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MUON-PROTON DEEP INELASTIC SCATTERING*

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ABSTRACT

We have measured the differential cross section for the deep inelastic scattering of 12 GeV/c muons on protons. Only the scattered muon was detected. Results are presented for $|q^2|$ (the square of the four-momentum transferred from the muon vertex) values of 0.3 to 4.0 $(GeV/c)^2$ and for muon energy losses up to 9 GeV. A brief interpretation of the data in terms of virtual photon cross sections and structure functions is presented.

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In this letter we report the results of the first large four-momentum transfer study of muon-proton inelastic scattering. Inelastic scattering differential cross sections, $d^2\sigma/dq^2d\nu$, have been measured for values of the square of the four-momentum transfer, $|q^2|$, extending to 4 $(\text{GeV}/c)^2$ and for energy transfers, ν, up to 9 GeV. The first purpose of this experiment, and the subject of this letter, was to study the interactions of virtual photons with protons. The second purpose was to compare muon-proton and electron-proton inelastic scattering in order to search in a hitherto unexplored region for possible differences between the muon and the electron. This is the subject of the following Letter.[1]

The experiment was carried out at the Stanford Linear Accelerator Center using a 12 GeV/c, pair-produced, positive muon beam[2] with a momentum resolution of $\pm$ 1.5%. The apparatus consisted of a 198 cm long, liquid-hydrogen target, a large analyzing magnet, optical spark chambers and scintillation counters. The small momentum width and small phase space (3×10^{-3} cm^2 sr) of the incident muon beam allowed inelastic events to be defined by measuring just the scattering angle and final momentum of the muon. The spark chambers which provided this information were triggered whenever three banks of scintillation counters indicated a muon scattering angle greater than 30 mr.

The beam at the hydrogen target contained less than 3×10^{-6} pions per muon. An additional pion rejection factor of 50 was obtained through the requirement that the scattered muon pass through a series of iron plates and spark chambers without nuclear interaction. Because of the high instantaneous flux, beam normalization was accomplished by using a scintillator array which sampled 1/30th of the beam. This array was regularly calibrated at low current. Detailed descriptions of the beam and apparatus, as well as preliminary results for low q^2 data, and a study of muon inelastic scattering on carbon and copper nuclei have already been reported.[2,3,4,5]

The data presented here result from 2.4×10^{10} muons incident upon the full hydrogen target. Empty target background subtraction runs were taken with 0.5×10^{10} incident muons. 264,000 photographs were taken of which 91% were with a full target. Most photographs were scanned three times, the remainder twice, and all events found in at least one scan were then measured. Events which did not reconstruct properly on first measurement were measured again and many events were measured three times. 10,950 inelastic events with target full (and 89 with target empty) were found in the kinematic region reported in this paper.

To obtain the differential cross sections from the primary data it is necessary to know the geometric detection efficiency of the apparatus as fixed by the apertures of the magnet and spark chambers. This detection efficiency, a function of the angle and momentum of the scattered muon, was calculated using a Monte Carlo technique to trace the paths of the scattered muons through the apparatus. Data is presented only for those kinematic regions whose detection efficiency is known with less than 1% uncertainty. The data was corrected by 2.5% for scanning, measuring and spark chamber inefficiencies and by about 2% for electronic dead time. In addition we found it necessary to assign different beam momenta to different blocks of data taken over several months and under varying beam momentum and trigger conditions. The beam momentum, as determined by a study of elastic events,[6] varied by as much as 1/2%. These shifts appear to be caused by several factors, such as slight irregularities in the film measuring machines, but we are unable to verify all of these factors. Accordingly we have allowed for a systematic error due to an uncertainty of 1/2% in the beam momentum. When we combine this uncertainty with estimated errors due to all other corrections and the uncertainty in the normalization procedure, we estimate a total systematic normalization uncertainty of ± 4%. However we find we must increase this estimate to ± 6% when we examine the internal consistency of our data and when we compare the 12 GeV/c measurements reported in this Letter with the smaller sample of 10 GeV/c measurements previously reported.[4] Therefore we use a total systematic normalization uncertainty of ± 6% although we cannot account for such a large uncertainty.

The inelastic scattering of charged leptons on protons leading to the production of hadrons takes place almost entirely through the exchange of a single virtual photon.[7] Lorentz and gauge invariance considerations then lead to the conclusion that the differential cross section $d^2\sigma/dq^2 d\nu$ depends on explicit factors involving all the kinematic variables and just two independent quantities which must be experimentally determined. These quantities, functions of only q^2 and ν, describe in a summary way the production of hadrons in the interaction of virtual photons with protons. The separation of the kinematic factors is to some extent arbitrary. One description, due to Hand,[8] defines $\sigma_T(q^2, K)$ and $\sigma_S(q^2, K)$ which may be thought of as the total cross sections for the interaction of transverse and scalar photons respectively with protons. Here $K = \nu - |q^2|/(2M)$, where M is the proton mass. Thus K is the energy that a real photon must have to give the same total energy in the photon-proton center-of-mass system. $\sigma_T(q^2, K)$ and $\sigma_S(q^2, K)$ are defined by

$$d^2\sigma/dq^2 d\nu = d^2\sigma/dq^2 dK = \Gamma_T(q^2, K, p)\, \sigma_T(q^2, K) + \Gamma_S(q^2, K, p)\, \sigma_S(q^2, K)$$

$$= \Gamma_T(q^2, K, p) \left[\sigma_T(q^2, K) + \epsilon(q^2, K, p)\, \sigma_S(q^2, K)\right] \qquad (1)$$

Γ_T and Γ_S are the virtual photon fluxes for transverse and scalar photons, respectively, and ϵ is the ratio of these fluxes as shown in the next two equations.

$$\Gamma_T = \left(\frac{\alpha}{2\pi |q^2|}\right)\left(\frac{K}{p^2}\right)\left(1 - \frac{2m^2}{|q^2|} + \frac{2EE' - |q^2|/2}{(E - E')^2 + |q^2|}\right) \qquad (2)$$

$$\epsilon = \Gamma_S/\Gamma_T = \left(\frac{2EE' - |q^2|/2}{(E-E')^2 + |q^2|}\right) \Bigg/ \left(1 - \frac{2m^2}{|q^2|} + \frac{2EE' - |q^2|/2}{(E - E')^2 + |q^2|}\right) \qquad (3)$$

Here p(p') and E(E') are the momentum and energy in the laboratory system of the incident (scattered) muon; m is the muon mass and α is the fine structure constant. As

q^2 goes to zero, $\sigma_S(q^2, K)$ goes to zero and $\sigma_T(q^2, K)$ goes to $\sigma_{\gamma p}(K)$ — the total cross section for the interaction of a physical photon of energy K with a proton. In this experiment we cannot separate σ_T from σ_S; therefore we report only the combination

$$\sigma_{exp}(q^2, K, p) = \sigma_T(q^2, K) + \epsilon\, \sigma_S(q^2, K) \tag{4}$$

In our data $\sigma_{exp}(q^2, K, p)$ is only weakly dependent on p because ϵ is always close to 1.

In Table I we list the values of $d^2\sigma/dq^2dK$ and σ_{exp} found in this experiment. The errors quoted are statistical only and must be combined with the overall normalization uncertainty of ± 6%. These cross sections have been corrected for radiative effects as previously described[4] and those corrections, in terms of the percentage by which the measured cross sections were reduced, are listed in Table I. σ_{exp} shows the same behavior versus $|q^2|$ and K as we observed in our preliminary results[4] and as observed in the extensive electron-proton inelastic scattering experiments[9] which have been carried out at the Stanford Linear Accelerator Center. For fixed K, σ_{exp} decreases smoothly as $|q^2|$ increases. Our data exhibits particularly well the smooth fall off of $\sigma_{exp}(q^2,K,p)$ from $\sigma_{\gamma p}(K)$ in the low $|q^2|$ region. At higher $|q^2|$ values, σ_{exp} decreases as fast or faster than $1/|q^2|$, but not as fast as $1/|q^2|^2$. For fixed $|q^2|$, σ_{exp} decreases as K increases.

It is also useful to express $d^2\sigma/dq^2dK$ in terms of the structure functions $W_1(q^2, \nu)$ and $W_2(q^2, \nu)$ defined by von Gehlen[10] and by Drell and Walecka,[11] which are the analogs of the proton form factors in elastic scattering. The structure functions are related to the total cross sections by the equations

$$W_2(q^2, \nu) = \frac{1}{4\pi^2\alpha}\left[\frac{K|q^2|}{\nu^2 + |q^2|}\right]\left[\sigma_T(q^2, K) + \sigma_S(q^2, K)\right]$$

$$W_1(q^2, \nu) = \frac{1}{4\pi^2\alpha}\, K\, \sigma_T(q^2, K) \tag{5}$$

It was suggested by Bjorken and others[12] that at large values of ν and q^2, νW_2 might be a function of only the combination $\omega = 2M\nu/|q^2|$. This "scaling law" has been generally confirmed by electron-proton inelastic experiments,[9] for values of $|q^2| \gtrsim 1\ (\mathrm{GeV/c})^2$.

In Fig. 1 we present our results for νW_2, assuming[13] $\sigma_S/\sigma_T = .18$, and plotting against $x = 1/\omega = |q^2|/(2M\nu)$. Our data also verifies the scaling law, and the continuity of our data allows us to make several additional observations. We divide these observations into two regions.

1. $0 < x < 0.1$. Our data show νW_2 rising with x from x = 0. This is a consequence of the fact that $\lim_{x \to 0} \nu W_2 \propto |q^2| \sigma_{\gamma p} \longrightarrow 0$ at fixed ν. We do not have data at small x, and large q^2, with which to test the behavior of νW_2 at small x in the scaling limit $\nu \to \infty$.

2. $0.1 < x \lesssim 0.3$. Our data show νW_2 going through a maximum at x = 0.2. Now at x = 0.2, $\nu W_2 = .26$ (Fig. 1) for the very <u>low</u> values of $|q^2| \sim 0.3\ (\mathrm{GeV/c})^2$ and K = 0.8 GeV. The electron-proton inelastic data[9] show that at x = 0.2 νW_2 does not rise above 0.35 at the <u>highest</u> $|q^2|$ reached to date. In other words, νW_2 has already reached 80% of its scaling limit value at $|q^2| \sim 0.3$, K = 0.8. Why should "scaling" be so good at such a low value of $|q^2|$, when all theoretical models which seek to discuss scaling are forced to take limits $|q^2| \longrightarrow \infty$, $\nu \longrightarrow \infty$? On any plausible assumption of the kind of mass which should be involved in departures from scaling, one would expect to see a substantial departure from scaling at $|q^2| \simeq 1\ (\mathrm{GeV/c})^2$, and we do not.

It seems to us much less surprising that this should be so when we remember that there are two thresholds to discuss. Take x fixed at 0.2 and decrease $|q^2|$. First there is the threshold behavior in $|q^2|$ which is expected to reduce νW_2 progressively as $|q^2|$ approaches zero. We see this effect clearly in the data for $x \lesssim 0.1$. Second, there is the threshold behavior in K. (For fixed x, as $|q^2|$ decreases, K decreases, because $x = |q^2|/(2M\nu)$.) As K decreases from, say, 2.0 GeV to 0.8 GeV, $\sigma_{\gamma p}(K)$ increases by almost a factor of two[14] as the resonance region is approached. Since we are at quite small $|q^2|$, we can expect the effects of this rapid rise in $\sigma_{\gamma p}(K)$ to be reflected in $\sigma_T(q^2, K)$ and hence in νW_2 (see Eq. (5)). Thus we suggest the possibility that "scaling" may be continued to low $|q^2|$ because of the two unrelated threshold effects working in opposition, and not because of a continuing operation of a "scaling" mechanism in the low $|q^2|$ region. Our observation at $x = 0.2$ of approximate scaling to the absurdly low value of $|q^2| \sim 0.3$ and our explanation, admittedly heuristic, in terms of threshold effects, suggests that it may not be necessary even at $|q^2| \sim 1\,(\mathrm{GeV}/c)^2$ to attribute the observed scaling entirely to mechanisms appropriate to the Bjorken limit. A smooth transition from the threshold region at low $|q^2|$ to an asymptotic region at substantially higher $|q^2|$ could produce the same behavior.

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6. In this experiment, if an event was a muon-proton elastic scattering, the recoil proton was detected in a large spark chamber just below the hydrogen target. This provided a three-constraint fit on elastic events; and also permitted, through a two-constraint fit, a direct determination of the momentum of the incident muon. The apparent beam momentum was determined by the study of these elastic events. The elastic events were acquired concurrently with the inelastic events; thus we have a continuous monitor of the beam momentum.
7. There are no comprehensive experimental studies of the validity of the one photon exchange assumption in inelastic scattering. The assertion that two photon exchange is less than a few percent effect in inelastic scattering is based on theoretical

considerations and on the lack of any detectable two photon contribution in elastic scattering. For a review of this subject in elastic scattering see: J. G. Rutherglen in Proceedings of the 4th International Symposium on Electron and Photon Interactions at High Energy, (Daresbury Nuclear Physics Laboratory, Daresbury, England, 1969). Also see L. Camilleri et al., Phys. Rev. Letters 23, 149 (1969). Some indication that two photon exchange in inelastic scattering is at most a few percent effect (but not necessarily an undetectable effect) has been given by experiments searching for T-violation in inelastic electron scattering, H. DeStaebler, (private communication); and Stephen Rock et al., Phys. Rev. Letters 24, 748 (1970).

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13. As shown in Ref. 9 of this paper, little change is produced in W_2 if a relatively larger value is picked for σ_S, for example if $\sigma_S/\sigma_T = 1$. Very little is known about the ratio σ_S/σ_T for $|q^2| < 1\ (\mathrm{GeV/c})^2$. For higher values of $|q^2|$ as shown in Ref. 9, σ_S/σ_T is less than 0.3.
14. See for example the compilation of $\sigma_{\gamma p}(K)$ data by F. J. Gilman in Proceedings of the 4th International Symposium on Electron and Photon Interactions at High Energy, (Daresbury Nuclear Physics Laboratory, Daresbury, England, 1969).

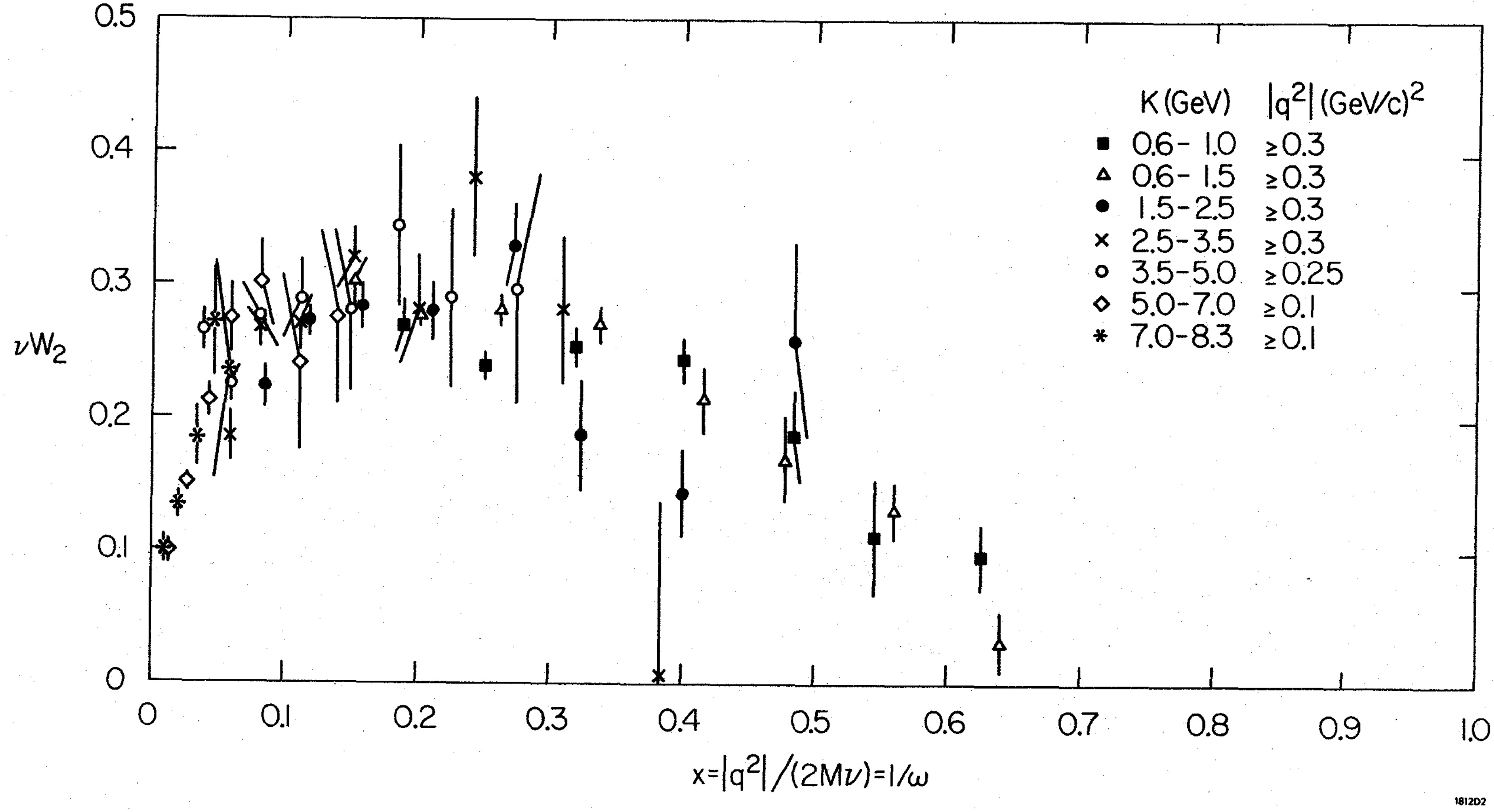
0.5
0.4
0.3
0.2
0.1
0
νW_2
K (GeV) |q²| (GeV/c)²
0.6- 1.0 ≥0.3
0.6- 1.5 ≥0.3
1.5-2.5 ≥0.3
2.5-3.5 ≥0.3
3.5-5.0 ≥0.25
5.0-7.0 ≥0.1
7.0-8.3 ≥0.1
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
x=|q²|/(2Mν)=1/ω
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Fig. 1