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THEORETICAL IDEAS ON INELASTIC ELECTRON-NUCLEON SCATTERING*

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PROLOGUE FOR THE SECOND PRINTING

The sole purpose of this second printing is to fulfill requests that arrived after the original supply of preprints was exhausted.

Since the published form of this talk was never proof-read, it contained several typographical errors which have been corrected for this printing. In this printing, I have also included reference to a review paper by F.J. Gilman that appeared as a preprint during the typing of the original manuscript. The most important addition, however, is an acknowledgement to those from whom the author has learned so much.

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Introduction

The impressive data on inelastic electron-proton scattering[1] represent a study of the proton under conditions of extreme violence. The energies are high and the momentum transfers very large. Under these circumstances we expect to learn something about the structure of the proton, that is, we expect to learn something about the basic mechanism, which produces the observed structure of the inelastic form factors. This is the main theme of the talk.

The presentation is divided into three parts. We first review the fundamental contributions,[2),3)] which together with the data provided the impetus for all subsequent work. The second part reviews several models[5)-9)] that have been proposed and makes a contrast among their predictions. It is pointed out that the underlying structure of the parton model can be used to describe the asymptotic limits for several of the models. The last part discusses the consequences of the data and of the models for other processes and it is presently a very active field of research.

I. Fundamental Contributions

The processes that we are dealing with are shown schematically in Fig. 1. An initial electron, with energy E, hits a proton giving a final state of an electron with energy E′, which is detected, and an unobserved final hadronic state. All the interesting structure is hidden in the black circle at the lower part of the diagram. The relevant variables in describing the form factors are the energy loss of the electron

$$\nu = E - E' = q_o = \frac{q \cdot p}{M_N} \tag{I.1}$$

and the square of the four-momentum transfer

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$$-q^2 = 4EE' \sin^2\left(\frac{\theta}{2}\right) \equiv Q^2 \qquad \text{(I.2)}$$

The differential cross section for such a process in the one photon approximation is given by

$$\frac{d\sigma}{dQ^2 d\nu} = \left(\frac{E'}{E}\right) \frac{4\pi\alpha^2}{q^4}\left[W_2(Q^2,\nu)\cos^2\frac{\theta}{2} + 2W_1(Q^2,\nu)\sin^2\frac{\theta}{2}\right] \qquad \text{(I.3)}$$

The inelastic form factors $W_1(Q^2,\nu)$ and $W_2(Q^2,\nu)$ can be expressed in terms of the total photoabsorption cross sections $\sigma_\ell(\nu)$ and $\sigma_t(\nu)$, corresponding to longitudinally and transversely polarized photons respectively (more details on kinematics can be found in the previous article by Dr. DeStaebler and in Refs. 10 and 11). The ratio of the form factors is bounded, as is seen when expressed in terms of the total cross sections:

$$\frac{W_1}{W_2} = \left(1 + \frac{\nu^2}{Q^2}\right)\frac{\sigma_t(\nu)}{\sigma_t(\nu) + \sigma_\ell(\nu)} \qquad \text{(I.4)}$$

An important contribution in this field was made by Bjorken[2] almost a year ago. He observed that:

(i) By combining the $q_0 \to i\infty$ limit with the infinite momentum limit, $P_z \to \infty$, he was able to relate the structure functions W_1 and W_2 to matrix elements of almost equal-time commutators at infinite momentum.[12]

(ii) By assuming that the limit is finite and nonvanishing he succeeded in showing that for large Q^2 and ν we must observe scale invariance. Namely, although $W_1(Q^2,\nu)$ and $W_2(Q^2,\nu)$ are in general functions of two variables in the above limit they become functions of a single dimensionless variable

$$\omega = \frac{P\cdot q}{Q^2} = \frac{M\nu}{Q^2} \qquad \text{(I.5)}$$

i.e.

$$\nu W_2(Q^2,\nu) \to F_2(\omega) \qquad \text{(I.6)}$$

$$W_1(Q^2,\nu) \to F_1(\omega) \qquad \text{(I.7)}$$

This suggestion of the theorem was consistent with the preliminary 6° data and it predicted scaling for the data at larger angles. Today we saw that the 6° and 10° data are in very good agreement with this law. Figure 2 shows the combined 6° and 10° data plotted under the assumption $\sigma_\ell/\sigma_t << 1$ and the condition $Q^2 \geq 1$. They all seem to fall on a universal curve[13] with nontrivial form.

The next step was taken by Feynman,[3] who gave an intuitive but very powerful interpretation of the infinite momentum limit. He proposed to describe the process in a frame where the proton moves with infinite momentum. At high energies the electron-proton center-of-mass system is to a good approximation such a frame. In this frame the proton, as it is shown in Fig. 3, is contracted into a thin pancake and its internal motion is slowed down. As a result the lifetime of its constituents, which Feynman calls partons, is very long in comparison to the time of interaction. We can, more precisely, describe the lifetime of the constituents as follows: Let one of the constituents have a fraction x of the total proton momentum P, as it is shown in Fig. 3:

$$p = xP \tag{I.8}$$

The rest of the system must have momentum $(1 - x)P$. The lifetime of the states is given by

$$T \approx \frac{1}{E_1 + E_2 - E_{inc.}} = \frac{1}{\sqrt{(xP)^2+\mu_1^2}+\sqrt{(1-x)^2 P^2 + \mu_2^2} - \sqrt{P^2+M^2}}$$

$$\approx \frac{2P}{\frac{\mu_1^2 + P_{1\perp}^2}{x} + \frac{\mu_2^2 + P_{2\perp}^2}{1-x} - M} \tag{I.9}$$

In the same frame the lifetime of the virtual photon is

$$\tau \approx \frac{4P}{2M\nu - Q^2} \tag{I.10}$$

If we require that $\tau << T$, then we can consider the partons within the proton as free during the time of the interaction. Under such conditions we can think of the scattering as taking place from a point like constituent, while the rest of them remain undisturbed. The cross section then is given by the point cross section between electron and parton averaged over the parton distributions. For spin zero partons with unit charge it is given by

$$\frac{d\sigma}{dq^2 d\nu} = \frac{4\pi\alpha^2}{q^4}\int_0^1 \delta\left(\nu - \frac{Q^2}{2Mx}\right) f(x)dx = \frac{4\pi\alpha^2}{q^4}\,\frac{x}{\nu}\, f(x) \tag{I.11}$$

The function f(x) gives the probabilty of finding at infinite momentum a parton which has a fraction x of the proton's momentum. The probability is normalized as follows:

$$\int f(x)\, dx = 1 \tag{I.12}$$

The δ-function in (I.11) guarantees that the parton is elementary and that its mass does not change during the collision. Using the notation of Fig. 4:

$$\delta[(xP + q)^2 - (xP)^2] = \delta(q^2 + 2p\cdot qx) = \delta(q^2 + 2M\nu x) \tag{I.13}$$

The rearrangement in the argument of the δ-function in (I.11) was performed so that the point cross section is normalized as follows:

$$\lim_{E \to \infty} \frac{d\sigma}{dq^2} = \frac{4\pi\alpha^2}{q^4} \tag{I.14}$$

From Eqs. (I.3) and (I.11) we read off:

$$\nu W_2 = x f(x) \tag{I.15}$$

This picture of the proton is perhaps too simple. The partons can in general have arbitrary charge and spin. We denote by Q_i the fraction of the charge of the electron carried by the ith parton. We also consider partons with different spins, s:

$$s = 0 \quad \sigma_t = 0; \; W_1 = 0 \tag{I.16}$$

$$s = \tfrac{1}{2} \quad \sigma_\ell = 0; \; W_1 = \frac{\nu^2}{Q^2} W_2 \tag{I.17}$$

$$s = 1 \qquad ? \tag{I.18}$$

Furthermore, the proton may be an admixture of configurations with different number of partons. If we denote the probability, that the proton is in a configuration of N constituents, by P(N) and the momentum distribution in this configuration by $f_N(x)$ we obtain a more general form for Eq. (I.15) discussed more fully in Ref. 5:

$$\nu W_2 = \sum_N P(N) \left\langle \sum_i Q_i^2 \right\rangle_N x f_N(x) \equiv F(x) \tag{I.19}$$

Therefore in the parton model $\frac{\nu W_2}{x}$ has a physical meaning: It is the probability of finding a parton carrying a fraction x of the proton's momentum, and it can be determined unambiguously by experiment. From the fact that the probability functions $f_N(x)$ are normalized to 1, we can obtain sum rules by taking different moments of Eq. (I.19) with respect to x.†

$$\int \frac{\nu W_2}{x} dx = \sum_N P(N) \left\langle Q_i^2 \right\rangle_N \tag{I.20}$$

By assuming the slightly stronger assumption that all partons in a configuration have the same distribution of longitudinal fraction $f_N(x)$, we obtain the sum rule:

†This sum rule borrowed from nuclear physics has been applied to the proton by K. Gottfried, Phys. Rev. Letters 18, 1174 (1967). J. D. Bjorken, Proceedings of Internl. School of Physics "Enrico Fermi," Varenna, 55 (1967).

$$\int \nu W_2 \, dx = \sum_N P(N) \frac{\langle Q_i^2 \rangle_N}{N}$$

$$= \text{Mean square charge per parton.} \qquad \text{(I.21)}$$

The first sum rule depends critically on the value of νW_2 at $x = 0$ and it may diverge. Sum rule (I.21) is not very sensitive on the values of νW_2 in the small x region and essentially it has already been determined by the experiments.

II. Models for Inelastic Electron-Proton Scattering

Parton Model: A particular quark-parton model has been studied by Bjorken and myself.[5] We viewed the proton as consisting of configurations of 3-quarks in an infinite sea of quark-antiquark pairs. The mean-square charge of the cloud was assumed to be statistical:

$$\frac{\Sigma \langle Q_i \rangle_N^2}{N} = \tfrac{1}{3} [(\tfrac{2}{3})^2 + (\tfrac{1}{3})^2 + (\tfrac{1}{3})^2] = \tfrac{2}{9} \qquad \text{(II.1)}$$

The main conclusions of such a model are:

(i) Scale invariance comes out naturally, as it is the case in all parton models. The shape of the curve νW_2 is in fair agreement with experiment and it could be improved by suppressing the three "unpaired" quark contribution. The overall normalization is off as it is discussed in (v).

(ii) The ratio $R = \sigma_\ell / \sigma_t$ depends on the admixture of spin $0, \frac{1}{2}, 1, \ldots$ partons. In this particular quark-parton model it is zero. Under such an assumption the scaling law works very well. A small value of $R = \sigma_\ell / \sigma_t$ is consistent with the preliminary experimental results shown in Fig. 5. It is seen there that it is consistent with zero and it is certainly smaller than 1.

(iii) The quantum numbers of the secondary particles emerging in the direction $\vec{q}$ depend on the admixture of spin zero and spin $\frac{1}{2}$ partons. In case that $R = 0$ they should have the quantum numbers of quarks.

(iv) Sum rules (I.20) and (I.21) follow naturally. In comparing theory with experiment we again assume $R \ll 1$:

$$\int_0^1 \frac{F(x)}{x} dx > \int_{0.05}^1 \frac{F(x)}{x} dx = 0.72 \pm .05 \text{ (experiment)} \qquad \text{(II.2)}$$

The theoretical value is ∞ since $F(0) =$ constant.

(v) A much better check is obtained from (I.21)

$$\int_0^1 F(x) dx \geq \int_{0.05}^1 F(x) dx = .17 \pm .01 \qquad \text{(II.3)}$$

The theoretical value is $\frac{2}{9} + \frac{1}{3}\langle\frac{1}{N}\rangle \geq .22$. Even with quark charges the mean-square charge per parton is larger than what is observed experimentally. This is the main reason for the overall normalization to be off by $\approx 50\%$.

(vi) The ratio $(\nu W_2)_{neutr.}/(\nu W_2)_{proton} \approx .8$, over a large range of x, although it approaches 1 as $x \to 0$.

Field Theoretical Model: A canonical field theoretical model has been developed by Drell, Levy and Yan.[6] The formal manipulations of this model are performed in the interaction picture, where the fully interacting electromagnetic current is replaced by the bare current. Then by assuming that the particles that are emitted or absorbed at any strong vertex have finite transverse momenta they argue that in the limit of Q^2, $M\nu \to \infty$ the scattering process can be represented by the classes of diagrams shown in Figs. 6a and 6b. In these diagrams, the bare current scatters one of the constituents (pions and protons)and imparts to it a large transverse momentum. In the additional limit of $x \ll 1$, the nonvanishing contribution comes from the rainbow diagrams of Fig. 7, where the electromagnetic current lands on the proton. In this limit it reduces to a parton model where the proton consists of a single parton: the bare proton. The main conclusions of the model are:

(i) Scale invariance

(ii) $R = \sigma_\ell/\sigma_t = 0$ in the limit $x \ll 1$.

(iii) Secondary particles energy along $\vec{q}$ are protons provided $x \ll 1$.

(iv) (νW_2)proton$/(\nu W_2)$neutron $= 1$ in the limit $x \ll 1$.

(v) $\int \frac{\nu W_2}{x} dx > 1$.

Diffraction Models: It has been argued by Abarbanel, Goldberger and Treiman[7] and by Harari[8] that the ν-dependence of the electroproduction data suggest that the dominant dynamic mechanism for the large ν/Q^2 region is exchange of the Pomeranchuk trajectory.[14] Although this picture does not provide any apparent reason for scaling it has other observable consequences. They follow from the observation[15] that the direct channel resonances seem to be building up, in the sense of finite energy sum rules, all the Regge trajectories, except for the Pomeranchuk which is postulated to come from the background. Now, since the form factors for the electroexcitation of the resonances are experimentally observed to decrease like the elastic form factor, we expect to observe a flatter and flatter ν-dependence of $\sigma(\gamma N)$ as q^2 increases. Other characteristic predictions of the same observation are the equality of ep and en cross sections and likewise νp, νn, $\bar{\nu} p$, $\bar{\nu} n$ cross sections.

We may now ask how can one understand the equality of these cross sections in the parton model? The difference between protons and neutrons in the quark-parton picture arises from the presence of three extra quarks besides the sea. Therefore, if we assume that the excess mean-square charge in the proton vanishes rapidly as $Q^2 \to \infty$ we obtain the equality of ep and en cross sections. Quantitatively

$$\left\langle \sum_i Q^2{}_i \right\rangle_N = \tfrac{2}{9}N + \tfrac{1}{3} \text{ for the proton} \qquad \text{(II.4)}$$

$$\left\langle \sum_i Q^2{}_i \right\rangle_N = \tfrac{2}{9}N \text{ for neutron} \qquad \text{(II.5)}$$

Therefore the difference

$$\left\langle \sum_i Q^2_i \right\rangle_{\text{proton}} - \left\langle \sum_i Q^2{}_i \right\rangle_{\text{neutron}} = \tfrac{1}{3} \qquad \text{(II.6)}$$

cannot contribute at large values of Q^2 for reasons not completely understood. This means that in the diffraction model we may associate the Pomeron not only with the infinite $q\bar{q}$-sea but also with a little extra mean-square charge but not as much as in an ordinary proton.[16]

Table I summarizes the results of the three previous models. The entries of the diffraction models for the sum rules, are obtained in the way just described.

Vector Meson Dominance: There have been several attempts trying to explain the data in terms of vector meson dominance. Berman and Schmidt[17] found a reasonably good fit of the early data for $Q^2 = 1-2\,(\text{GeV}/c)^2$ by using ρ-dominance only with $\sigma_\ell \sim 3\sigma_t$. Sakurai[9),18] observed that the ρ-meson model could give the observed slow variation of the form factors for large values of Q^2 provided that the scalar and transverse cross sections satisfy

$$\sigma_S/\sigma_T = \xi(k)\,\frac{Q^2}{M^2_\rho}\left[1 - \frac{Q^2}{2M_\rho \nu}\right] \qquad \text{(II.7)}$$

where the parameter ξ characterizes the ratio of the total ρ cross section with different helicities:

$$\xi(k) = \frac{\sigma_{\rho p}(\lambda = 0)}{\sigma_{\rho p}(\lambda = \pm 1)} \qquad \text{(II.8)}$$

Comparison of this model with the preliminary data shown in Fig. 5 shows that the experimental value is less than 1 and the Sakurai model predicts a value close to $6 \sim 7$.

The various theoretical models described so far try to explain several features of the data, but none of them is totally satisfactory. They all have in common the ability to make predictions for future experiments. There is one more approach with such predictive power. This approach relates integrals over the data to equal-time commutators of currents and their time derivatives. The main results of this approach for inelastic electron-proton scattering have been reviewed by Bjorken.[4] In the next section, we summarize the main conclusions of this model for inelastic neutrino-nucleon scattering and we point out its connection with the previous results obtained by Cornwall and Norton,[19] and by Callan and Gross.[20]

III. Other Physical Implications

The pleasant aspect of this subject is the many implications that it has for other branches of high energy physics. In the last few months the domain of implications of the data and of the models for other processes has been a very active field of research. An interesting feature that seems to become clear is the observation that at very high energies, where most of the 2-body inelastic channels will be too small to be detected experimentally, the inelastic cross sections will be sizable and should attract a good deal of experimental interest.

A. Inelastic Neutrino Scattering

Closely related are the inelastic processes induced by neutrinos (or antineutrinos) yielding a final state of a μ^- (or μ^+) which is detected and an unobserved final hadronic state. In such processes we can study not only the vector current but also the axial current. Figure 8 shows the process and it also defines the kinematics to be used in this section.[21)]

The V-A form of the weak current determines (in the high energy limit) the polarization state of the final muon (as well as the incident neutrino) and defines a "virtual W" of pure polarization state. It is therefore natural to describe the process in terms of total cross sections corresponding to the three polarization states of the "virtual W" σ_R, σ_L, σ_S, corresponding to right-handed, left-handed and longitudinally polarized W. Such formulas are in direct correspondnece to the formulas widely used in inelastic electron-proton scattering[11),22)] and in the limit $M_\mu = 0$, $\nu \gg 2M \approx$ 2 BeV, $Q^2 \ll \nu^2$ they reduce to

$$\frac{d\sigma}{dQ^2 d\nu} = \frac{G^2}{2\pi}\frac{E'}{E}\beta(Q^2,\nu)\left[1 + \frac{\nu}{E'}(L) - \frac{\nu}{E}(R)\right] \qquad \text{(III.1)}$$

where the kinematic variables have the same meaning as in electroproduction and

$$(L) = \frac{\sigma_L}{\sigma_R + \sigma_L + 2\sigma_S} \leq 1; \quad (R) = \frac{\sigma_R}{\sigma_R + \sigma_L + 2\sigma_S} \leq 1 \qquad \text{(III.2)}$$

is a short hand for the cross sections. The function $\beta(Q^2,\nu)$ is the structure function, corresponding to the $W_2(Q^2,\nu)$, for weak processes and it is related to the cross sections as follows:

$$\beta(Q^2,\nu) = \underline{W}_2(Q^2,\nu) = \frac{1}{2\pi}\frac{Q^2}{\nu}\left(\frac{1}{1+\frac{Q^2}{\nu^2}}\right)\left(1 - \frac{Q^2}{2M\nu}\right)\left(2\sigma_S + \sigma_R + \sigma_L\right) \qquad \text{(III.3)}$$

The cross section for processes induced by antineutrinos is obtained from (III.1) by interchanging σ_R and σ_L. The interference terms between the vector and the axial current are included in

$$\beta(Q^2,\nu)\frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L + 2\sigma_S} \tag{III.4}$$

which is proportional to $\sigma(\nu p) - \sigma(\bar{\nu}n)$. Therefore neutrino-antineutrino comparisons in D_2 or light nuclei is an excellent way to test for Vector-Axial current interference. For other interesting consequences which follow from the locality of the weak current we refer to Refs. 23 and 21.

We now show that under the assumption that $\nu\beta(Q^2,\nu)$ is scale invariant, we can show that the total neutrino (antineutrino) total cross section rises linearly with energy. We use (III.1) and integrate over Q^2:

$$\frac{d\sigma}{d\nu} \cong \frac{G^2}{2\pi}\frac{E'}{E}\int_{\sim 0}^{2M\nu}\frac{dQ^2}{\nu}\,\nu\beta(Q^2,\nu)\left[1 + \frac{\nu}{E'}(L) - \frac{\nu}{E}(R)\right]$$

$$= \frac{G^2 M}{\pi}\frac{E'}{E}\left[1 + \frac{\nu}{E'}\langle L\rangle - \frac{\nu}{E}\langle R\rangle\right]\int_0^1 dx\nu\,\beta(Q^2,\nu)$$

where $\langle R\rangle$ and $\langle L\rangle$ implies the appropriate averages over x have been taken. Then the total cross section is

$$\sigma_{tot} \cong \frac{G^2 M}{\pi} E\int_0^1 dx\,\nu\beta\left\{\tfrac{1}{2} + \tfrac{1}{2}\langle L\rangle - \tfrac{1}{6}\langle R\rangle\right\}$$

Noting that the values of the curly bracket range from $\frac{1}{3}$ to 1 and using reasonable estimates for the integral we can obtain the cross section and compare it to the experimental result:

$$\sigma_{tot} = \frac{G^2 M}{\pi} E(0.6 \pm 0.15)$$

The agreement is satisfactory although inconclusive in view of the poor statistics of the neutrino data and the theoretical ambiguities.

We next turn to the results that have been obtained using methods of current algebra. These are mainly sum rules which have already been discussed in the literature in some detail. We catalogue them here to emphasize the richness of this field with regard to the quantum numbers of the projectile, the internal quantum numbers of the target and the helicity state of the "virtual W." Some of the sum rules may be written as follows:

$$\int_0^\infty d\nu\,[\bar{\beta}(\nu,Q^2) - \beta(\nu,Q^2)] = I_{00} \tag{III.5}$$

$$\lim_{Q^2\to\infty}\int_0^\infty d\nu[\bar{\beta}(\nu,Q^2)(\bar{R}+\bar{L}) - \beta(\nu,Q^2)(R+L)] = I_{xx} \tag{III.6}$$

$$\lim_{Q^2\to\infty}\int_0^\infty d\nu\,[\bar\beta(\nu,Q^2)(\bar L-\bar R)+\beta(\nu,Q^2)(L-R)] = iJxy \qquad \text{(III.7)}$$

The superscript bar refers to antineutrino-induced processes. The right hand side of these sum rules are equal time current commutators evaluated as $p_z \to \infty$; in particular

$$J_{\mu\nu} = \lim_{P_z\to\infty} \int d^3x\,\langle P_z|[J_\mu(\underset{\sim}{x},0),\,J_\nu(0,0)]|P_z\rangle \qquad \text{(III.8)}$$

Equation (III.5) is the Adler[24] (Fubini-Gell-Mann-Dashen[25]) sum rule and it depends on the reliable current commutator J_{00}. Equation (III.6) is Bjorken's "backward" neutrino sum rule.[26] (III.3) is a sum rule of Gross and Llewellyn-Smith.[27] The right hand sides of the last two sum rules are model dependent. It is of great interest to know what the values of the commutators are. We catalogue below the results for $J_{\mu\nu}$ in the "naive" quark model.

	Proton Target		Neutron Target	
	$\Delta S=0$	$\|\Delta S\|=1$	$\Delta S=0$	$\|\Delta S\|=1$
J_{00}	$2\cos^2\theta c$	$4\sin^2\theta c$	$-2\cos^2\theta c$	$+2\sin^2\theta c$
J_{xx}	$2\cos^2\theta c$	$4\sin^2\theta c$	$-2\cos^2\theta c$	$+2\sin^2\theta c$
iJ_{xy}	$6\cos^2\theta c$	$4\sin^2\theta c$	$6\cos^2\theta c$	$2\sin^2\theta c$

An additional new set of sum rules has been invented which involves commutators of space-components of the current with various time-derivatives of the current at infinite momentum. A prototype was given in the case of inelastic electron proton scattering by Cronwall and Norton[19] and also Callan and Gross.[20]

$$\lim_{Q^2\to\infty}\int_0^1 [\nu\bar\beta(\nu,Q^2)(\bar R+\bar L)+\nu\beta(\nu,Q^2)(R+L)] = \dot J_{xx}$$
$$\equiv \lim_{P_z\to\infty}\frac{d^3x}{P_0}\left\langle P_z\left|\left[i\frac{\partial Jx}{\partial t}(x,t),\,J_x^{+}(0,0)\right]\right|P_z\right\rangle_{t=0} \qquad \text{(III.9)}$$

The values of this commutator is a theoretical terra incognita.

Neutrino processes can also be discussed in the context of the parton model.[27),21] Such a treatment is very convenient in terms of Eq. (III.1) and it leads to sum rules similar to the ones that we have already discussed. The predictions of the quark parton model are identical to those of current commutators when they are evaluated in the quark model.

The point cross section between neutrinos and partons follows from (III.1):

(i) For spin $\frac{1}{2}$ partons $\sigma_S = 0$

(ii) For a given kind of parton (p, $\bar{p}$, n, $\bar{n}$-type quarks) only one of the two cross sections σ_R, σ_S contributes. To see this we observe that we can always find a Breit frame, where the parton is relativistic before and after the collision. The (V-A) form of the weak current guarantees that the parton is always "left-handed." Therefore a right-handed "virtual-W" cannot contribute as is illustrated in Fig. 9. In (III.1) there is only $\sigma_L \neq 0$ and the point cross section is

$$\frac{d\sigma}{dQ^2 d\nu} = \frac{G^2}{\pi} \delta(\nu - \frac{Q^2}{2M}) \quad . \tag{III.10}$$

In a similar manner we obtain the point cross sections for neutrinos on antipartons, as well as for antineutrinos on partons or antipartons. Then repeating the methods from Sec. III.A, we obtain the following neutrino cross sections:

$$\begin{aligned} \beta(R) &= \sum_N P(N)\, N_{\bar{p}'} \int_0^1 dx\, f_N(x) 2\delta(\nu - \frac{Q^2}{2Mx}) \\ &= \frac{2}{\nu} \sum_N P(N)\, N_{\bar{p}'}\, x f_N(x) \end{aligned} \tag{III.11}$$

$$\nu\beta(L) = 2 \sum_N P(N) \left[N_{n'} \cos^2\theta c + N_{\lambda'} \sin^2\theta c\right] x f_N(x) \tag{III.12}$$

$$\nu\bar{\beta}(\bar{R}) = 2 \sum_N P(N) \left[N_{\bar{n}'} \cos^2\theta + N_{\lambda'} \sin^2\theta c\right] x f_N(x) \tag{III.13}$$

$$\nu\bar{\beta}(\bar{L}) = 2 \sum_N P(N)\, N_p\, x\, f_N(x) \tag{III.14}$$

where N_i is the number of the ith-type quarks within the proton and (R), (L), ($\bar{R}$), ($\bar{L}$) have the same meaning as in (III.2). By taking linear combinations of (III.11)-(III.14) and then integrating over different moments of x, we obtain sum rules. In this way we can illustrate that sum rules (III.5)-(III.7) and (III.9) have a simple physical meaning in the (quark) parton model. Table II summarizes the results. Column 1 gives the integrals over the data, while column 2 gives the commutator to which they are related. For a proton target the values of the commutators in the quark model are given in column 3. The results for the parton model are in column 4. It is of interest to note that the last two columns are identical.

B. Neutron-Proton Mass Difference

Some time ago Cottingham[28] obtained a formula for the electromagnetic mass shifts of hadrons, and consequently for the electromagnetic mass difference between neutron and proton, in terms of the inelastic form factors and subtraction constants. We can now use the new data to study the mass shift of the proton and to estimate the contribution of the form factors to the n-p mass difference. By using scale invariance Pagels[29] rewrote the Cottingham formula in the form:

$$\frac{\Delta M}{M} = \frac{3}{16}\frac{\alpha}{\pi} \int_0^{-\infty} \frac{dq^2}{q^2} \left\{ -3q^2\ T_1(Q^2, \infty) + \int [\omega W_1(\omega) + \nu W_2(Q^2, \omega)]\, d\omega \right\} \quad \text{(III.15)}$$

where $\omega = \frac{Q^2}{\nu}$. It is clear that any contribution to (III.15) from the form factors, which scales, diverges logarithmically. Furthermore the contribution to the mass difference has the wrong sign provided that the proton form factors are larger than the neutron form factors over a sizable region of ω. This will be tested by experiment. The neutron-proton mass difference though may still converge if either the integrals over ω in (III.1) are identical for neutron and proton or if these integrals are cancelled in the $Q^2 \to \infty$ limit by the nonscaling contribution of the form factors and the subtraction constants. In either case good knowledge of the subtraction constants is essential.

Recent attempts[30] to determine the subtraction constants by assuming Regge asymptotic behavior (with $\alpha > 0$) and using finite energy sum rules are discouraging, since they do not even give the correct sign for the n-p mass difference. Therefore, any determination of the n-p mass difference by using (III.15) and experimentally determined quantities seems rather remote.

The indirect determination of the subtraction constants using finite energy sum rules can be further complicated by the presence of a fixed pole at $\alpha = 0$. A test for the presence of a real constant in the forward Compton amplitude with real photons coming from a pole at $\alpha = 0$ was proposed[31] using finite energy sum rules. It was also estimated that knowledge of the total photoabsorption cross section up to 10-15 GeV and to an accuracy of 5% could detect such a contribution whose magnitude is larger than 30% of the Thomson limit. What is of interest to this conference is the fact that several new measurements of the total photoabsorption cross section[32] have been or will soon be completed. Re-evaluations[33] of the sum rules, using the data for the total photoabsorption cross section from group A at SLAC, indicate that a real constant part in the Compton amplitude of the same size as the Thomson limit can be present, but the uncertainty due to the experimental errors is ± (Thomson limit). The presence of a fixed pole at $\alpha = 0$ in virtual Compton scattering will make any indirect test of the Cottingham formula also very difficult.

C. Inelastic Electron-Positron Scattering

From the many interesting processes that can be studied with colliding rings the following one is of special interest to this subject:

$$e^+ + e^- \rightarrow \text{proton} + \text{anything}, \tag{III.16}$$

since it is a crossing symmetry process to inelastic electron proton scattering. To be more precise, this process is described by two structure functions $\overline{W}_1(Q^2,\nu)$ and $\overline{W}_2(Q^2,\nu)$ which are related to the structure functions of inelastic e-p scattering by the substitution law:

$$\overline{W}_1(Q^2,\nu) = -W_1(Q^2,-\nu) \tag{III.17}$$

$$\nu\overline{W}_2(Q^2,\nu) = (-\nu)W_2(Q^2,-\nu) \tag{III.18}$$

In the field theoretical model of Drell, Levy and Yan[6] crossing can be studied explicitly. They find that the structure functions $\overline{W}_1$ and $\overline{W}_2$ have a Bjorken limit, i.e., they become universal functions of a single dimensionless variable $2M\nu/Q^2$ in the region of large Q^2 and $2M\nu$. Furthermore they argue that with a mild assumption of smoothness the inelastic electron-proton data near $2M\nu/Q^2 \gtrsim 1$ predict the process (III.16) in the region $2M\nu/Q^2 \lesssim 1$. That is, in the region $\frac{2M\nu}{Q^2} \lesssim 1$ the process $e^+ + e^- \rightarrow$ proton + "anything" must have a point-like cross section.

D. Inelastic Compton Scattering

Other processes for which the point interaction is known, should also be described in the parton model. Inelastic Compton scattering

$$\gamma + p \rightarrow \gamma + \text{anything} \tag{III.19}$$

is such a process. In the limit of high incident energies and large momentum transfers this process has been analyzed by Bjorken and myself.[5] We found that inelastic Compton scattering can be predicted from existing electron scattering data, through the relation

$$\left(\frac{d^2\sigma}{d\Omega dE'}\right)_{\gamma p} = \left(\frac{d^2\sigma}{d\Omega dE'}\right)_{ep} \frac{(k-k')^2}{kk'} \frac{\langle \Sigma_i Q_i^4 \rangle}{\langle \Sigma_i Q_i^2 \rangle} \tag{III.20}$$

which may permit one to tell fractional from integrally charged partons. The most formidable difficulty from the experimental point of view is the background coming from the decay of neutral pions. A related experiment with perhaps better signature is to consider the "Compton" terms in inelastic photoproduction of μ-pairs. In this case the background of muons comes from (i) the inelastic Bethe-Heitler diagrams, which can be calculated explicitly once we know accurately W_1 and W_2, and (ii) from the accidental decays of π^+, π^- into muons and neutrinos. Both experiments are hard but perhaps not impossible at the presently available electron accelerators.

In conclusion, it seems that there is a strong indication for a point like structure, and a Thomson picture of the proton may be emerging at energies one billion times larger. To paraphrase a quotation from J. J. Thomson, these experiments and the theoretical ideas I have described have opened up new fields for experiments which we hope with confidence will throw much light on that fundamental question, "What is the nature of the proton?"

FIGURE CAPTIONS

1. Kinematics of inelastic electron-nucleon scattering.
2. Plot of the experimental values of νW_2 as a function of $x = \frac{Q^2}{2m\nu}$ assuming $R \ll 1$ and taking data points with $Q^2 \geq 1$ (GeV/c)^2.
3. Kinematics of lepton-nucleon scattering in the parton model.
4. Detailed kinematics for the photon-parton interaction.
5. Preliminary experimental results on the separation of the structure functions. These data were presented at this conference by Dr. DeStaebler. The ratio σ_S/σ_T used in the figure is equal to the ratio σ_ℓ/σ_t used in the text.
6. Diagrams that are included in the field theoretic model. The cross indicates interaction with the electromagnetic current.
7. The rainbow diagrams are the only diagrams of the field theoretic model which survive in the limit $x \to 0$.
8. Kinematics for inelastic neutrino-nucleon scattering.
9. Detailed description of the W-parton interaction showing that only the left-handed W can contribute.

Acknowledgements

Very informative and stimulating discussions with J.D. Bjorken, S.D. Drell and R.P. Feynman have contributed greatly to the preparation of this talk. I also thank D.J. Levy and T.M. Yan for many helpful conversations.

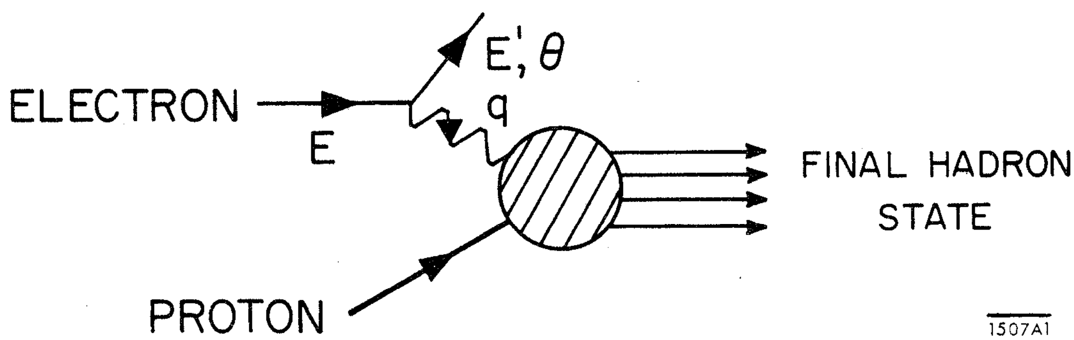


Fig. 1

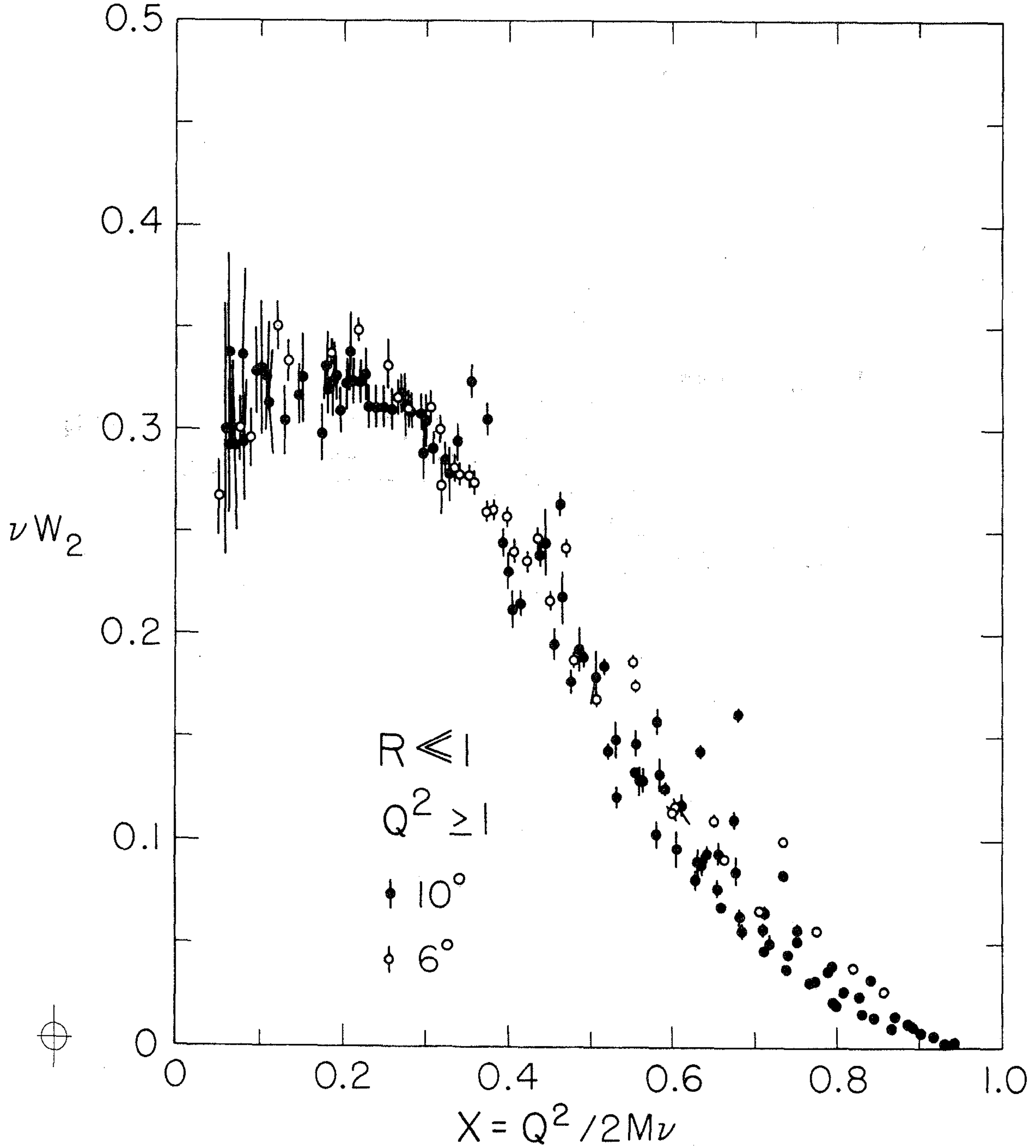


Fig. 2

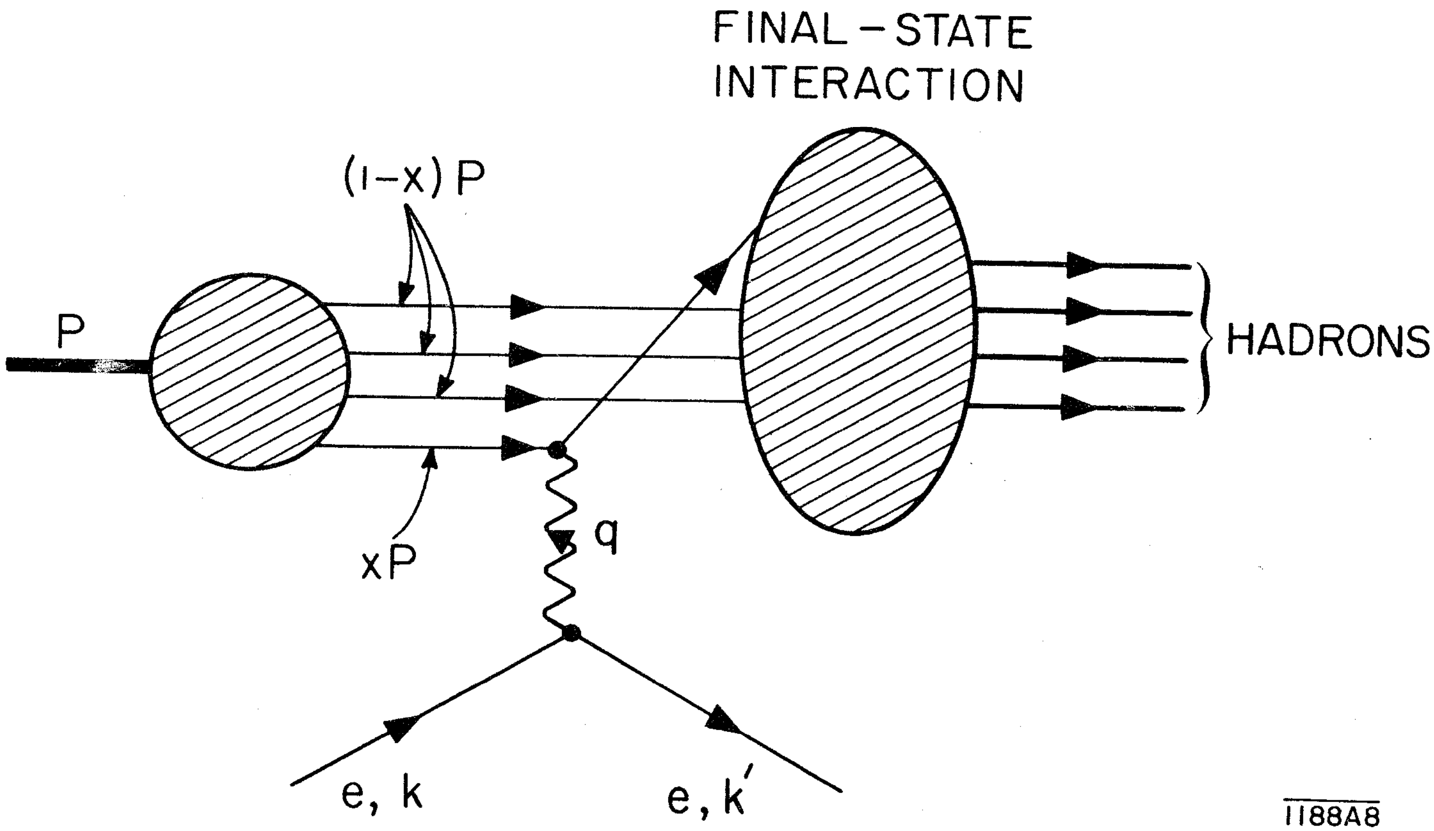
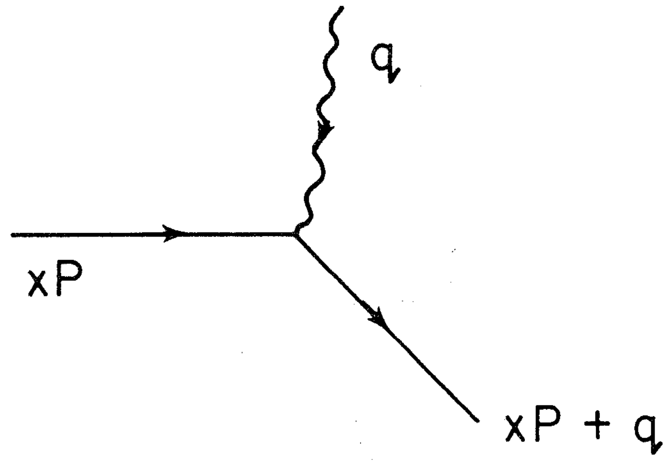
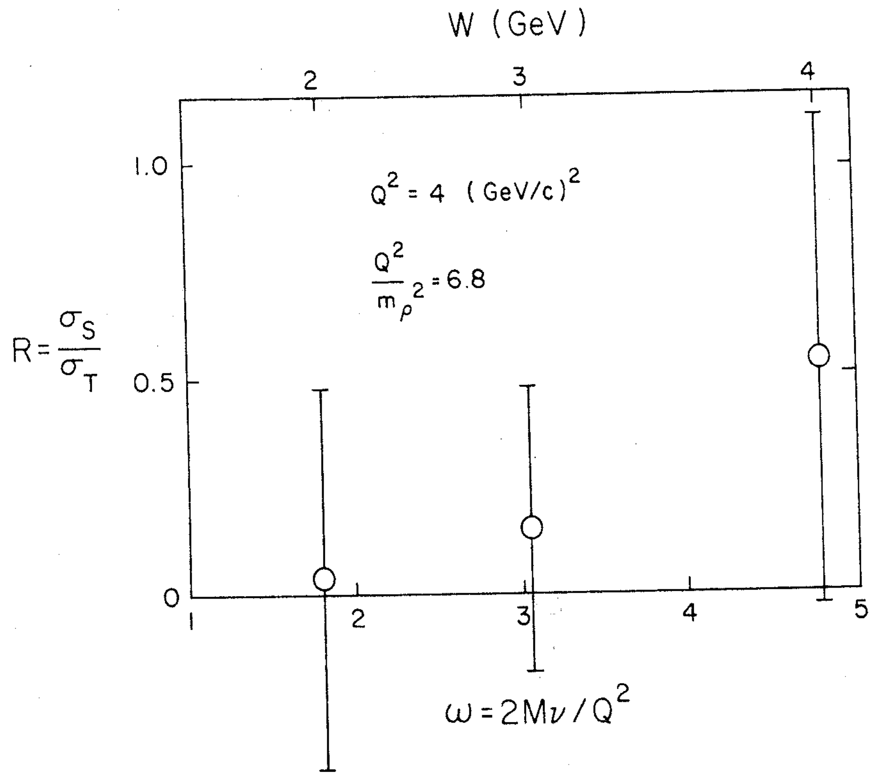


Fig. 3



q
xP
xP + q

Fig. 4



W (GeV)
2
3
4
1.0
$Q^2 = 4 \ (\mathrm{GeV}/c)^2$
$\frac{Q^2}{m_\rho{}^2} = 6.8$
$R = \frac{\sigma_S}{\sigma_T}$
0.5
0
1
2
3
4
5
$\omega = 2M\nu / Q^2$

Fig. 5

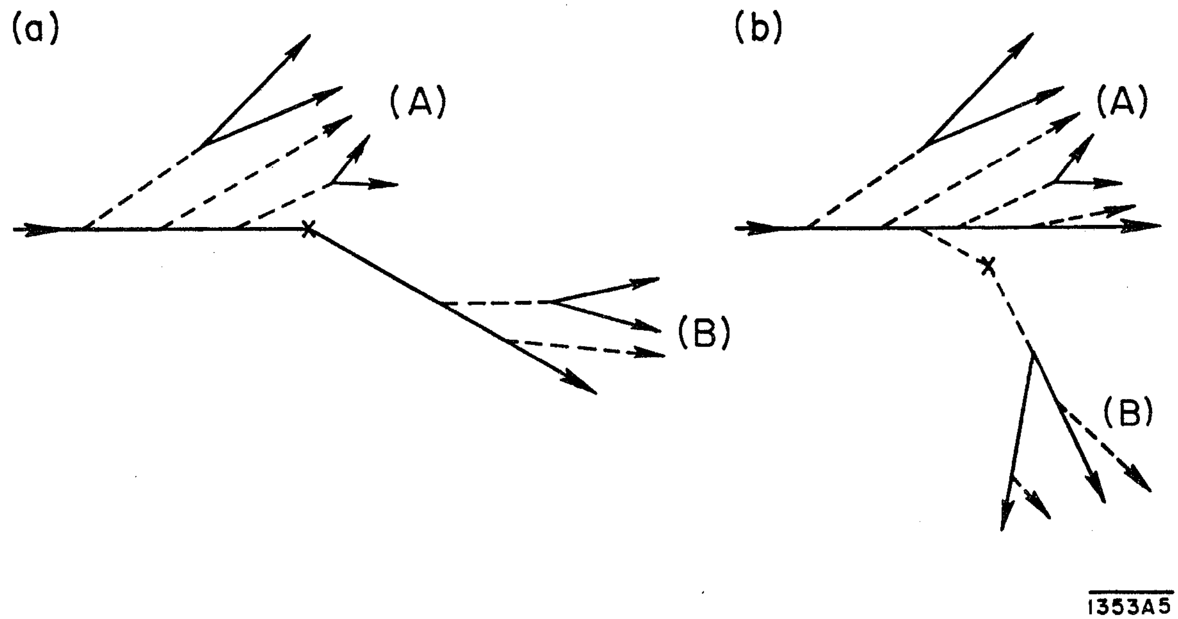


Fig. 6

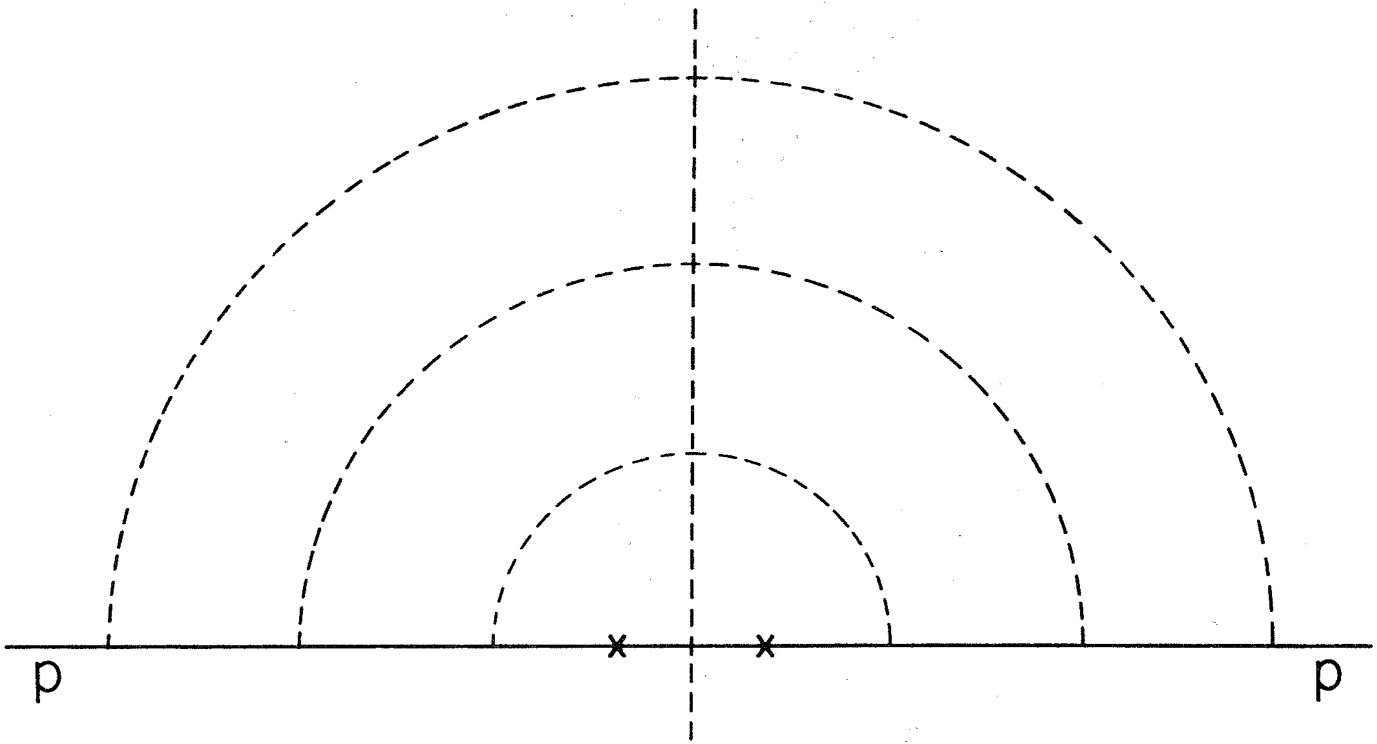


Fig. 7

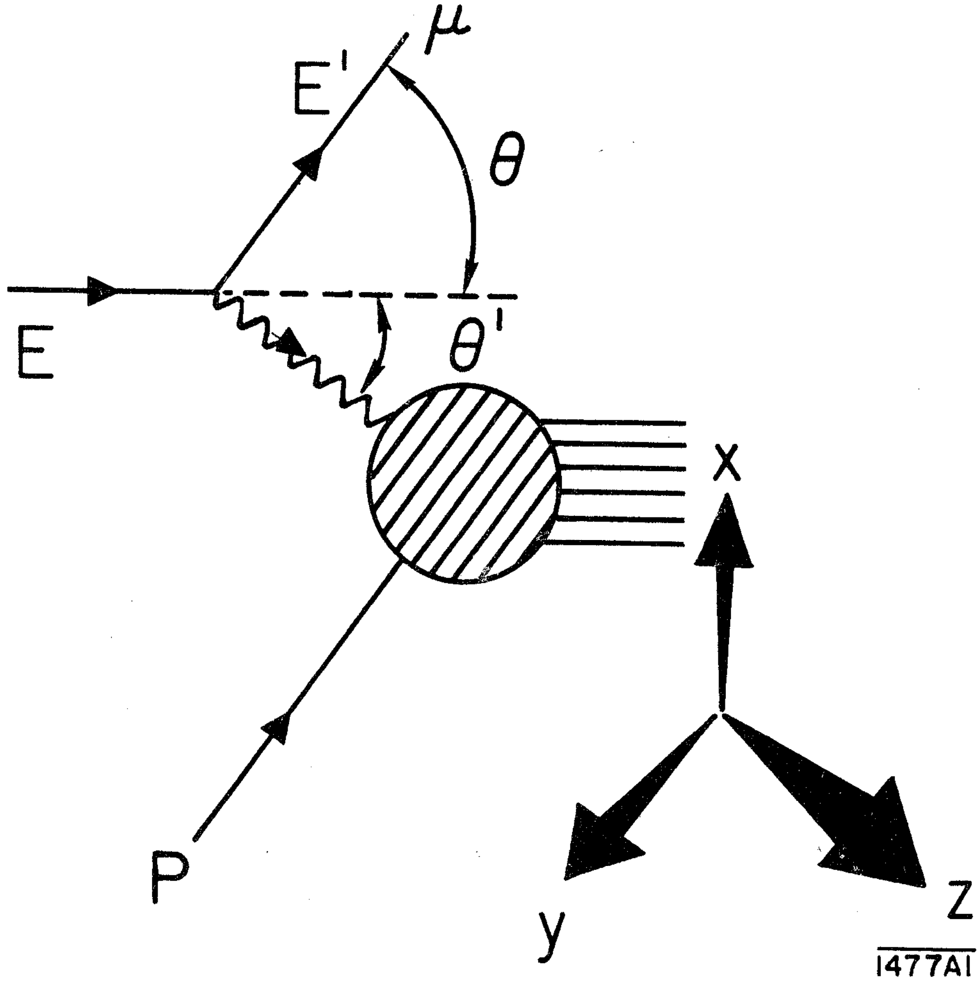


Fig. 8

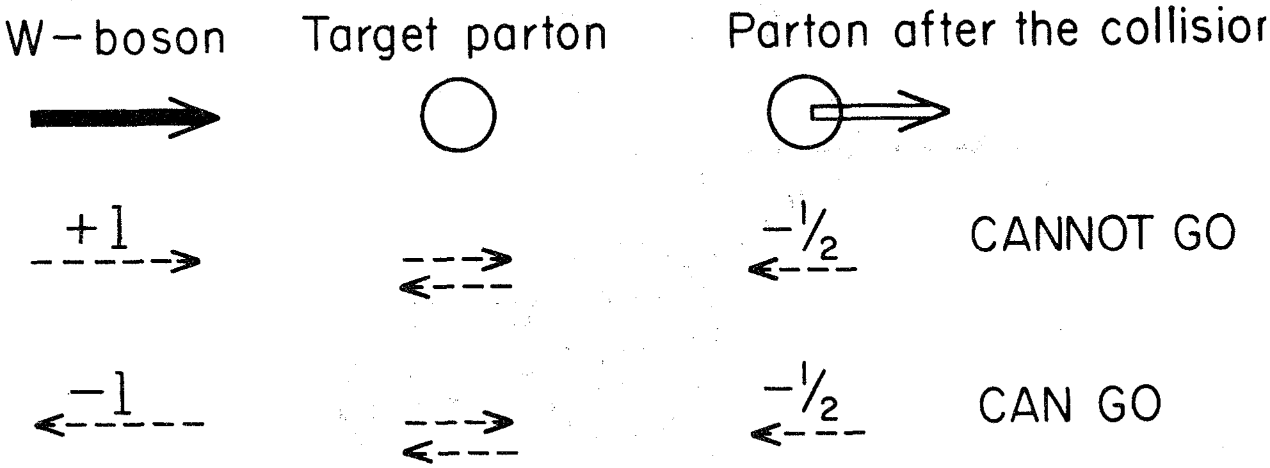

W-boson
Target parton
Parton after the collisior
+1
$-1/2$
CANNOT GO
-1
$-1/2$
CAN GO

Fig 9

TABLE I

MODEL	PARTONS	FIELD THEO. MOD.	DIFFR.	EXPER.
	Bjorken, Paschos	Drell, Levy, Yan	Harari	
FEATURE				
Scaling	Yes	Yes	--	Yes
$\int \nu W_2 \, dx$	$\gtrsim .22$	--	.22	$.17 \pm .01$
$\int \frac{\nu W_2}{x} \, dx$	∞	> 1	∞	$> .72 \pm .01$
σ_ℓ/σ_t	$S = 0 \rightarrow \infty$ $S = \frac{1}{2} \rightarrow 0$	0	?	$\lesssim 1 \pm ?$
Particles along $\vec{q}$	Depends on σ_ℓ/σ_t	Mostly Protons for $x \ll 1$	--	?
σ_n/σ_p	.8 Quark 1 as $\frac{\nu}{Q^2} \rightarrow \infty$	1 for $x \ll 1$	1	?
$\nu_p/\bar{\nu}_p$	~ 1	3	1	?

Table I. Comparison among the predictions of three models with the experimental results. The last row gives the ratio of total cross sections for neutrinos and antineutrinos on protons.

Table II

SUM RULE	COMMUTATOR	QUARK COMMUTATOR	PARTONS
$\int_0^\infty d\nu[\bar{\beta}(\nu,Q^2) - \beta(\nu,Q^2)]$	J_{oo}	$2(\cos^2\theta_c+2\sin^2\theta_c)$	$2\langle N_{p'}+N_{\bar{n}'}\cos^2\theta_c-N_{\bar{p}'}-N_{n'}\cos^2\theta_c$ $+ (N_{\bar{\lambda}'}+N_{\lambda'})\sin^2\theta_c\rangle$
$\int_0^\infty d\nu[\bar{\beta}(\nu,Q^2)(\bar{R}+\bar{L}) - \beta(\nu,Q^2)(R+L)]$	J_{xx}	$2(\cos^2\theta_c+2\sin^2\theta_c)$	$2\langle(N_{p'}-N_{\bar{p}'})+(N_{\bar{n}'}-N_{n'})\cos^2\theta_c$ $+ (N_{\bar{\lambda}'}-N_{\lambda'})\sin^2\theta_c\rangle$
$\int_0^\infty d\nu[\bar{\beta}(\nu,Q^2)(\bar{L}-\bar{R}) + \beta(\nu,Q^2)(L-R)]$	J_{xy}	$2(3\cos^2\theta_c+2\sin^2\theta_c)$	$2\langle(N_{p'}-N_{\bar{p}'})+(N_{n'}-N_{\bar{n}'})\cos^2\theta_c$ $+ (N_{\lambda'}-N_{\bar{\lambda}'})\sin^2\theta_c\rangle$
$\int_0^1 dx[\nu\bar{\beta}(\nu,Q^2)(\bar{R}+\bar{L}) + \nu\beta(\nu,Q^2)(R+L)]$	$\dot{J}_{xx}$	----	$2\langle\frac{1}{N}[(N_{p'}+N_{\bar{p}'})+(N_{n'}+N_{\bar{n}'})\cos^2\theta_c$ $+ (N_{\lambda'}+N_{\bar{\lambda}'})\sin^2\theta_c]\rangle$
$\int_0^1 dx[\nu\bar{\beta}(\nu,Q^2)(\bar{L}-\bar{R}) - \nu\beta(\nu,Q^2)(L-R)]$	$\dot{J}_{xy}$	----	$2\langle\frac{1}{N}[(N_{p'}+N_{\bar{p}'})-(N_{n'}+N_{\bar{n}'})\cos^2\theta_c$ $- (N_{\lambda'}+N_{\bar{\lambda}'})\sin^2\theta_c]\rangle$
$\int_0^1 dx[\nu\bar{\beta}(\nu,Q^2)(\bar{R}+\bar{L}) - \nu\beta(\nu,Q^2)(R+L)]$	----	----	$2\langle\frac{1}{N}[(N_{p'}-N_{\bar{p}'})+(N_{\bar{n}'}-N_{n'})\cos^2\theta_c$ $+ (N_{\bar{\lambda}'}-N_{\lambda'})\sin^2\theta_c]\rangle$
$\int_0^1 dx[\nu\bar{\beta}(\nu,Q^2)(\bar{L}-\bar{R}) + \nu\beta(\nu,Q^2)(L-R)]$	----	----	$2\langle\frac{1}{N}[(N_{p'}-N_{\bar{p}'})+(N_n-N_{\bar{n}'})\cos^2\theta_c$ $+ (N_{\lambda'}-N_{\bar{\lambda}'})\sin^2\theta_c]\rangle$

Table II. Sum rules suggested by current commutators and by the parton model.

⅃

REFERENCES

1. W. K. H. Panofsky in the Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, 1968 (CERN, Geneva, 1968), pp. 36-37. E. D. Bloom, D. H. Coward, H. DeStaebler, J. Drees, G. Miller, L. W. Mo, R. E. Taylor, M. Breidenbach, J. I. Friedman, G. C. Hartmann and H. W. Kendall, "High Energy Inelastic e-p Scattering at 6° and 10°," Report No. SLAC-PUB-642, Stanford Linear Acclerator Center, Stanford, California (1969), and M. Breidenbach, J. I. Friedman, H. W. Kendall, E. D. Bloom, D. H. Coward, H. DeStaebler, J. Drees, L. W. Mo, R. E. Taylor, "Observed Behavior of Highly Inelastic Electron-Proton Scattering," Report No. SLAC-PUB-650, Stanford Linear Accelerator Center, Stanford, Stanford, California (1969).
2. J. D. Bjorken, Phys. Rev. 179, 1547 (1969).
3. R. P. Feynman (unpublished).
4. For an earlier review see J. D. Bjorken, Report No. SLAC-PUB-571 (1969).
5. J. D. Bjorken and E. A. Paschos, SLAC-PUB-572 (1967) (to be published in Phys. Rev.).
6. S. J. Drell, D. J. Levy, and T. M. Yan, Phys. Rev. Letters 22, 744 (1969) and SLAC-PUB-606 (1969)(to be published in Phys. Rev.).
7. H. D. Abarbanel, M. L. Goldberger and S. Treiman, Phys. Rev. Letters 22, 500 (1969).
8. H. Harari, Phys. Rev. Letters 22, 1078 (1969).
9. J. J. Sakurai, Phys. Rev. Letters 22, 981 (1969).
10. S. D. Drell and J. D. Walecka, Ann. Phys. (N.Y.) 28, 18 (1964).
11. L. N. Hand, Phys. Rev. 129, 1834 (1963).
12. The currents that are used in this paper have an operational meaning--they are defined as the currents involved in the matrix element $M_{\mu\nu}$ which when multiplied by the leptonic currents gives a covariant S-matrix. Such a qualification is necessary when we use the $q_0 \to i\infty$ limit, because as it has been noted by R. Jackiw and G. Preparata (Phys. Rev. Letters 22, 975 (1969)) and by S. L. Adler and W. K. Tung (Phys. Rev. Letters 22, 978 (1969)) the Bjorken limit breaks down in second order perturbation theory. In fact, in second order perturbation theory the limit, $\lim_{q^2\to\infty} \nu W_2 (q^2, x)$ diverges like $\log q^2/\Lambda^2$. However, there is no compelling reason to believe that this argument is relevant for the currents used in this paper and in particular in the high energy limit. It seems that we learn once again that nature is less singular than the most singular parts of field theory. See F. Gilman on ref. 33
13. Due to the assumption $\sigma_\ell/\sigma_t \ll 1$, the statistical errors and the limited range of Q^2 where data are available, we cannot presently eliminate the presence of $\log (Q/\Lambda)^2$ terms. Such terms have been mentioned in Ref. 12 and they are also present in the calculation of H. Cheng and T. T. Wu, Phys. Rev. Letters 22, 1409 (1969).

14. The authors of Ref. 7 found that forward Compton scattering scales, provided that the Regge residue goes like $\beta \ (q^2) \to \frac{1}{q^2}$ as $q^2 \to \infty$. This is shown to be true in ladder diagrams of φ^3-theory. The case of ladder diagrams coupled to two external vector mesons has been studied by G. Altarelli and H. Rubinstein. A diffractive model using the eikonal method has been studied by R. W. Griffith, preprints ANL-HEP, 1969 (unpublished).
15. See H. Harari, Phys. Rev. Letters 20, 1395 (1968); P. Freund, Phys. Rev. Letters 21, 235 (1968); F. J. Gilman, H. Harari, and Y. Zarmi, Phys. Rev. Letters 21, 323 (1968).
16. This connection, of course, is not unique. One can also assume that a larger fraction of the proton's charge does not contribute in the $Q^2 \to \infty$ limit. One can assume, for instance, that only the infinite sea, or a fraction of it, is contributing in this limit. Such assumptions are much more model dependent.
17. S. Berman and W. Schmidt, private communication, 1968 (unpublished).
18. C. F. Cho, G. J. Gounaris, and J. J. Sakurai, University of Chicago preprint, 1969 (unpublished). See also Y. S. Tsai, SLAC-PUB-600, 1969 (unpublished); J. D. Sullivan, University of Illinois preprint, 1969 (unpublished); and G. A. Piketty and L. Stodolsky in Proceedings of the Topical Conference on Weak Interactions, CERN, Geneva, 1969, (CERN 69-7), p. 75.
19. J. M. Cornwall and R. E. Norton, Phys. Rev. 177, 2584 (1969).
20. C. G. Callan, and D. J. Gross, Phys. Rev. Letters 22, 156 (1969).
21. We follow rather closely the discussion of J. D. Bjorken and E. A. Paschos (to be published).
22. F. J. Gilman, Phys. Rev. 167, 1365 (1968).
23. A. Pais and S. Treiman, contributed to the Anniversary Volume "Problems of Theoretical Physics" dedicated to Nikolai Nikolaievitch Bogoliubov, Publishing House "Nauka," Moscow 1969, pp. 257-260.
24. S. L. Adler, Phys. Rev. 143, 1144 (1966).
25. S. Fubini, Nuovo Cimento 43A, 161 (1966). R. Dashen and M. Gell-Mann, Phys. Rev. Letters 17, 340 (1966).
26. J. D. Bjorken, Phys. Rev. 163, 1769 (1967).
27. D. J. Gross and C. H. Llewellyn-Smith, preprint CERN-TH. 1043(1969).
28. W. N. Cottingham, Ann. Phys. (N.Y.) 25, 424 (1963).
29. H. Pagels, preprint, The Rockefeller University, 1969.
30. M. Elitzur and H. Harari, Weizmann Institute preprint, 1969 (unpublished); R. Chanda, University of Maryland preprint, 1969 (unpublished); D. J. Gross and H. Pagels, Phys. Rev. 172, 1381 (1968).
31. M. J. Creutz, S. D. Drell and E. A. Paschos, Phys. Rev. 178, 2300 (1969).
32. E. D. Bloom et al (SLAC)
D. O. Caldwell et al (Santa Barbara)
H. Meyer et al (DESY)
J. Ballam et al (SLAC) and J. Ballam et al, Phys. Rev. Letters 21, 1541 (1968).

33. There are several re-evaluations of the sum rules. The re-evaluation mentioned in the text refers to the calculation by M. Creutz and myself at SLAC, using the data of group A. An independent re-evaluation by F. J. Gilman concludes that there is an extra real, constant part in the Compton amplitude. It will be of interest to re-evaluate the sum rule using all the data, once they become available. The importance of a fixed pole at $\alpha = 0$ and the determination of its contribution to the mass difference has been emphasized by F.J. Gilman, Proceedings of the 4th International Symposium on Electron and Proton Interactions at High Energy, Daresbury, England (1969).