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EIKONAL APPROXIMATION IN PSEUDOSCALAR THEORY*

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ABSTRACT

In an effort to extend the semiclassical intuitions developed in infrared dominated field theories (quantum electrodynamics, ϕ^3, etc.) to field theories more likely characteristic of strong interactions, we have considered the high energy — small momentum transfer limit of the ordinary linearly coupled fermion-pseudoscalar (nucleon-pion) interaction. In contrast with the infrared theories, we find (without using a cutoff) dominance of "hard" exchanged pions but with a dynamically forced pair-wise correlation resulting in an s-dependent effective potential appearing in the eikonal phase. The correspondence of the nonfixed-pole "soft" two-pion structure thus developed to a nonlocal generalization of Wentzel pair theory is made and the qualifications on the quantum field theory necessary to produce this semiclassical picture are explored. The role played by internal structure such as spin, isospin, and chiral symmetry is particularly interesting. The asymptotic behavior of the theory is extracted and we find that the dominant N-N amplitude is s-channel helicity nonflip.

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I. INTRODUCTION

Recently much effort has been devoted to the study of high energy limits in relatively well-defined field theories such as quantum electrodynamics[1] and ϕ^3.[2] As a consequence of the iteration of lowest order connected t-channel exchanges, these calculations lead to eikonal-like results in which the eikonal phase is a simple power of the coupling constant times a two-dimensional integral over an effective potential. A major hope of this approach has been that it would be generalizable to hadronic interactions. However, there may be even simple qualitative differences between the predictions of common hadronic field theories and the predictions of theories in which perturbation theory is a more reasonable calculational tool.

In particular the theories thus far considered have either possessed,[1,2] or forced[3] dominance of the infrared region of exchanged particle momenta. This has insured correspondence with one's classical intuitions about locally smooth (quasi-) potentials[4] and straight-line paths for external particles[2,5] but has required neglect of quantum-mechanical effects such as pair creation and annihilation and radiative corrections, except where these effects are only a minor qualification to the basic picture (e.g., building up a form factor).[6] This effective neglect of "ultraviolet" structure within field theories may well be incorrect when strong interactions are considered.[7]

As a first step in studying such structure, we have considered the high energy limit of the pseudoscalar interaction of fermions. We shall call the exchanged particles "pions" and the fermions "nucleons" for convenience. As is well known, scalar exchange in the generalized eikonal ladder summation is dominated by the Born term. This is a result of the spin of the exchanged particle being zero <u>and</u> of the lack of correlations. In a pseudoscalar theory this last requirement is not met if one allows "hard" pions to be exchanged (hard pion exchange is not allowed in the various cutoff models proposed for deep inelastic ep scattering[8] but is preferred by a γ_5 vertex).

We have not imposed a cutoff but instead have included the minimal correlation substructure in summing the leading (after cancellations) behavior of the generalized ladder graphs. These correlations arise from the successive helicity flips preferred by the high energy nucleon or, in other language, from the periodic dominance of contact terms in the infinite momentum[9] nucleon propagator. Pair effects are treated only in the sense in which the fermion lines contain these dominant contact terms — terms which do not arise in scalar theories.[10]

In contrast with theories possessing Lagrangian chiral symmetry,[11] the forward amplitude in our case does not vanish. If we had summed only soft pions and embedded the theory in the nonlinear σ-model this would have happened.[3] There are two simple ways to see this. The first is that, in effect, we have kept the scalar projection of an iterated derivative coupling term (obtained by a Dyson transformation on our interaction Lagrangian). Or, second, one can impose chiral symmetry on our theory in the total Lagrangian just by taking the nucleon mass to zero. Then, however, the nucleon is in an eigenstate of helicity and the contact terms mentioned above do not contribute. From our point of view, then, it would be wise to regard chiral symmetry in hadronic interactions as a truly asymptotic symmetry not to be built into an effective Lagrangian. A similar statement can be made later about the isospin symmetry — the breaking of which does not seem to have such drastic consequences.

II. CALCULATION OF AMPLITUDES

We shall sketch the calculation, indicating the basic approximations and then examine qualifications to the picture evolved as well as giving a simple interpretation of the method used and of the result obtained. We begin with the usual interaction

Lagrangian

$$\mathscr{L}_I = -ig\overline{\psi}\gamma^5 \psi\phi \tag{1}$$

and for simplicity put $g\phi(x) \equiv V(x)$ as an effective meson potential. One may proceed by consideration of Feynman graphs or by considering the equation of motion. We choose the latter course, checking results by evaluating high energy limits of sums of low order Feynman graphs. We first calculate the amplitudes for a nucleon scattering in an external "potential" and then obtain the various nucleon-nucleon amplitudes by functional techniques.

The nucleon wave function in an external potential obeys the equation

$$\left[i\not{\partial} - m - i\gamma^5 V(x)\right]\psi(x) = 0 \tag{2}$$

Utilizing the infinite momentum[1] or Sudakov variables (for a vector A, $A_+ = A_0 + A_3$, $A_- = A_0 - A_3$, $\partial_- = 2\partial/\partial x_-$, etc.) the projection operators for large and small components

$$P_+ = \frac{1}{4}\gamma_-\gamma_+ \qquad P_- = \frac{1}{4}\gamma_+\gamma_-$$

and putting

$$P_\pm\psi = \psi_\pm = e^{-ip\cdot x}\rho_\pm$$

we have the coupled equations

$$(O - i\gamma_5 V)\rho_+ + \frac{1}{2}(p_+ + i\partial_-)\gamma_-\rho_- = 0 \tag{3}$$

$$\frac{1}{2}(p_- + i\partial_+)\gamma_+\rho_+ + (O - i\gamma_5 V)\rho_- = 0 \tag{3'}$$

where the transverse operator $O = (\overrightarrow{i\partial_\perp - p_\perp})\cdot\vec{\gamma}_\perp - m$. From Eq. (3) we can write the "small" solution

$$\rho_- = -\frac{1}{2p_+}\gamma_+\left(1 + i\,\frac{\partial_-}{p_+}\right)^{-1}(O - i\gamma_5 V)\rho_+ \tag{4}$$

and with $\gamma_\pm O = \bar{O}\gamma_\pm$, write the equation for the "large" solution ρ_+ as

$$(m^2 + ip_+\partial_+)\rho_+ + (\bar{O} + i\gamma_5 V)\left(1 + i\,\frac{\partial_-}{p_+}\right)^{-1}(O - i\gamma_5 V)\rho_+ = 0 \qquad (5)$$

which we rearrange as

$$\left[ip_+\partial_+ - \frac{1}{2}\left\{V, (1 + i\partial_-/p_+)^{-1}V\right\}_+\right]\rho_+ = N\rho_+ \qquad (6)$$

where N contains derivatives with respect to transverse variables of the potential and a term $V^2(1 + i\partial_-/p_+)^{-1}(\partial_-/p_+)\rho_+$. The latter will be small under the conditions discussed below. The other terms can be shown not to contribute in the high energy, small momentum transfer limit. The operator factor $(1 + i\partial_-/\rho_+)^{-1}$ provides the non-localization of paired pion emissions necessary for convergence in momentum space. Its origin is the nucleon propagator which has otherwise been eikonally approximated. That is, the usual procedure of dropping quadratic terms in denominators renders loop integrals divergent and convergence must be restored. One could simply cut off the integrals,[3] but the operator above restores convergence in the correct, unambiguous s-dependent ($p_+ \approx \sqrt{s}$) fashion without affecting the solvability of the problem. This is because the operator produces an averaging in the x_- coordinate, while the differential equation to be solved is in the independent coordinate x_+.[12] This operator is interesting also in that it selects only the scalar (and later, isoscalar) state of two pions. If we had kept other components of the original multipole operator (for example, dependence on transverse coordinates), the other states contributing would yield lower order (by powers of $\ell n(s/\mu^2)$) asymptotic contributions to the eikonal phase.

Treating the right-hand side of (6) perturbatively, we can integrate formally and to lowest order find

$$\rho_+(x) = e^{-i\chi(x)} \cdot (\text{spinor factor})_+ \qquad (7)$$

where

$$\chi(x) = \int_0^\infty d\tau\ U(x - 2p\tau) = \frac{1}{2}\int_0^\infty d\tau \left\{V(y),(1 + i\partial_-/p_+)^{-1} V(y)\right\}_{+(y \to x - 2p\tau)} \tag{8}$$

The spinors are chosen in an appropriate infinite-momentum[1] basis as eigenstates of the projection operators. Note that, in contrast to the usual eikonal potential, the effective potential U is now s-dependent. Working in a frame in which the momentum transfer 2Δ is purely transverse, the helicity nonflip amplitude for nucleon potential scattering is ($p_+ = p_0 + p_3 \approx \sqrt{s} \approx p'_+$)

$$f_{\text{nonflip}} = \frac{ip_+}{4\pi}\int d^2x_\perp dx_- e^{2i\Delta \cdot x}\left\{e^{-i\int_{-\infty}^{\infty} d\tau\ U(x - 2p\tau)} - 1\right\} \tag{9}$$

while the helicity-flip amplitude is

$$f_{\text{flip}} = \frac{i\Delta}{2\pi}\int d^4x\ V(x) e^{2i\Delta \cdot x - i\chi(x)} \tag{10}$$

As expected, this is down by a factor of Δ/P_+ over the nonflip amplitude, vanishes in the forward direction, and has an odd number of interactions with the potential.

Nucleon-nucleon scattering amplitudes do not emerge as easily as they would with a simpler coupling and no correlations. Utilizing functional techniques, however, we can write

$$f_{nn} = C_{12} f_1(V(x)) f_2(V'(y)) \underset{V \to 0}{=} (4\pi)^2 f_1\left(ig^2\int d^4\eta\ D(x-\eta)\frac{\delta}{\delta V'(\eta)}\right) f_2(V'(y)) \tag{11}$$

where C_{12} is the "connector" operator which insures correct counting

$$C_{12} = (4\pi)^2 \exp ig^2\int d^4\xi\ d^4\eta\ \frac{\delta}{\delta V(\xi)}\ D(\xi - \eta)\frac{\delta}{\delta V'(\eta)} \tag{12}$$

and is a simple displacement operator as indicated in Eq. (11), where we have also taken $V \to 0$ after the shift is made. D is the boson propagator.

Let us work in the c.m. frame where incoming nucleons have momenta $p - \Delta$ and $q + \Delta$ and outgoing nucleons have momenta $p + \Delta$ and $q - \Delta$ with the momentum transfer 2Δ purely transverse. Then the on-shell amplitude for neither nucleon suffering a helicity flip is $(p_+ \approx \sqrt{s} \approx q_-)$,

$$f_{nn} = -\frac{2p_+q_-}{(4\pi)^2}(2\pi)^4\delta^4((p+q)-(p+q))\int dT_\perp e^{2i\Delta\cdot z}\left\{\exp -\frac{1}{2}\mathrm{Tr}\,\ell n(1-K(z_\perp,\omega,\omega'))-1\right\} \tag{13}$$

where

$$K(z_\perp,\omega,\omega') = \frac{q^4}{2}\int_{-\infty}^{\infty} d\tau \left\{ D(z_\perp - 2p\tau + 2q\omega)\overleftrightarrow{O}_+\overleftrightarrow{O}_- D(z_\perp - 2p\tau + 2q\omega') \right.$$
$$\left. + D(\tau \leftrightarrow \omega)\overleftrightarrow{O}_+\overleftrightarrow{O}_- D(\tau\leftrightarrow\omega')\right\} \tag{14}$$

and where

$$\overleftrightarrow{O}_- = (\overrightarrow{1 + i\partial_+/q_-}) + (\overleftarrow{1+i\partial_+/q_-})$$
$$\overleftrightarrow{O}_+ = (\overrightarrow{1 + i\partial_-/p_+}) + (\overleftarrow{1+i\partial_-/p_+})$$

In taking the trace and logarithm, K is treated as a matrix with continuous indices ω and ω' (which play the role of proper times along the eikonal direction). This amplitude (13) corresponds to the graphs shown in Fig. (1a). These are Feynman graphs in which internal propagators have been eikonally approximated consistent with the permutation symmetry (all possible crossings of boson lines) and the correlations discussed above. This is indicated by a dot. If we include isospin, and assume that this inclusion does not alter the nonlocal averaging at each pair of vertices, then the only change above is a multiplicative factor of $\delta^\alpha_\alpha = 3$ (pions) in the eikonal phase. The amplitude for scattering with both nucleons changing helicity can be similarly derived. If we write $\exp(\mathrm{Tr}\,\ell n(1-K)) = \det(1-K)$, the result is the minor of this determinant. The corresponding graphs are shown in Fig. (1b).

The compactness of Eq. (13) for f_{NN} belies the difficulty of evaluating it numerically. However, since at high energies the characteristic interaction time is very small, one may expect K to have the form of a sharply peaked distribution in $(\omega-\omega')$.

That is, the matrix may be nearly diagonal and, for example, $\mathrm{Tr}(K \cdot K) \approx (\mathrm{Tr}K)^2$ when one expands. We believe this will be true as long as the singularity structure of higher order Feynman graphs is simple. Cut contributions, large contributions from nonend-point regions in Feynman parameter space or from noneikonal "t-paths"[13] could prevent this simplification.

With the preceeding qualification in mind and faced with the difficulty of evaluating higher order terms in the expansion of

$$-\mathrm{Tr}\,\ln(1-K) = \mathrm{Tr}K + \frac{1}{2}\mathrm{Tr}(K \cdot K) + \frac{1}{3}\mathrm{Tr}(K \cdot K \cdot K) + \ldots \tag{15}$$

we have evaluated the asymptotic N-N amplitude when only TrK is kept in the exponent. This has the further utility of insuring only small corrections to the lowest order solution of Eq. (6). The lowest order term in the expansion of the exponential yields the asymptotic behavior of the sum of box and crossed-box Feynman graphs, in agreement with an explicit analysis of these graphs. In momentum space

$$\mathrm{Tr}K = \frac{g^4(2\pi)^2}{4s}\int d^4k \int d^4k' \, \frac{\delta(k_+ + k'_+)\,\delta(k_- + k'_-)\,e^{-iz_\perp \cdot (k+k')_\perp}}{(k^2-\mu^2+i\epsilon)(k'^2-\mu^2+i\epsilon)\left(1-k_+^2/p_+^2\right)(1-k_-^2/q_-^2)} \tag{16}$$

To evaluate the forward amplitude, we expand the exponential and do the loop integrals by rotating to a euclidean basis. Only the leading behavior in s is retained in each order. One should note, however, that substantial cancellation has already taken place in the sum over all crossings and permutations. The T-matrix corresponding to the amplitude[13] then has the form

$$T_{NN} = \int d^2z_\perp \, e^{2i\Delta \cdot z} \left\{ e^{iA\,\ln(s/\mu^2)\,K_0^2(z_\perp)} - 1 \right\} \tag{17}$$

where $A = 2^{-4}(2\pi)^{-3}g^4$. In the forward direction we can do the integrals and find asymptotically (i.e., $g^4 \ln s/\mu^2$ very large)

$$T_{NN}(\Delta = 0) \sim i\pi \, \ln^2(A \ln(s/\mu^2)) \tag{18}$$

with a resultant total cross section which behaves like $\sigma_{tot} \sim \ln^2(A \ln s/\mu^2)/s$. It is somewhat surprising that the sum of leading logarithms yields a cross section with this behavior. Indeed one suspects from this result that one must carefully consider the effect of the nonclassical contributions mentioned above.

III. DISCUSSION

The model discussed here bears an interesting resemblence to the static pair theory of Wentzel[14] in that, in the extreme relativistic case, each center-of-mass proton has a very large effective "longitudinal" mass and is able to act as an instantaneous fixed source of pion pairs (a retarded classical field density) with which the other proton can interact. Our purpose has been to show how this picture can arise even in the relativistic field-theoretic formalism and to consider the qualifications on its accuracy. In particular, the two-pion vertex is smeared out over a region which is, in general, dependent on both s and t (though we kept only the s-dependence in looking at the near-forward amplitude) and the scalar, I= 0, object thus constructed must be absorbed in a similar region on the other proton (otherwise the "butterfly" graphs of Fig. 1(a) corresponding to higher order terms in the expansion, Eq. (15) must be considered). The two-pion structure obtained is found to be "soft." That is, its total four-momentum is small even though each individual pion has large momentum components and is far off-shell.

We note in passing that we could have generated higher angular momentum states of two pions than the "sigma," whose contributions to the eikonal phase have a nonleading behavior in s, by using a more accurate form of the operation in Eq. (5). We kept only the essential part of the operator — the part which generates an expansion about the light cone $x_- = 0$, in powers of $1/\sqrt{s}$, of the product of pion fields. There is cancellation in this expansion when one sums over permutations. These permutations at the same time generate the isospin symmetry. Because of the non-local s-dependent coupling, any higher angular momentum states would not produce fixed poles. Further structure in these states, as in the scalar case, will result from the interaction of internal pions. The imaginary parts arising in these internal amplitudes will also be important in any realistic statement of unitarity. We have in mind, for example, the pion "tower" diagrams resulting from the insertion of nucleon loops.

We emphasize that these results should be taken seriously only for the qualitative features revealed. In particular, we have found that the emergence of a semi-classical picture from a hadronic field theory requires not only the space-time averaging one might expect but also the suppression or cancellation of the more delicate singularity structure one believes the field theory to possess. More importantly, we have found that some aspects of the field theory, the helicity and isospin correlations arising from internal structure for example, are consistent with a semiclassical interpretation.

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REFERENCES

1. J. B. Kogut and D. E. Soper, Phys. Rev. D1, 2901 (1970); and references quoted therein.
2. H. D. I. Abarbanel and C. Itzykson, Phys. Rev. Letters 23, 53 (1969); M. Levy and J. Sucher, Phys. Rev. 186, 1656 (1969); S. J. Chang and T. M. Yan, Phys. Rev. Letters 25, 1586 (1970).
3. H. M. Fried, Phys. Rev. D2, 3035 (1970).
4. B. M. Barbashov, S. P. Kuleshov, V. A. Matveev, A. N. Sissakian and A. N. Tavkhelidze, Phys. Letters 33B, 419 (1970).
5. R. Blankenbecler and R. L. Sugar, Phys. Rev. D2, 3024 (1970).
6. Y. P. Yao, to be published, (Phys. Rev. D3).
7. The opposite point of view may also be taken in that, as in the infrared dominated theories, one might expect the gross statistical properties of the hadronic theory to be most important at high energies.
8. S. D. Drell, D. Levy, and T. M. Yan, Phys. Rev. 187, 2159 (1969); D1 1035 (1970); D1, 1617 (1970); and S. D. Drell, T. M. Yan D1, 2402 (1970), to be published. Cutoffs, and the implied nonlocality induced, in this approach play the same role as the smearing out of the interaction region in our treatment — smoothing out or removing the large momentum behavior of the theory. The kinematical regions treated are, however, quite different. The infinite momomentum frame in which Drell *et al.*, apply a cutoff on transverse pion momenta implies a cutoff on longitudinal momenta in the c.m. system. That is, "hard" pions in our sense are not allowed.
9. The contact term in the propagator, when a proton with a large component p_+ emits a pion with momentum k and then absorbs another pion with the same momentum, has the form $(1-k_+/p_+)^{-1}\int dz_+\delta(z_+)$. The first factor is just the momentum space form of the operator in Eq. (6).

10. S. Weinberg, Phys. Rev. 150, 1313 (1966).

11. S. Weinberg, Phys. Rev. D2, 674 (1970).

12. The method of averaging is entirely suggested by the field theory itself and the constraint on internal symmetries is made consistent with this averaging and with the semiclassical correspondence.

13. G. Tiktopoulos and S. B. Treiman, to be published (Princeton preprint).

14. G. Wentzel, Helvetica Physica Acta 15, 111 (1942); 25, 569 (1952). This early work on the nuclear force problem considers the interaction of pion pairs with a static nucleon source density; this gives a local four-point interaction which is similar to our effective interaction in the relativistic limit. There are formal similarities in solutions such as the logarithmic dependence on the coupling constant in Wentzel's two-nucleon potential.

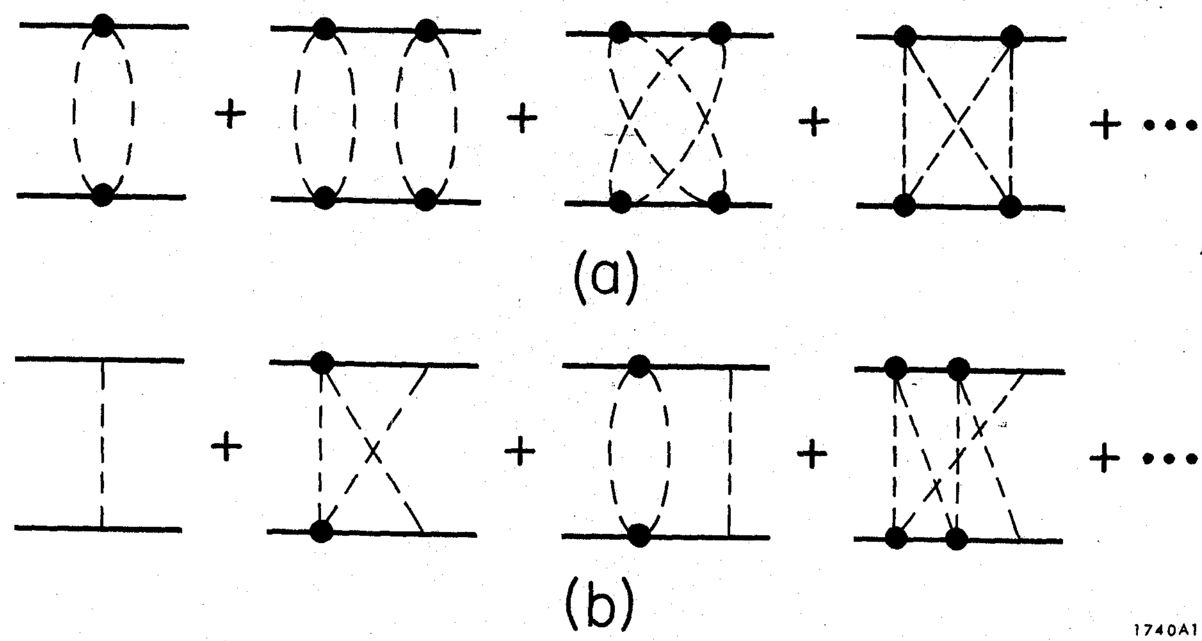

Fig. 1

Representative graphs considered in nucleon-nucleon scattering, (a) for the nonflip-nonflip helicity amplitude, (b) for the flip-flip helicity amplitude. The heavy dot indicates the correlations described in the text.