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PHYSICAL BASIS FOR
AN EXPANDING HIGH-ENERGY INTERACTION RADIUS*

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ABSTRACT

A simple argument concerning bremsstrahlung shows how the presence of many "soft" particles increases the radius of a system. This suggests there is a natural basis for a growth of a particle's effective radius at high energy in terms of the angular momentum fluctuations of a system containing increasingly larger numbers of particles. A formula which results for the radius in terms of the multiplicity, $R = \sqrt{N}\langle r\rangle$, appears to be compatible with experiment with a reasonable value for the parameter $\langle r\rangle$.

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Imagine a reasonably energetic particle scattering on a smooth potential of definite range ℓ. It will typically be scattered through a momentum transfer $\Delta \approx 1/\ell$. In impact parameter space, the scattering amplitude will be significant up to values of impact parameter $b \approx \ell$.

Now imagine the particle (thought of as a kind of "electron") coupled to a very light but nonetheless finite mass particle (a kind of "photon") which however, does not interact with the potential. When the "electron" scatters now it will radiate a few photons by "bremsstrahlung." In fact if the "photon" is light enough the "electron" will find it very difficult to escape the momentum exchange $\Delta \approx 1/\ell$ unaccompanied by one or more "photons" and thus the elastic scattering region in Δ becomes smaller than $1/\ell$.

This quite unexceptional and entirely reasonable state of affairs becomes highly mysterious when viewed in the impact parameter representation for the elastic scattering amplitude. <u>For now there must necessarily be scattering for $b > \ell$</u>, where none existed before, in order to confine the elastic scattering to the smaller region in Δ. The coupling to the "photons" has somehow spread out the position of the "electron," apparently, so that "electrons" coming in at high impact parameter can "touch" the potential. This compelling argument was made (essentially) by Hassoun and Yennie[1] and Yennie[1] in the course of investigating certain "paradoxes" of the infrared bremsstrahlung problem. They also went on to show the logical continuation to an equally bizarre result: An "electron" in a wave packet aimed to pass well outside the potential will nevertheless interact with it, due to the delocalizing effects of the "photons." An "electron," it seems, can suddenly acquire a large size when attached to "photons." Any subtle objections, (concerning perhaps how the total cross section is redistributed among the different multiplicities) may be laid to rest by considering the <u>imaginary</u> part of the elastic scattering amplitude. Since the total cross section is not changed by these "infrared" effects, the optical theorem says that $\sigma^{tot} \sim \mathrm{Im} f(0^0) \sim \mathrm{Im}\int f(b)\, b db$ is unchanged. On the other hand the sharpening of the scattering peak due to the possibility of radiation applies here, too. So Imf(b), which is essentially the cross section at b, extends to higher b, while its total integrated amount is constant, thus there must be more scattering at high b.

These observations are intriguing in connection with high-energy reactions where the possibility of an expanding interaction radius at high energy has been entertained for years, since the introduction of Regge poles. In the absence of a clear physical motivation for this phenomenon, however, many workers have been loath to abandon the simple geometric-optical picture of high-energy diffraction from an object of fixed radius, the complex ℓ-plane or selective graph summation notwithstanding. It is of interest then, that the arguments from the infrared problem may be interpreted so that a "growth" in the effective size of a particle appears, in a wide class of situations, to be a natural consequence of the many body aspect of the problem.

The infrared effect gives an exponential damping of elastic scattering by a factor $\int d\omega/\omega = \int dn$, i.e., the number of photons present; is it possible that at higher and higher energies more and more particles serve the role of the "photons" so that the "radius" of a high energy particle grows?

For example in the classic diffraction-dissociation point of view[2] a relativistic particle can make virtual fluctuations to higher and higher mass states as the energy increases. The time dilation slows down the motion "inside" the particle so that if it takes a time Δt to cross the target, virtual masses up to M are reached where

$$\Delta E \lesssim 1/\Delta t \qquad (1)$$

or by relativistic kinematics

$$\frac{M^2 - m^2}{2p} \lesssim 1/\Delta t$$

with m and p are the mass and lab. momentum of the incident particle. The virtual states M form a "cloud" and contain increasing numbers of particles which should then manifest themselves in an increasing multiplicity since once a fluctuation "lives" the traversal time it can easily be materialized by transferring a small amount of energy momentum to the target. In the multiperipheral model[3] it may be more appropriate to attach the "cloud" to the reaction as a whole rather than to the entering particles, but the picture is similar.

The connection between the number of particles effectively present and the "size" of the object can be seen as an effect of angular momentum fluctuations in a many body system: even though the system has a relatively low total angular momentum, with many particles present $\langle J^2 \rangle$ can get big, even if $\langle J \rangle$ is small, and large internal angular momenta means finding particles far from the center. If we have ten particles, say, there is a certain probability that a total angular momentum zero will be composed of 9 particles with angular momentum one and one particle with angular momentum 9; this last particle is likely to be far from the center of gravity of the system.

It is essential, however that we deal with a large number of <u>independent</u> virtual emissions. If instead, for example, we had an ever-ascending series of resonances, each one with its definite "wavefunction," or if all the emissions were constrained somehow to come from the same space-time "point" these statistical notions would not be applicable. But if the emissions are independent and each one, say, spreads out the system by an amount $\langle r \rangle$, a latter emission cannot undo the effect of an earlier one and the spreading will build up randomly. The position spread $\langle r \rangle$ should not be thought of as a mechanical recoil effect, associated with the propagation between emissions. It is an intrinsic quantum mechanical uncertainty due to the new degrees of freedom from the emitted field.

Since with a very high energy system all constituents are energetic and move forward at small angles, we can simplify the angular momentum argument by using an impact parameter representation. Figure 1 shows the incident system coming in at impact parameter $\underline{B}$ with momentum K; the transverse position of the virtual constituents with $\underline{b}_i$, K are represented on a plane perpendicular to the motion.

Conservation of momentum for the system is

$$K = \sum k_i$$

and of angular momentum

$$K\underline{B} = \sum k_i \underline{b}_i$$

or

$$\underline{B} = \sum \eta_i \underline{b}_i \tag{2}$$

with $\eta_i = k_i/K$.

As the number of constituents grows, we expect, if the virtual emissions are independent, that the dimensions of the populated region in Fig. 1 expands. If the distribution of the η's becomes energy independent at high energy[4] then the only thing changing with increasing energy is the number N of particles present and we expect under rather general conditions the transverse dimension to spread out as in a random walk:

$$R = \sqrt{N}\, \langle r \rangle . \tag{3}$$

Since we say the typical number of particles virtually present should also be the number produced, this relation can be roughly tested. The radius of interaction in hadron reactions is characterized experimentally by the slope in momentum transfer in elastic scattering

$$\frac{d\sigma}{dt} \sim e^{-Bt} \sim e^{-\frac{R^2 t}{2}} . \tag{4}$$

So this suggests for large N

$$\frac{d\sigma}{dt} \sim e^{-Nt\langle r\rangle^2/2} \tag{5}$$

where we interpret N as the average multiplicity. (The r's are written so that the interaction amplitude is $\sim e^{-b^2/R^2}$.)

Although it is not at all clear that "shrinkage" will be the rule at very high energy,[5] and although measurements of the slope parameter B only exists at accelerator energies where N can hardly be considered a large number, it is interesting that both the increase observed in B[6] and in N[7] in p-p scattering may have the same behavior, viz. ~ logs. Taking this behavior for B to continue as the energy increases, then, it is possible that Eq. (5) is correct and we can use the data of Ref. 7 to find $\langle r^2\rangle/2$. In Fig. 5a of Ref. 5 we have roughly (in GeV^{-2})

$$B = \text{const.} + (1 - 2)\ \ln s \ .$$

While if we multiply the charged multiplicity of Ref. 7 by 3/2 (assuming $N_{\pi} = N_{\pi^-} = N_{\pi^0}$) to attempt to account for the missing neutrals we get

$$N = \text{const.} + (.9 - 1.2) \ln s,$$

(we neglect the difference between $\sqrt{s}$ and the variable Q of Ref. 7). Assuming that the const. terms are to be ignored at high energy we equate the coefficients of the logarithms to get

$$\frac{\langle r^2\rangle}{2} = (.8 - 2.2)\ GeV^{-2} \tag{6}$$

or

$$\sqrt{\langle r^2\rangle} = (.9 - 1.5)\ GeV^{-1}$$
$$= (.18 - .30) f$$

which seems a not unreasonable size for a basic step length. A direct interpretation of the significance of $\langle r\rangle$ is difficult since it must stand for an averaging over many complicated effects, different kinds of emissions, spin effects, resonance production and so forth.

Our discussion clearly has much in common with the multiperipheral model[3] (in turn a kind of bremsstrahlung model) which has the additional virtue of giving a logarithmic increase for N. These arguments indicate, however, that independent of many of the specialized details of this or other models, a growing radius may arise naturally. Thus if the multiperipheral model is verified in detail our remarks are an explication of the physical content of that model; otherwise we may expect Eq. (5) to hold anyway, if uncorrelated emissions are at play.

A radius which really expands suggests some interesting speculations concerning the high-energy behavior of cross sections. In the bremsstrahlung problem we know that infrared corrections do not change the total cross section even though we have seen that the interaction density is rearranged in impact parameter. This is because the "photons" are taken not to interact with the target, so that even though a particle coming in a very large B may interact (Fig. 2(a)) it will be compensated by the identical configuration at low impact parameter which misses (Fig. 2(b)).

We might imagine something similar in high-energy reactions: a fixed amount of interacting "matter" as already observed at low energy is simply smeared out as the energy goes up. This essentially corresponds to the multiperipheral model; the more complicated diagrams where the "photons" would interact are neglected. Aside from perhaps some increase in the cross section at intermediate energies due to the disappearance of shielding effects[8] as the hadron makes the transition from a 3 dimensional to a 2 dimensional object, we then expect the total cross section to be constant at high energy as elastic scattering goes to zero.

On the other hand one might think that since in reality we are not dealing with noninteracting "photons" the configuration of Fig. 2b <u>does</u> lead to an additional interaction and the presence of large impact parameters gives rise to a growing total cross section. This would suggest, at least in its simplest version, where a constant "opacity" is reached, that ultimately

$$\sigma \sim R^2 \sim N \tag{7}$$

while at present energies we presumably have something like[9]

$$\sigma \sim \text{const.} + A \ln s.$$

Since we do not know the constants it is difficult to say anything concerning the possible experimental validity of such a relation.

The most novel feature of this picture is that higher multiplicity is associated with higher impact parameters, counter to the traditional prejudice that high multiplicity collisions being "harder," must be more central. Here an alternate language is that virtual configurations of large multiplicity, being very extended, are quite "fragile" and so can break up easily in a glancing collision.

At a given energy, of course, a high multiplicity event involves more transverse momentum than a low multiplicity event, but as the energy increases each particular channel "shrinks," so that the increasing multiplicity does not lead to more transverse momentum. In fact, we suspect that the only outcome consistent with both constancy of the average transverse momentum and a growing multiplicity is some kind of hidden effect of the type we are discussing; otherwise the momenta themselves should "random walk" with a consequent increase in the transverse momentum with multiplicity. In this sense the present picture offers an explanation for the basic fact of approximately constant transverse momenta.

A way to bring out the connection between the increase in multiplicity and glancing collisions might be to study coherent production on complex nuclei, where hard collisions are totally suppressed. If hard collisions are involved with high multiplicity then the multiplicity in coherent production should dive sharply; otherwise the multiplicity will go down, naturally, to account for the constraint that the target remain intact but perhaps by only 50% or so. This is also to be expected with direct production of resonances, but then the transverse momentum increases with multiplicity.

The most immediate experimental question, of course, is to what extent Eq. (5) is true in all elastic reactions (including the proton compton effect and photo-ρ production) and, if so, to what extent $\langle r \rangle$ is universal for the various reactions or follows some simple law of combination, perhaps like that for the variance of combining two "clouds."

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9. I thank H. Harari for a helpful discussion on this point.

FIGURE CAPTIONS

1. Virtual breakup of fast-moving particle.

2. An electron-photon configuration incident at high (a) or low (b) impact parameter.

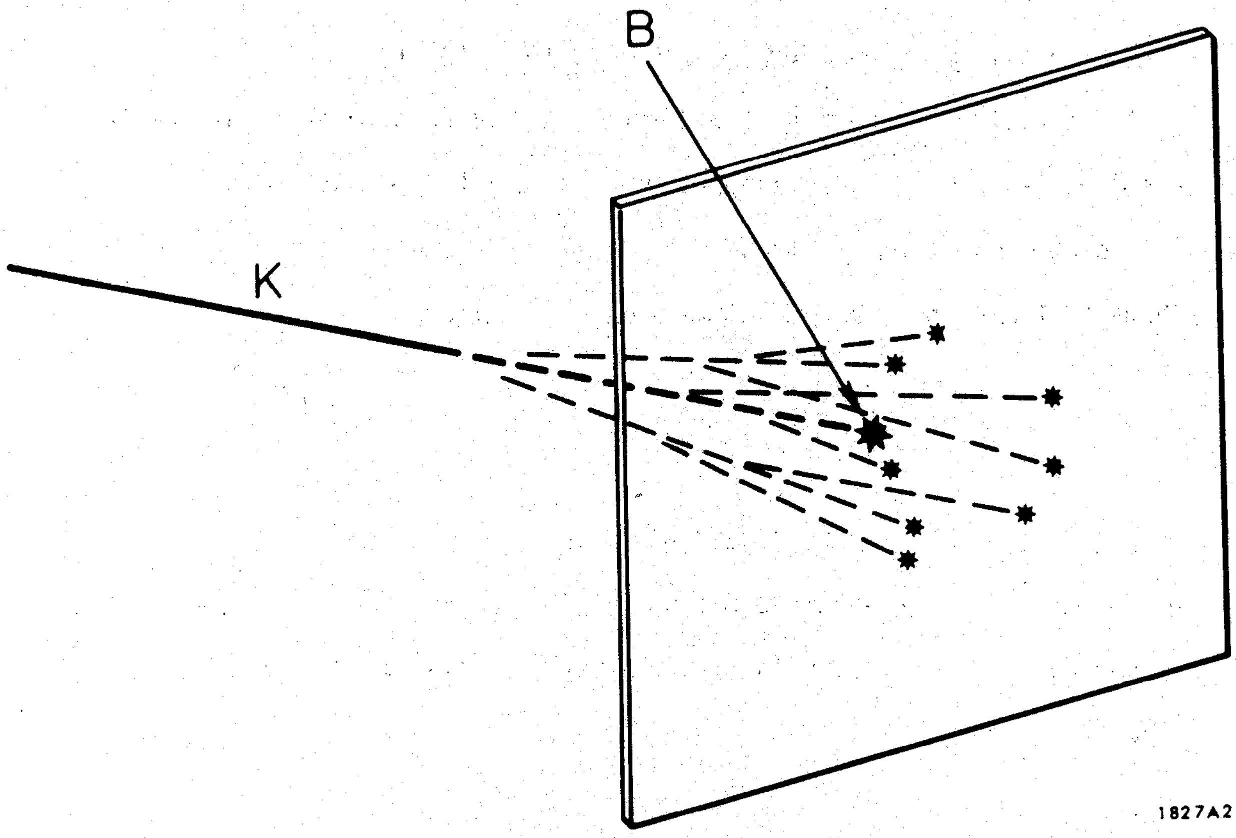

Fig. 1

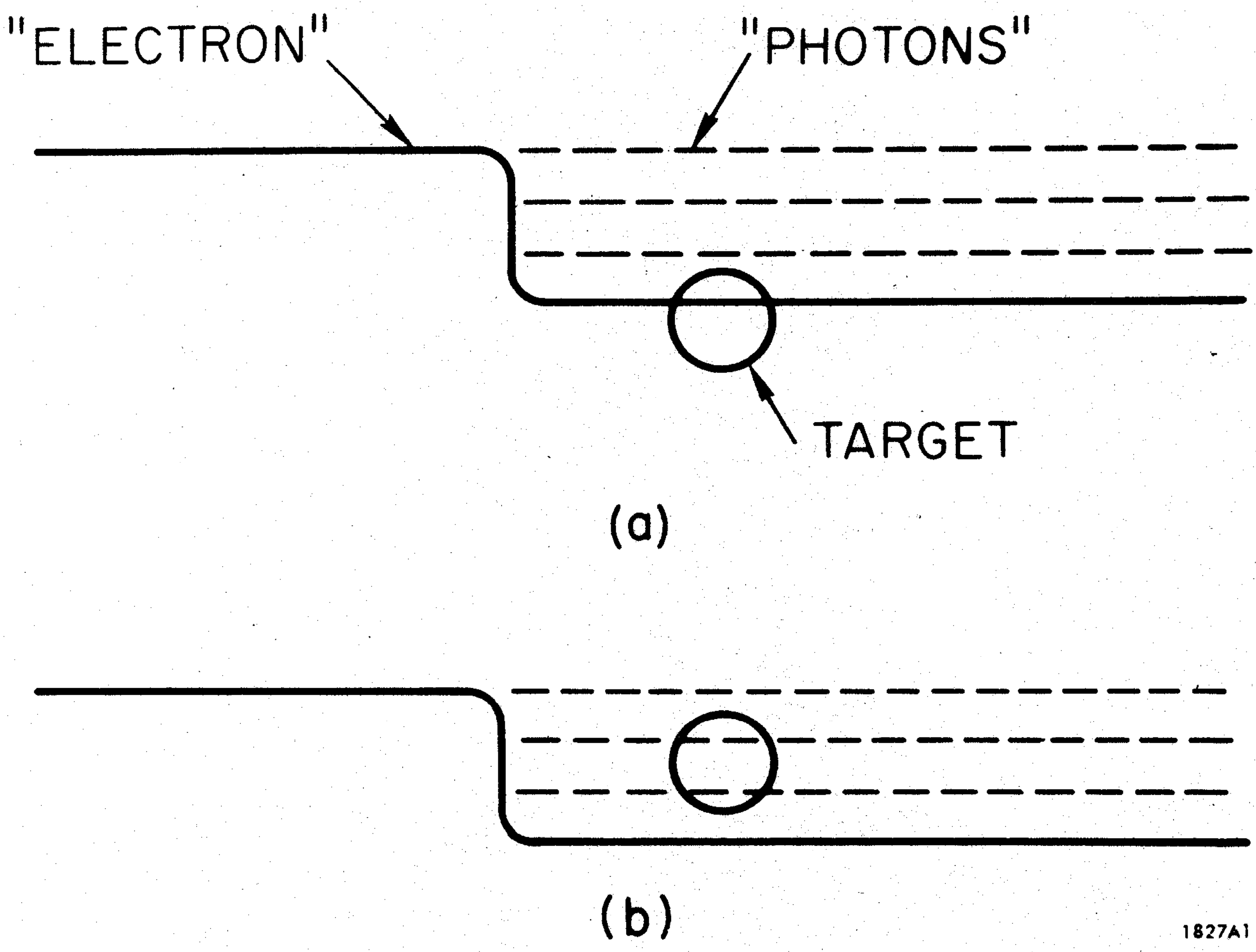

Fig. 2