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BEAM - RF CAVITY STABILITY WITH FEEDBACK CONTROL IN A CIRCULAR ACCELERATOR*

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Introduction

The stability of a beam in an RF accelerating cavity in a storage ring or circular accelerator has been studied by K. Robinson[1,2] and many others. Robinson has found that under certain assumptions the beam-RF system with no feedback control is stable against "center-of-charge" oscillations if the cavity resonance frequency, ω_0, lies in restricted ranges. In particular, if ω_0 is less than the RF frequency, ω, and the beam current is less than a critical value, I_{BC}, the system is always stable. It may be possible, however, to increase the value of I_{BC} and the range of stable frequency by the use of feedback. This possibility has been investigated for a beam-RF system with feedback control of the cavity voltage amplitude and phase.

Under Robinson's assumptions, the beam-RF cavity system is represented by a single parallel RLC circuit driven by ideal current sources $\vec{I}_B$ and $\vec{I}_g$ which characterize the beam and generator currents as shown in Fig. 1. The assumption that the beam may be represented by a single current source $\vec{I}_B$ restricts the treatment to center-of-charge motion of the whole beam. In a multibunch beam there are as many modes of oscillation as there are bunches. The system may be described by a set of modes in which the center-of-charge is stationary in all modes but one and it is that mode to which this analysis applies. The state of the system may be described by the variables $\vec{V}$, $\vec{I}_B$, $\vec{I}_g$ and E, where $\vec{V}$ is the cavity voltage and E is the particle energy. The amplitude of the beam current is assumed to be a constant and the system is assumed to be initially in steady state. At t=0 some arbitrary but small perturbations are applied to the system variables. The conditions for stability are then determined by an analysis of the perturbed system.

Feedback Equations

For a system without feedback, the value of $\Delta\vec{I}_g$ is independent of the value of $\Delta\vec{V}$. With feedback, however, $\Delta\vec{I}_g$ is related to $\Delta\vec{V}$ via some control functions. For this analysis, the time delay in the feedback loop is assumed to be sufficiently small so that we may neglect it. Then we take for the feedback equations

$$\frac{\Delta I_g}{I_{gs}} = k_v \frac{\Delta V}{V_s} \tag{1}$$

and

$$\Delta\phi_g = k_\phi \Delta\phi_v , \tag{2}$$

where k_v and k_ϕ are constants proportional to the open loop gains of the amplitude and phase feedback loops; I_g and V represent the amplitudes of $\vec{I}_g$ and $\vec{V}$ and ϕ_g and ϕ_v their phases; Δ denotes variation of any quantity from its steady state value which is labeled with a subscript s.

Normal Mode Frequency

Due to the perturbations applied at t=0, the system variables are assumed to vary as $e^{\gamma t}$ for $t > 0$. The frequency of oscillation, γ, satisfies a similar dispersion equation as obtained by Robinson

$$\gamma^4 + b_3\gamma^3 + b_2\gamma^2 + b_1\gamma + b_0 = 0 . \tag{3}$$

The coefficients in this dispersion equation are given by the coefficients in Robinson's equation plus terms which contain the feedback gains:

$$b_3 = \alpha\left[2 - \frac{V_{gs}}{V_s}\sin\phi_{gs}(k_v + k_\phi)\right]$$

$$b_2 = \omega_s^2 + \alpha^2\left[1 + \tan^2\phi_Y - \frac{V_{gs}}{V_s}(\sin\phi_{gs} - \cos\phi_{gs}\tan\phi_Y)(k_v + k_\phi) + \frac{V_{gs}^2}{V_s^2}k_v k_\phi\right]$$

$$b_1 = \omega_s^2 b_3$$

$$b_0 = \omega_s^2\left[b_2 - \omega_s^2 - \alpha^2\frac{V_{Bs}}{V_s}\frac{\tan\phi_Y}{\cos\phi_{Bs}} - \alpha^2\frac{V_{gs}V_{Bs}}{V_s^2}\Big(k_v\cos(\phi_{Bs} - \phi_{gs}) + k_\phi\tan\phi_{Bs}\sin(\phi_{Bs} - \phi_{gs})\Big)\right]$$

$$\omega_s^2 = \frac{2\pi f_0^2 h\alpha' V_s\cos\phi_{Bs}}{E_s}, \quad V_{gs} = I_{gs}R, \quad V_{Bs} = I_{Bs}R, \quad \alpha = \frac{1}{RC}$$

f_0 is the revolution frequency of the beam; h is the harmonic number and α' is the momentum compaction coefficient. The other symbols are defined in Figs. 1 and 2.

Stability Condition

For a stable solution the real part of γ must be negative. This condition will be fulfilled if all of the coefficients of the dispersion relationship are positive, and also if the Routh-Hurwitz criterion is satisfied:

$$\mathscr{R} = b_1 b_2 b_3 - b_1^2 - b_0 b_3^2 > 0 . \tag{4}$$

For negative feedback case b_1 and b_3 are positive since $\sin\phi_{gs} > 0$ (see Fig. 2). Also, substitution of $b_1 = \omega_s^2 b_3$ into Eq. (4) gives

$$\omega_s^2 b_2 = b_0 + \frac{\mathscr{R}}{b_3^2} + \omega_s^4$$

so that b_0 and $\mathscr{R}$ positive implies b_2 positive. Thus, for negative feedback case the requirements for a stable solution are:

$$b_0 > 0 \quad \text{and} \quad \mathscr{R} > 0 .$$

It has been found[3] that these conditions will be satisfied provided the cavity resonant frequency is chosen such that $\phi_c < \phi_Y < \pi/2$ and the beam loading is less than a critical value, I_{BC}, where

$$\phi_c = \tan^{-1}\frac{\cos\phi_{Bs}\left[k_\phi\sin\phi_{Bs} - k_v\left(\frac{V_{BC}}{V_s} + \sin\phi_{Bs}\right)\right]}{1 - k_\phi\sin^2\phi_{Bs} - k_v\cos^2\phi_{Bs}} \tag{5}$$

and

$$I_{BC} = \frac{2V_s}{R}\cos\phi_{Bs}\left(\frac{1 - k_v}{1 + k_v\sin 2\phi_{Bs}}\right) \tag{6}$$

For the case of no feedback, these equations give the results of Robinson, i.e., $\phi_C = 0$ and $I_{BC} = (2V_s/R)\cos\phi_{Bs}$.

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Effects of Feedback

The effect of feedback upon beam loading may be characterized by r, the ratio of I_{BC} with feedback to I_{BC} without feedback:

$$r = \frac{1-k_v}{1+k_v \sin 2\phi_{Bs}} \tag{7}$$

The value of r varies from unity to infinity as k_v varies from zero to its extreme value $-1/\sin 2\phi_{Bs}$. It is possible, therefore, to stabilize a system for any desired critical beam loading by using an sufficient amount of amplitude feedback.

The effect of feedback upon the stable tuning range of the cavity resonant frequency may be characterized by the value of ϕ_C. In particular, for the case with no feedback $\phi_C=0$ so that the system is stable for $I_{Bs} < I_{BC}$ if the cavity susceptance is capacitive. This limitation, however, can be removed by using a sufficient amount of phase feedback such that ϕ_C is negative.

The values of the feedback gains corresponding to given values of r and ϕ_C may be found from Eqs. (5) and (6):

$$k_v = -\frac{r-1}{r \sin 2\phi_{Bs}+1} \tag{8}$$

and

$$k_\phi = \frac{\tan\phi_C - \cos\phi_{Bs}\left(\frac{r-1}{r\sin 2\phi_{Bs}+1}\right)\left[(2r-\tan\phi_C)\cos\phi_{Bs}+\sin\phi_{Bs}\right]}{\sin\phi_{Bs}\,(\tan\phi_C \sin\phi_{Bs}+\cos\phi_{Bs})}. \tag{9}$$

As an illustration, the value of k_v and k_ϕ for different values of r and ϕ_{Bs} are plotted in Fig. 3 for the case of $\phi_C=0$.

Acknowledgement

Helpful discussions with Roger McConnell and John Rees are appreciated.

References

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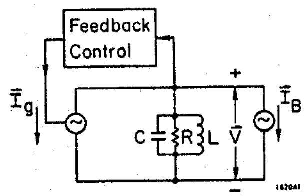

FIG. 1--System model.

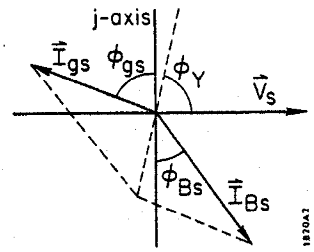

FIG. 2--Steady state conditions.

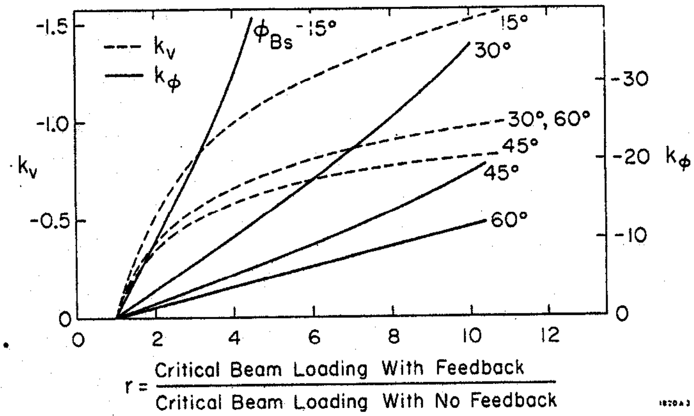

FIG. 3--Amplitude and phase feedback gains versus r for several values of synchronous phase angle, ϕ_{Bs}. ($\phi_C=0$).