R\otimes R)}$ and $\underline{R\cdot R}$. The relative strength of these terms is estimated by considering exotic cross-sections. These must be of the form $(R\otimes R)\cdot(R\otimes R)$. A few examples: (i) At $p_L = 7$ GeV/c, $\sigma(pn \to \Delta^{++}\Delta^-) = 1.1 \pm 0.2$ mb, $\sigma(pn \to \Delta^-\Delta^{++}) = 0.09 \pm 0.03$ mb.[16] $\eta^2 = (R\otimes R)\cdot(R\otimes R)/R\cdot R \sim 0.08$; $\underline{\eta_0 \sim 0.7}$.
(ii) At $p_L = 5.7$ GeV/c, $\sigma(\bar{p}p \to \bar{\Sigma}^+\Sigma^-)/\sigma(\bar{p}p \to \bar{\Sigma}^-\Sigma^+) \sim 0.05$[17], giving $\underline{\eta_0 \sim 0.5}$.
(iii) At $p_L = 5$ GeV/c, $\sigma(K^-p \to pK^-)/\sigma(K^+p \to pK^+) \sim 0.01$[18], giving $\underline{\eta_0 \sim 0.25}$.

We see that, in all cases, the parameter η_0 is somewhere between 0.25 and 0.8. For all practical purposes, any quantity (cross-section, polarization, etc.) can be approximated by the two leading non-vanishing terms in the hierarchy. If we use only the leading term we may face serious difficulties which are well known in the case of neglecting R with respect to P, but are equally dangerous and less known when neglecting $R\otimes R$ with respect to R. (We may, however, neglect $R\otimes R$ with respect to P or $R\cdot R$ with respect to $P\cdot P$.) It would be very gratifying if some future theory of hadron dynamics will give physical meaning to our parameter η and would transform our "hierarchy" into an honest expansion in this parameter. Until such a time we must, however, recognize the importance of this hierarchy in any phenomenological analysis.

A final remark concerning duality. The P-term is dual to s-channel background. The R-term is dual to s-channel resonances. What about the $R \otimes R$ term? We can quote here three hints which, unfortunately, point in different directions and leave us undecided: (i) The $R \otimes R$ term in $\sigma_{tot}(pp)$ (discussed above) cannot be dual to resonances, and seems to relate to the background. (ii) Any simple argument based on duality diagrams would hint that DPE is not dual to s-channel resonances. (iii) The success of the analysis of the Pomeron + resonance model for low-energy πN scattering[19] indicates, however, that the t-channel I = 1 exchange is a remarkably pure sum of resonances, possibly hinting that the $f^0 \otimes \rho$ and $\rho \otimes \rho$ terms are dual to resonances. We cannot give a definite answer to this question and we suspect that the correct answer may not be very simple.

I thank my colleagues in SLAC for helpful discussions.

Figure Caption

Figure 1: $\sigma_{tot}(pp)$ is plotted against $1/\nu$. Data are from reference 6. The straight line is $\sigma_{tot} = 38.3 + 15/\nu$. Deviations from the straight line for $\nu < 4$ BeV (not shown) actually tend to add an extra positive term, falling faster than ν^{-1}. For comparison we also plot the same data against $\nu^{-\frac{1}{2}}$. It is clear that the ν^{-1} behavior is preferred.

Footnotes and References

1. Throughout this paper the term "single particle exchange" refers to the contribution of a single non-Pomeron t-channel pole (a Regge pole or an "old-fashioned" one-particle-exchange pole) and to the contributions of a cut generated by an ordinary pole and the Pomeron. "Double particle exchange" refers to the cut generated by the exchange of two (non-Pomeron) poles.
2. This number as well as all other estimates of η or η_0 in this paper should be considered, at best, as being reliable up to a factor of two. Our only purpose in quoting these values is to give representative estimates of different terms in the hierarchy of contributions to hadronic scattering amplitudes.

3. A. Dar, T. L. Watts and V. F. Weisskopf, Nuclear Physics B13, 477 (1969); M. Ross, F. S. Henyey and G. L. Kane, Michigan preprint, 1970; H. Harari, SLAC-PUB-821, Annals of Physics, to be published.
4. See our discussion below, and the recent reviews of J. L. Rosner, Proceedings of the Philadelphia Meson Conference, 1970; C. W. Akerlof, Proceedings of the Austin meeting, November 1970; H. Harari, Proceedings of the Brookhaven Summer School, 1969.

5. $\sigma_{tot}(\overline{N}N) - \sigma_{tot}(NN)$ is dominated by ω exchange. If the dominant single pole exchange in $\overline{N}N \to \overline{N}\Delta$ and $NN \to N\Delta$ is, say, π-exchange, the largest absorption correction is the $\pi \otimes P$ cut, which contributes equally to $\sigma(\overline{N}N \to \overline{N}\Delta)$ and $\sigma(NN \to N\Delta)$. The difference in absorption is then due to the $\pi \otimes \omega$ cut.

6. K. J. Foley, et al., Phys. Rev. Letters 19, 857 (1967); D. V. Bugg et al., Phys. Rev. 146, 980 (1966).
7. H. Harari, SLAC-PUB-821, Annals of Physics, to be published.
8. N. E. Booth, Proceedings of the International Conference on High Energy Physics, Chania, Crete, Greece, 1969.

9. A. M. Boyarski et al., Phys. Rev. Letters 25, 695 (1970) and SLAC-PUB-803, to be published.

10. When estimating η-values from interference terms we use the following procedure. If, say, the R·R term predicts $\sigma_1 = \sigma_2$ and any deviation is due to the R·(R⊗R) term, we assume η = R·(R⊗R)/R·R = $(\sigma_1 - \sigma_2)/(\sigma_1 + \sigma_2)$.

11. T. C. Bacon et al., Phys. Rev. 162, 1320 (1967).

12. F. Bomse et al., Phys. Rev. 158, 1298 (1967).

13. G. W. London et al., Phys. Rev. 143, 1034 (1966).

14. For a review see e.g. H. Harari, reference 4. It is possible that deviations from these predictions are caused by R⊗R terms as well as by other effects.

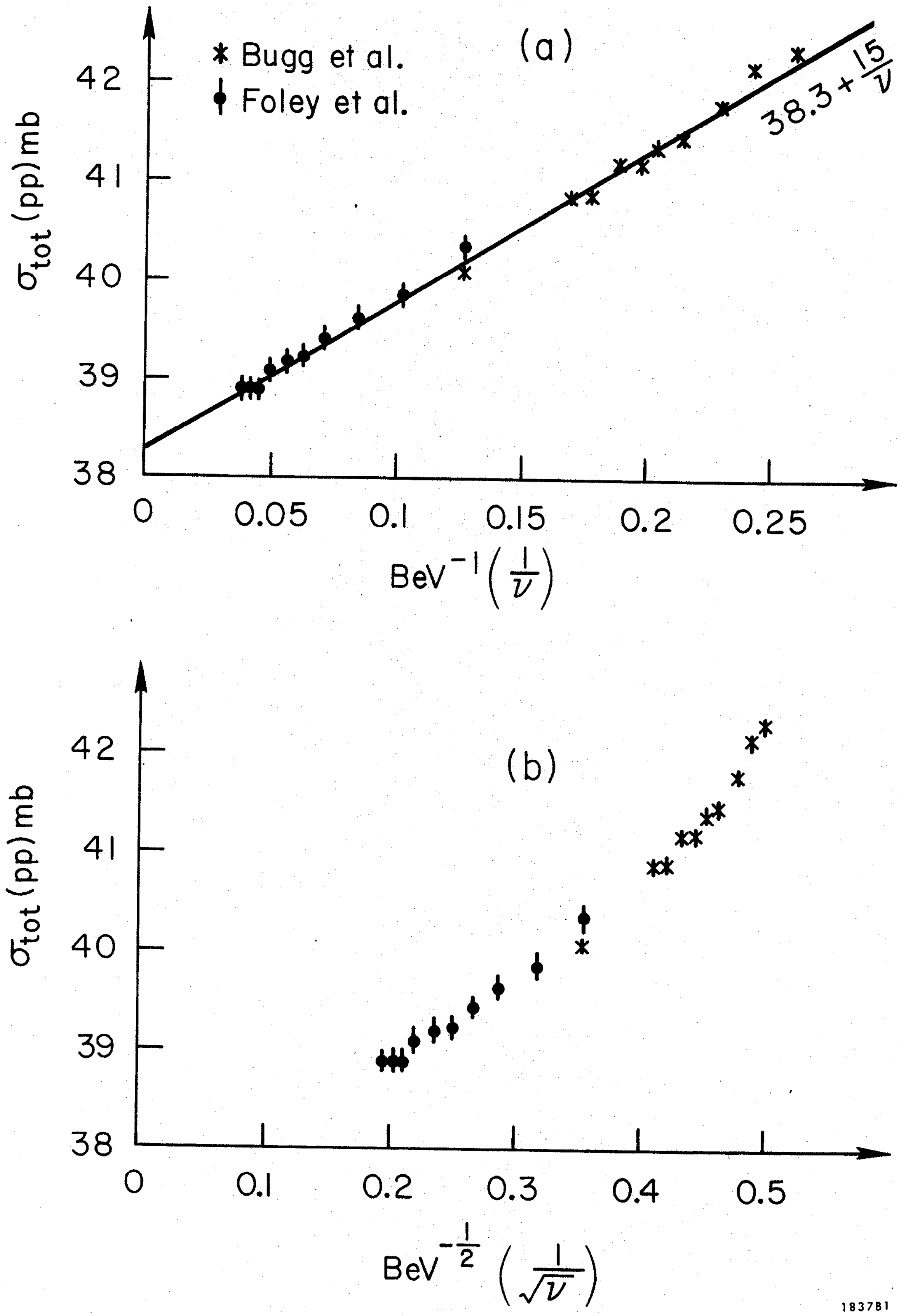
15. For a review of recent $\pi^+ p \to K^+ \Sigma^+$ data see J. V. Allaby, Rapporteur talk at the Kiev conference, 1970, CERN preprint.

16. G. Yekutieli et al., Phys. Rev. Letters 25, 184 (1970).

17. H. W. Atherton et al., Phys. Letters 30B, 494 (1969).

18. C. Baglin et al., CERN-ORSAY-PARIS-STOCKHOLM collaboration, CERN preprint, to be published. This experiment also indicates indirectly that the backward $K^- p$ elastic peak is due to DPE rather than an exotic Z*-type resonance.

19. H. Harari and Y. Zarmi, Phys. Rev. 187, 2230 (1969).



(a)
Bugg et al.
Foley et al.
$38.3 + \frac{15}{\nu}$
42
41
40
39
38
σ_{tot} (pp) mb
0
0.05
0.1
0.15
0.2
0.25
$BeV^{-1} \left(\frac{1}{\nu}\right)$
(b)
σ_{tot} (pp) mb
0
0.1
0.2
0.3
0.4
0.5
$BeV^{-\frac{1}{2}} \left(\frac{1}{\sqrt{\nu}}\right)$
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Fig. 1