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THE FREQUENCY DEPENDENCE OF SUPERCONDUCTING CAVITY Q AND MAGNETIC BREAKDOWN FIELD*

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Abstract

A theoretical explanation is given to account for the unexpected observation that L-band Nb superconducting cavities were found to have lower Q and lower magnetic breakdown field than those of the higher X-band frequencies. Both effects can be related to the trapping of magnetic flux in the cavity walls. The frequency dependence arises from the frequency dependence of the resistivity of oscillating fluxoids. Calculations based on this model are in agreement with experimental observations.

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According to established theories[1,2] of rf superconductivity, one would expect the superconducting surface resistance, R_s, to be approximately proportional to the square of the cavity angular frequency, ω. Hence, in going to the lower frequency L-band Nb cavities, it was expected that the Q's would be as high or higher than with comparably processed X-band cavities. However, as we shall see, this may not be the case when the surface resistance is dominated by the trapped flux at the operating temperature. A discussion of the trapping of the flux due to an incomplete Meissner-Ochsenfeld effect, and the related power dissipation in an rf superconductor, is given by Rabinowitz.[3]

Assuming that the only non-superconducting loss is due to trapped flux, the total average power loss for a magnetic field $H_p \cos \omega t$ at the cavity surface is:

$$P = \frac{1}{2} \int R_n H_p^2 \, dA_n + \frac{1}{2} \int R_s H_p^2 \, dA_s \quad , \tag{1}$$

giving an effective surface resistance for the cavity,

$$R = R_n \left(\frac{A_n}{A_t} \right) + R_s \left(1 - \frac{A_n}{A_t} \right) \quad , \tag{2}$$

where R_n is the surface resistance of the fluxoids, A_n is the normal area, R_s is the superconducting surface resistance, A_s is the superconducting area, and A_t is the total cavity area.

$$\begin{aligned} Q &= \frac{\left[\frac{1}{2} \int_0^V \mu_0 H_p^2 \, dV \right]}{P/\omega} \\ &= \frac{G V \omega}{R A_t} \\ &= \frac{G V \omega}{R_n A_n + R_s \left(A_t - A_n \right)} \quad , \end{aligned} \tag{3}$$

where V is the cavity volume, and G is a constant related to cavity geometry. The trapped flux is proportional to the total flux intercepted by the cavity, so that

$$A_n \doteq d \frac{B}{B_0} A , \tag{4}$$

where B is the magnetic flux density, $H_0 = B_0/\mu$ is the corresponding critical field for type I or II, A is the cross-sectional area of the cavity normal to the flux, and d is a proportionality constant. Therefore

$$Q = \frac{G V \omega}{R_n\left(dBA/B_0\right) + R_s\left(A_t - dBA/B_0\right)} \quad . \tag{5}$$

If the power loss is dominated by the trapped flux,

$$R_n\left(dBA/B_0\right) \gg R_s\left(A_t - dBA/B_0\right) , \tag{6}$$

and

$$Q \doteq \frac{G V \omega B_0}{R_n d B A} \quad . \tag{7}$$

$$R_n = \frac{\rho}{2\lambda} , \tag{8}$$

where λ is the penetration depth, and ρ is the effective resistivity of an oscillating fluxoid as derived by Rabinowitz.[3]

$$\rho = \left[\frac{\omega^2 \phi^2 H H_0 \mu^2}{\rho_n^2\left(\omega^2 M - p\right)^2 + \omega^2 \phi^2 H_0^2 \mu^2}\right] \rho_n \; . \tag{9}$$

ϕ is the flux trapped in a fluxoid, H is the magnetic field in the fluxoid of permeability μ, H_0 is the appropriate critical field, ρ_n is the normal state resistivity, M is the fluxoid mass/length, and p is the pinning constant/length.

As previously pointed out,[3] when the viscous damping force is negligible,

$$\rho = \left[\omega^2 \phi^2 H H_0 \mu^2 / \left(\omega^2 M - p\right)^2\right] \left(1/\rho_n\right) . \tag{10}$$

When the viscous damping force dominates,

$$\rho = \left(H/H_0\right)\rho_n . \tag{11}$$

Therefore, if the power loss due to trapped flux dominates and a cavity is being operated under the conditions of Eq. (10), less material purity, within 2λ, yielding a higher ρ_n, may be desirable to reduce the power loss. When a cavity is being operated under the conditions of Eq. (11), then higher material purity would be desirable.

Let us consider the effect on Q, when cavities are operated in the two regions given by Eq. (10) and in the third region given by Eq. (11). When $\omega^2 M \gg p$, Eq. (10) and (8) yield

$$R_n = \left[\phi^2 H H_0 \mu^2 / 2\lambda \rho_n M^2\right] \omega^{-2} . \tag{12}$$

When $p \gg \omega^2 M$,

$$R_n = \left[\phi^2 H H_0 \mu^2 / 2\lambda p^2 \rho_n\right] \omega^2 . \tag{13}$$

Combining Eq. (12) and (7),

$$Q \doteq \left[\frac{2 G V B_0 \lambda \rho_n M^2}{d B A \phi^2 H H_0 \mu^2}\right] \omega^3 . \tag{14}$$

If we were to compare two geometrically similar cavities of different frequency in the same field orientation, assume that they have the same ρ_n, preparation and processing history, neglect any differences in their ability to exclude flux, and any differences in the topography of the trapped flux, then Eq. (14) gives

$$Q \propto \omega^2 B^{-1} , \tag{15}$$

since $V \propto \omega^{-3}$ and $A \propto \omega^{-2}$ for geometrically similar cavities.

Therefore, if one cavity is operated at 8.6 GHz (X-band), the cavity operated at 1.3 GHz (L-band) will have its Q lower by a factor of 44 in the same field. The highest reported Q $> 5 \times 10^{11}$ for a 10.5-GHz Nb cavity was measured at SLAC.[4] Turneaure and Viet[5] reported Q $> 10^{11}$ at 8.6 GHz. Equation (15) would then predict Q $> 2 \times 10^{9}$ at 1.3 GHz, all the other factors being similar. This is in good agreement with the HEPL results of Turneaure et al.[6] at 1.3 GHz, though some of the Q's were as low as 10^{8}. All the factors are not necessarily equal or similar; in particular the ambient magnetic flux density, B, has been significantly different. Typically, for X-band measurements in shielded, de-gaussed, dewars, B $\sim 10^{-5}$ to 10^{-4} gauss. In the HEPL L-band accelerating structure, B $\sim 10^{-3}$.

Combining Eq. (13) and (7),

$$Q \doteq \left[\frac{2GVB_0\lambda\rho_n p^2}{dBA\phi^2 HH_0\mu^2}\right]\omega^{-1} \quad . \tag{16}$$

Making the same kind of comparison as before, Eq. (14) gives

$$Q \propto \omega^{-2} B^{-1} \quad . \tag{17}$$

This has the same dependence on ω as expected from the superconducting surface resistance, and from stationary non-magnetic normal regions.[7] It appears to be quite advantageous to operate in this region of negligible viscous damping, and dominant pinning, if possible.

Now to consider the region where the viscous damping force dominates, then Eq. (8) and (11) yield

$$R_n = H\rho_n/2\lambda H_0 \quad . \tag{18}$$

Combining Eq. (7) and (18),

$$Q \doteq \left[\frac{2GVB_0\lambda H_0}{dBAH\rho_n}\right]\omega \quad . \tag{19}$$

Making the same kind of comparison again, Eq. (19) gives

$$Q \propto B^{-1} . \tag{20}$$

This region has no frequency dependence.

The most general relationship comes from combining Eq. (7), (8), and (9):

$$Q \doteq \frac{2GV\omega B_0 \lambda}{dBA\rho_n}\left[\frac{\rho_n^2\left(\omega^2 M - p\right)^2 + \omega^2\phi^2 H_0^2\mu^2}{\omega^2\phi^2 H H_0 \mu^2}\right] \tag{21}$$

Now that we have considered the frequency dependence of Q when it is dominated by the trapped flux power loss, let us also consider the magnetic breakdown field, H_p', in this case. As derived by Rabinowitz,[3] when breakdown is dominated by fluxoid power loss,

$$H_p' = \frac{\frac{-k_{1n}}{2B}\frac{T_c^2}{H_0} + \left[\left(\frac{k_{1n}}{2B}\frac{T_c^2}{H_0}\right)^2 - 2NRF_1^2 b\left\{\frac{1}{2}k_{1n}\left[T_b^2 - \frac{T_c^2}{B}\left(1 - \frac{H_a}{H_0}\right)\right]\right\}\right]^{1/2}}{NRF_1^2 b} , \tag{22}$$

for the case of a fluxoid perpendicular to the surface. An equation of the same form is derived for a parallel fluxoid.[3, 8] For the present purposes, in which we are primarily concerned with the frequency dependence of H_p', let us substitute the combination of Eq. (8) and (10) into Eq. (22), representing most of the non-frequency dependent terms by k_i.

$$H_p' = \frac{-k_1 + \left[k_1^2 + \frac{k_2\omega^2}{\left(\omega^2 M - p\right)^2 + k_3\omega^2}\right]^{1/2}}{\left[\frac{k_4\omega^2}{\left(\omega^2 M - p\right)^2 + k_3\omega^2}\right]} \tag{23}$$

Let us consider the three regions of R_n again. If $\left(\omega^2 M - p\right)^2 \gg k_3 \omega^2$ and $\omega^2 M \gg p$, then

$$H_p' = \left[-k_1 + \left(k_1^2 + k_2 M^{-2} \omega^{-2}\right)^{1/2}\right] k_4^{-1} M^2 \omega^2 \, . \tag{24}$$

This is for the case of negligible viscous damping, and negligible pinning. When $k_2 M^{-2} \omega^{-2} \gg k_1^2$, then

$$H_p' \propto \omega \, . \tag{25}$$

Therefore, in going from 8.6 GHz to 1.3 GHz, the magnetic breakdown field could be reduced by a factor of 6.6. Tureaure and Viet[8] reported a breakdown field of 1080 Oe at 8.6 GHz. So, if the conditions governing Eq. (25) were met, H_p' would be ~ 160 Oe for a 1.3-GHz cavity. Values of ~ 300 Oe have been obtained at 1.3 GHz.[6] Even aside from the question of whether the conditions of Eq. (25) apply, it must be borne in mind that breakdown is dominated by the fluxoid in the most vulnerable position.[3,8] It is not too likely that two cavities will have the dominant fluxoid in the same position. Nevertheless, it is significant that Eq. (25) gives H_p' within a factor of 2 of the experimental value.

When the damping is negligible and the pinning is dominant, $\left(\omega^2 M - p\right)^2 \gg k_3 \omega^2$ and $p \gg \omega^2 M$, then Eq. (23) becomes

$$H_p' = \left[-k_1 + \left(k_1^2 + k_2 p^{-2} \omega^2\right)^{1/2}\right] p^{+2} k_4^{-1} \omega^{-2} \, . \tag{26}$$

This would be a nice region to work in, if possible, both for high H_p' and high Q.

When the viscous damping force dominates, $k_3 \omega^2 \gg \left(\omega^2 M - p\right)^2$ and H_p' has no frequency dependence.

In conclusion, it would appear that calculations based on the model of trapped flux dominating the frequency dependence of cavity Q and magnetic breakdown field are in good accord with experimental observations.

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