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EFFECTS OF TRANSVERSE COUPLING IN THE SLAC STORAGE RING*

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Introduction

In order to achieve the maximum luminosity from the SLAC storage ring "SPEAR" it is necessary to vary the effective vertical beam height at the interaction regions. The minimum height is determined by field errors in the ring which couple the horizontal motion to the vertical motion. Since the horizontal extent is normally larger than the vertical extent of the beam, it is possible to increase the vertical beam height by increasing the coupling between the horizontal and vertical motion.

We will discuss a method of controlling the coupling by introducing four skew quadrupoles of length ℓ symmetrically into the storage ring lattice. The coupling and hence the beam height is varied by varying the field gradient in these skew quadrupoles.

Eigenfrequencies

We will designate the equilibrium-orbit path length in the storage ring by s. The radial excursion of a particle and its derivative with respect to s will comprise the first two components of the four-dimensional vector $\vec{x}$, while the vertical excursion and its derivative will comprise the third and fourth components respectively. The transformation matrix from point s_1 to another point s_2 is defined by the matrix equation

$$\vec{x}(s_2) = P(s_2, s_1)\,\vec{x}(s_1) \tag{1}$$

and the matrix $M(s_1)$ is the transformation matrix from point s_1 once around the ring back to s_1. The matrices P and M depend upon the magnet lattice of the ring including the field gradient g of the skew quadrupoles.

For some $s=s_0$ we find the modal matrix $E(s_0)$ and the eigenfrequencies ν_i (which are independent of s_0) such that[1]

$$M(s_0)\,E(s_0) = E(s_0)\begin{vmatrix} e^{i2\pi\nu_1} & 0 & 0 & 0 \\ 0 & e^{i2\pi\nu_2} & 0 & 0 \\ 0 & 0 & e^{i2\pi\nu_3} & 0 \\ 0 & 0 & 0 & e^{i2\pi\nu_4} \end{vmatrix} \tag{2}$$

where we have numbered the eigenfrequencies so that $\nu_1=-\nu_2$ and $\nu_3=-\nu_4$.

Figure 1 shows the variation of these frequencies with the skew quadrupole strength for one of the two proposed SPEAR operating modes.[2]

The performance of electron storage rings are limited by the incoherent disruptions of one beam by the other. The onset of instability is found empirically to be characterized by the linear tune shifts $\Delta\nu_x$ and $\Delta\nu_y$. In the presence of coupling these shifts produce the eigenfrequency shifts $\Delta\nu_1$ and $\Delta\nu_3$ which probably also characterize the onset of instability, and are given by

$$\begin{aligned} \Delta\nu_1 &= a_x\Delta\nu_x + a_y\Delta\nu_y \\ \Delta\nu_3 &= b_x\Delta\nu_x + b_y\Delta\nu_y \end{aligned} \tag{3}$$

Figures 2 and 3 show the variation of the eigenfrequency shift coefficients a_x, a_y, b_x, and b_y with skew quadrupole strength for the two SPEAR operating modes. It is important that the coefficients not be appreciably larger than one and this will restrict the magnitude of the skew quadrupole strength.

Beam Size

The mean square beam width and height at any point are the sums of the mean square betatron excursions and the transverse excursions due to the mean square momentum spread in the beam:

$$\langle w^2\rangle = \langle x_1^2\rangle + \eta_1^2\left\langle\left(\frac{\Delta P}{P}\right)^2\right\rangle, \tag{4}$$

$$\langle h^2\rangle = \langle x_3^2\rangle + \eta_3^2\left\langle\left(\frac{\Delta P}{P}\right)^2\right\rangle, \tag{5}$$

where the four-dimensional vector $\vec{\eta}(s)$ defines the transverse excursion for an off-momentum particle of $\Delta P/P=1$ and is a function of the magnet lattice of the ring including the skew quadrupoles.

In order to determine the size of the betatron oscillations we define the matrices A(s) and B(s) by

$$\begin{aligned} A_{ij}(s_0) &= \frac{1}{2}\left[E^{-1*}_{1i}(s_0)\,E^{-1}_{1j}(s_0) + E^{-1}_{1i}(s_0)\,E^{-1*}_{1j}(s_0)\right] \\ B_{ij}(s_0) &= \frac{1}{2}\left[E^{-1*}_{3i}(s_0)\,E^{-1}_{3j}(s_0) + E^{-1}_{3i}(s_0)\,E^{-1*}_{3j}(s_0)\right] \end{aligned} \tag{6}$$

$$\begin{aligned} A(s) &= \tilde{P}^{-1}(s,s_0)A(s_0)P^{-1}(s,s_0) \\ B(s) &= \tilde{P}^{-1}(s,s_0)B(s_0)P^{-1}(s,s_0) \end{aligned} \tag{7}$$

where a superscript (-1) denotes the inverse, a star the complex conjugate and a tilde the transpose of a matrix.

In the absence of synchrotron radiation damping or fluctuations the following quantities would be constants of the motion

$$W_1 = \Sigma_{ij}\,A_{ij}(s)x_i(s)x_j(s) \tag{8}$$

$$W_3 = \Sigma_{ij}\,B_{ij}(s)x_i(s)x_j(s) \tag{9}$$

The average value of the quantities W_1 and W_3 are determined by the synchrotron radiation fluctuations and radiation damping. For SPEAR, with damping rates for the horizontal and vertical motion equal to one half of the damping rate of the energy oscillations, the average values of W_1 and W_3 are given approximately by[3]

$$\langle W_1\rangle = \left(\frac{55}{2^3\,3^{3/2}}\right)\frac{\epsilon_c}{E}\left[\overline{\Sigma A_{ij}\eta_i\eta_j}\right] \tag{10}$$

$$\langle W_3\rangle = \left(\frac{55}{2^3\,3^{3/2}}\right)\frac{\epsilon_c}{E}\left[\overline{\Sigma B_{ij}\eta_i\eta_j}\right] \tag{11}$$

where ϵ_c is the critical energy of the radiation, E the particle energy, and the bar over the sums designates the average over all of the bending magnets. All the bending magnets have the same field strength and no gradient.

The distribution function for the betatron motion is given by

$$\psi(\vec{x}) = k\,\exp\left\{-C_{ij}\,x_ix_j\right\} \tag{12}$$

where k is a normalization constant and the matrix C is given by

$$C_{ij} = \frac{A_{ij}}{\langle W_1\rangle} + \frac{B_{ij}}{\langle W_3\rangle} \tag{13}$$

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The mean square value of the betatron excursion[4] is

$$\langle x_i^2 \rangle = \frac{1}{2} (C^{-1})_{ii} \tag{14}$$

Figures 4 and 5 show the root mean square value for w and h at the interaction region versus the skew quadrupole strength for the two SPEAR operating modes at 1 GeV. The rms width and height scale linearly with energy.

From the above figures it appears that it is possible using a reasonable skew quadrupole strength to increase the height sufficiently to achieve the desired luminosity without driving one of the eigenfrequencies to a resonant value or producing large eigenfrequency shift coefficients.

References

1. F. B. Hildebrand, Methods of Applied Mathematics (Prentice-Hall, Englewood Cliffs, New Jersey, 1952).
2. By changing the strengths of the quadrupoles in the lattice it is possible to vary the operating conditions for SPEAR. There are two major modes of operation: one where the dispersion through the insertion is zero, and another where the dispersion function η is rather large at the interaction regions. For more detail see paper D-1 presented at this conference by Burton Richter.
3. Matthew Sands, "The physics of electron storage rings," Report No. SLAC-121 (November 1970), p. 113.
4. It is a pleasure to thank G. Golub and S. Howry for bringing this fact to our attention.

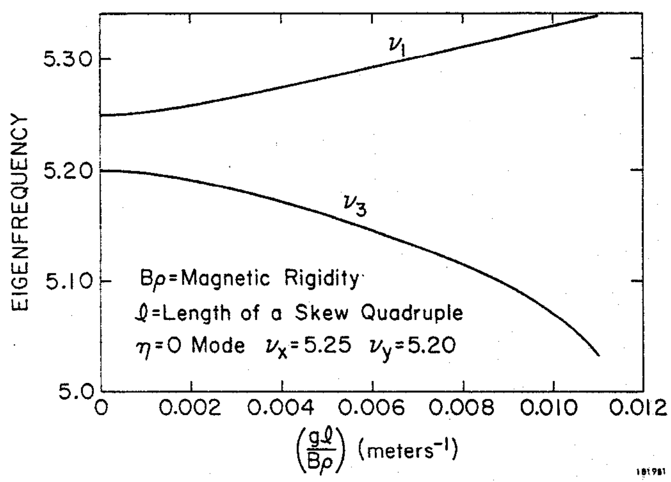

FIG. 1--Variation of eigenfrequencies (ν_1 and ν_3) with skew quadrupole gradient (g).

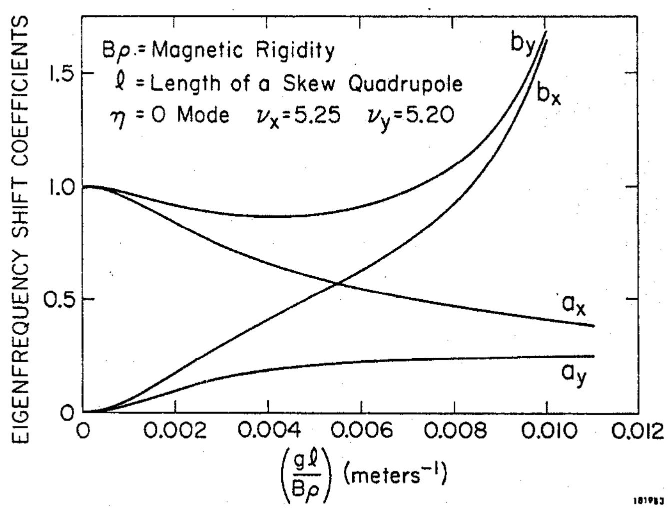

FIG. 2--Variation of eigenfrequency coefficients with skew quadrupole gradient (g).

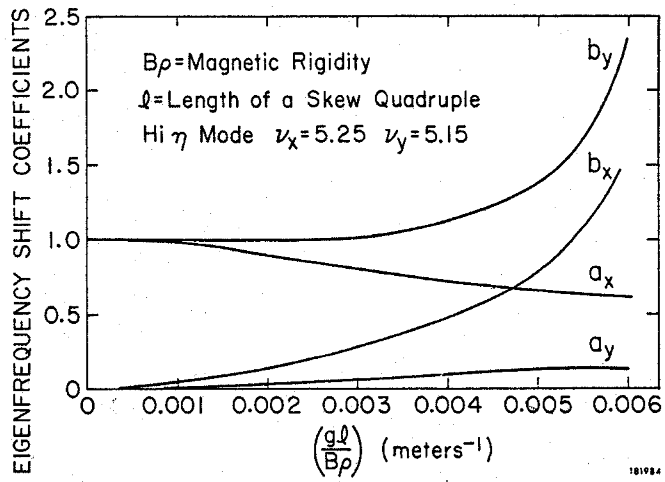

FIG. 3--Variation of eigenfrequency coefficients with skew quadrupole gradient (g).

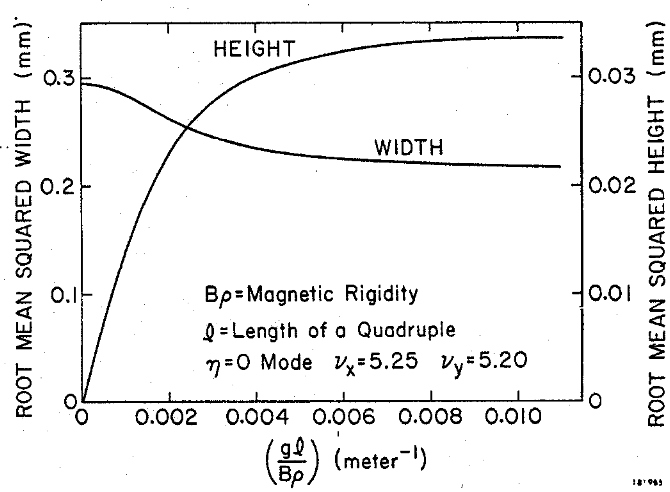

FIG. 4--Variation of beam width and height at interaction region with skew quadrupole gradient (g).

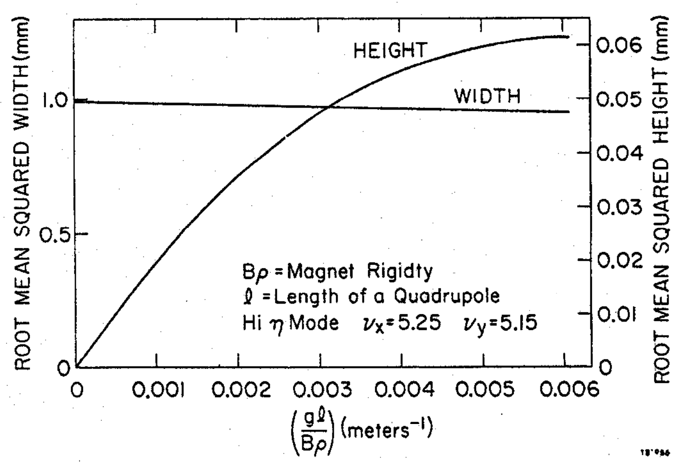

FIG. 5--Variation of beam width and height at interaction region with skew quadrupole gradient (g).