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THEORIES OF HIGHLY INELASTIC ELECTRON SCATTERING†

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Introduction

Although some of the models proposed for deep inelastic electron scattering were sufficiently scientific to be tested and eliminated by the new data[1], so many viable models remain that it would be impossible to give a systematic review in thirty minutes. I shall therefore concentrate on a few topics; the omissions may be rectified slightly by my concluding remarks in which I shall attempt to summarise the present situation. I shall assume that the kinematics and the main features of the data are well known, as are the basic ideas of the most popular models, which have been adequately advertised at innumerable conferences and seminars.[2,3,4]

More specifically my plan is to discuss:

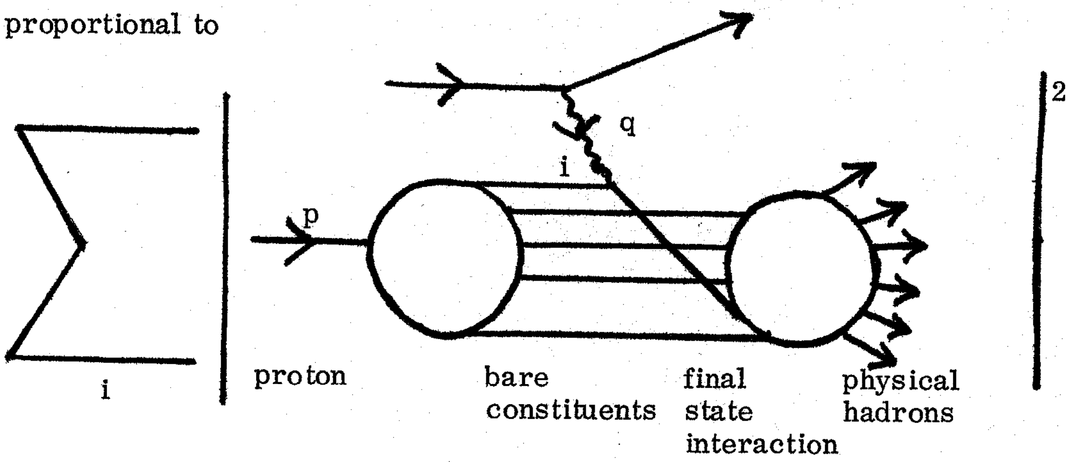
1) The parton model.
2) The possible survival of resonance contributions at large $|q^2|$.
3) Regge behavior, FESR and fixed poles.
4) What has been learned about e^-N recently (summary and conclusions).
5) Related processes.

(In the written version of this talk the discussion has been considerably extended.)

In this talk I shall take the preliminary SLAC-MIT data literally. Once and for all, let me make the necessary qualification that all conclusions based on this data should be treated cautiously. I shall assume that, to a good approximation, the deuteron structure functions are the sums of the proton and neutron structure functions, although subtle corrections may occur due to high momentum components in the deuteron wave function.[5]

The parton model

In the parton model it is supposed that, in the deep inelastic region, the nucleon behaves effectively as a free gas of point like constituents (or "partons") from which the electron scatters incoherently. Pictorially the cross section is proportional to

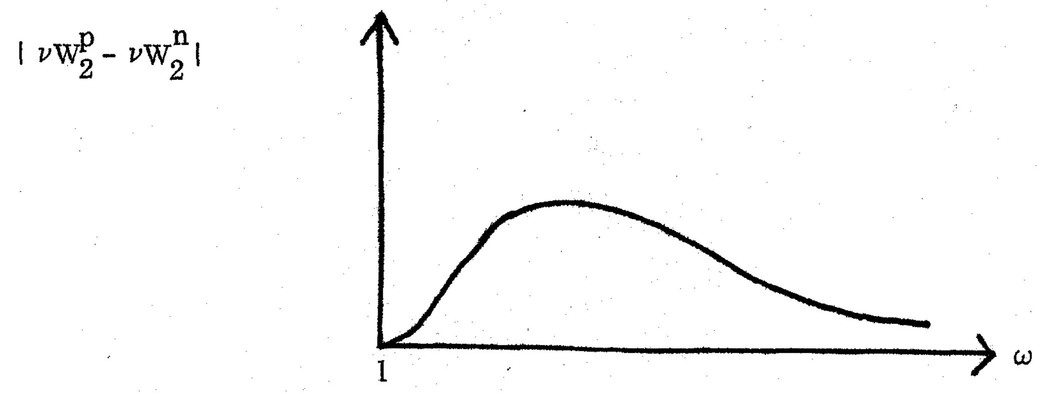


The final state interaction is normally ignored on the grounds that it turns one complete set of states into another. The model is supposed to apply in a frame where $|\vec{P}| >> M$ in which, it is argued, the partons are almost free (i. e. on mass shell). The partons are assumed to have small momentum transverse to the proton's three momentum $\vec{P}$ (as does the debris observed when the proton is broken up by collision with another hadron). Then if the i^{th} parton carries a fraction x_i of $\vec{P}$ we find $P_\mu^i \simeq x_i P_\mu$. Because the partons are nearly on mass shell, the photon parton cross section $\sim \delta(x 2q \cdot P + q^2)$. It is this delta function together with the assumption that the partons are point like which gives scale invariance, i. e. the result that the structure functions W_1 and νW_2 depend only on $\omega = \frac{1}{x} = \frac{2\nu}{-q^2}$.

In order to see qualitatively the sort of prediction the model makes, suppose that the proton's momentum is symmetrically distributed among the partons on average so that

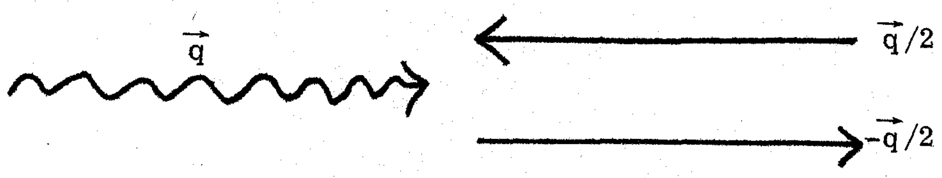
$$< x >_N = \frac{1}{N}$$

in a configuration with N partons. Hence at small ω ($= 1/x$) we are effectively examining small N configurations while at large ω we are probing large N configurations. Since some of the constituents in the proton and neutron must be different, they will appear different when ω is small (N small) but presumably they will look the same at large ω (large N). Combining this with the fact that νW_2 vanishes kinematically at $\omega = 1$, we see that $\nu W_2^p - \nu W_2^n$ will have the shape:



as is observed.

The first thing we learn from the data about the nature of the partons is their spin. To see this consider the photon parton interaction in the Breit frame:



If the parton has spin 0 it carries no helicity in or out along $\vec{q}$; hence it cannot absorb a transverse photon and $\sigma_T = 0$. If the parton has spin $\frac{1}{2}$ its helicity is conserved by the electromagnetic vertex when it is highly relativistic but, since its direction is reversed, it must absorb a unit of helicity from the photon; scalar photons are therefore impotent in this case and $\sigma_S = 0$. In fact,

$$\frac{\sigma_S(\omega)}{\sigma_T(\omega)} = \frac{\text{effective number of spin 0 partons}}{\text{effective number of spin } \frac{1}{2} \text{ partons}}$$

Experimentally σ_S/σ_T is small at all ω so that spin 0 partons cannot play an important role; the model of Drell, Levy and Yan[6] (DLY), in which the partons are bare pions and nucleons, is therefore in trouble.

We shall assume henceforth that the charged partons are quarks[7], which is the simplest choice compatible with the sacrosanct principles of current algebra. One of the most pertinent experimental results for the quark parton model is that

$$\Delta = \int_0^\infty \frac{d\omega}{\omega} \left(F_2^{\gamma p}(\omega) - F_2^{\gamma n}(\omega) \right) = 0.19 \pm 0.08$$

where Regge theory has been used for $\omega > 12$, so that the true error is possibly infinite (F_2 is the scale invariant limit of νW_2). In the parton model

$$\int F_2 \frac{d\omega}{\omega} = \sum_N P_N \sum_{i=1}^{N} Q_i^2$$

where P_N is the probability of there being N partons and Q_i their charges. This result follows directly from the fact that the distribution functions for the parton's momentum ($f_i^N(x)$) are normalized to one. Note that if F_2 has the Pomeron dominated Regge behavior $F_2(\omega) \xrightarrow{\omega \to \infty}$ const. the left hand side is infinite so that $\sum_N$ must go to infinity.

From a mixture of quarks and antiquarks there are two ways to make an isodoublet:

1. proton $\sim$ p quark $\times$ SU(2) scalar

$$\left(\sum_i Q_i^2 |_{\text{proton}} - \sum_i Q_i^2 |_{\text{neutron}} = \tfrac{1}{3} \right)$$

2. proton $\sim$ $\bar{n}$ quark $\times$ SU(2) scalar

$$\left(\sum_i Q_i^2 |_{\text{proton}} - \sum_i Q_i^2 |_{\text{neutron}} = -\tfrac{1}{3} \right) .$$

Assuming fractions $1-\epsilon$ and ϵ of the two configurations we get[8]:

$$-\tfrac{1}{3} \leq \Delta = \tfrac{1}{3} - \frac{2\epsilon}{3} \leq \tfrac{1}{3} .$$

Experimentally $\epsilon = 0.22 \pm 0.12$. Although, strictly, the error in ϵ is infinite, let us assume that indeed $\epsilon \neq 0$ in which case the second configuration is required and all models previously considered are excluded.

Another sum rule[9] follows in models in which $< x >_N = 1/N$:

$$< Q^2 > = \int F_2 \frac{d\omega}{\omega^2} \simeq 0.17 \qquad \text{(Expt.)}$$

where we have again used Regge theory at large ω. If only quarks and antiquarks are present

$$< Q^2 > \geq 2/9 .$$

Neutral particles must therefore be introduced to reduce this number which is not unreasonable since neutral "gluons" are present in quark models based on renormalizable Lagrangians.

The necessity for configuration mixing ($\epsilon \neq 0$) and the presence of gluons gravely reduce the predictive power of the quark parton model. Relations remain but their main content can be obtained from more profound considerations in most cases[10]. An exception is[11]:

$$6\int\frac{d\omega}{\omega^2}\,(F_2^{\gamma p}-F_2^{\gamma n})+\frac{8}{27} > \left(\frac{G^2 ME}{\pi}\right)^{-1}(\sigma^{\nu n}-\sigma^{\nu p}) > 6\int\frac{d\omega}{\omega^2}\,(F_2^{\gamma p}-F_2^{\gamma n})$$

Combining this with the CERN and SLAC data we get

$$3.3\begin{pmatrix}+16.0\\-\ 1.6\end{pmatrix} > \frac{\sigma^{\nu n}}{\sigma^{\nu p}} > 1.6\begin{pmatrix}+0.6\\-0.4\end{pmatrix} \quad .$$

The errors are left to indicate that a reduction of the input errors could give a very tight constraint. The upper (but not the lower) bound on $\sigma^{\nu n}-\sigma^{\nu p}$ involves the assumption $< x >_N = 1/N$ which gives $2 > \frac{\sigma^{\nu n}}{\sigma^{\nu p}} > 1$. The limits on $\frac{\sigma^{\nu n}}{\sigma^{\nu p}}$ are therefore

$$2 > \frac{\sigma^{\nu n}}{\sigma^{\nu p}} > 1.2 \quad .$$

This indicates that high energy ν (as opposed to $\bar{\nu}$) experiments in a hydrogen bubble chamber may be more productive than has sometimes been thought.[12] This expectation is sustained by the CERN experiment which indicates that $\frac{\sigma^{\nu n}}{\sigma^{\nu p}} \approx 1$ (for $|q^2| > 0.5\ \mathrm{GeV}^2$) and is unlikely to be > 2 anywhere in the deep inelastic region.[13]

While it is easy to destroy, the quark parton model is hard to verify and may not be very useful except as an heuristic device in inelastic lepton scattering.[14]

Survival of Resonant Contributions in the Deep Inelastic Limit

Resonance contributions are expected to fall off rapidly with increasing $|q^2|$ at fixed missing mass. Nevertheless the resonances can contribute in the scaling limit provided their density increases sufficiently rapidly; this occurs in the Veneziano type model of Landshoff and Polkinghorne[15], for example. To appreciate this phenomenon, consider scattering from a spin zero nucleon in a world consisting of spin zero resonances whose excitation form factors are all equal to $G(q^2)$. In this case:

$$\nu W_2 = 2\nu G^2(q^2) \sum_i \delta(s-m_i^2) \simeq 2\nu G^2(q^2)\, \rho(s)$$

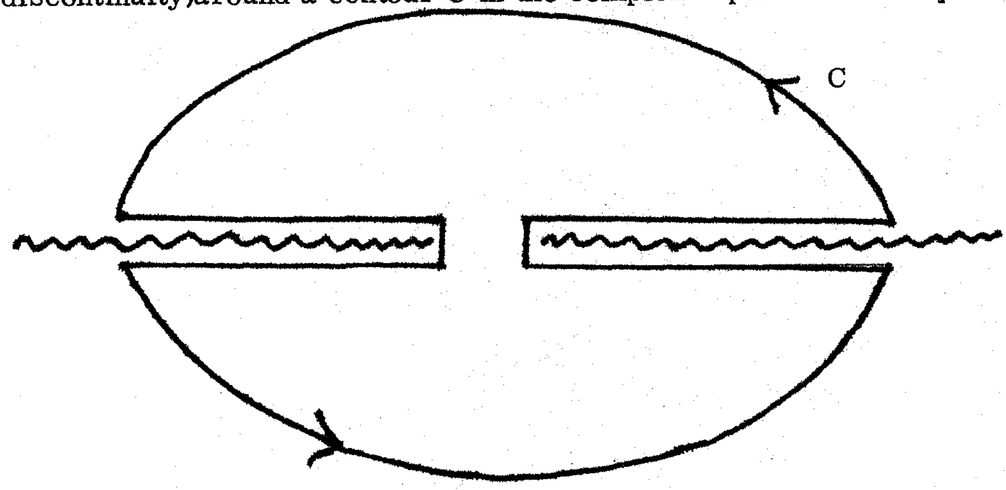
$$= -q^2 \omega\, G^2(q^2) \rho(-q^2(\omega' -1))$$

where we have assumed that the level density $\rho(s)$ is large so that $\sum_i \simeq \int \rho(m^2) dm^2$ ($\omega' = \omega + M^2/-q^2$). If $G^2(q^2) \sim (1/q^2)^n$ at large q^2, scaling is achieved provided $\rho(s) \sim s^{n-1}$, and we deduce that:

$$\nu W_2 \sim \omega(\omega' - 1)^{n-1} ,$$

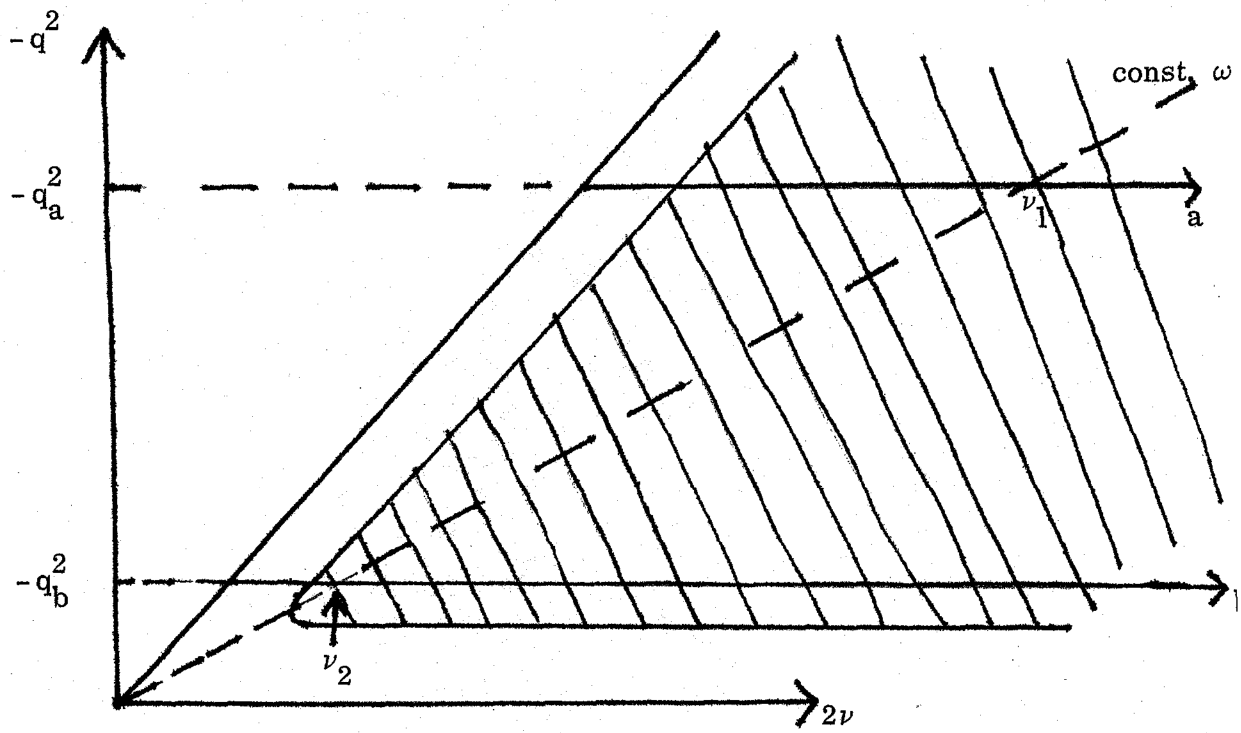
a relation first obtained by DLY[6], on quite different grounds, which fits the data near threshold with $n = 4$. (Away from threshold this crude model presumably fails due to finite resonance widths — just as the Veneziano model gives a misleading sum of δ functions for Im A on the real axis but Reggeises if $\nu \rightarrow \infty$ at an infinitesimal angle to the real axis.)

Bloom and Gilman[16] have plotted the data in a way which suggests that the resonances do survive in the scaling limit. They considered a sum rule which is easily derived by writing an integral of the function νT_2 (of which νW_2 is the discontinuity)around a contour C in the complex ν plane at fixed q^2:



$$\oint_C \nu T_2 d\nu = 0$$

This integral is considered at two different values of q^2, e.g. on the lines a and b:

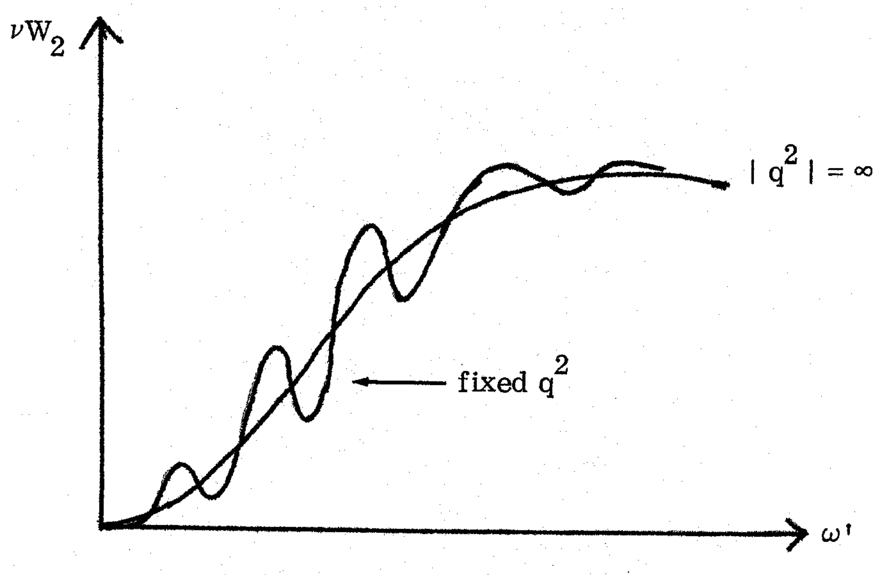


If (empirically) νW_2 scales in the shaded region and if $\mathrm{Re}\,\nu T_2$ also scales there, then the contributions from $|\nu| > \nu_1$ and $|\nu| > \nu_2$ are the same in the two cases. Therefore

$$\int_0^{\nu_1} \nu W_2(\nu, q_a^2) d\nu = \int_0^{\nu_2} \nu W_2(\nu, q_b^2) d\nu \ .$$

The first integral is essentially entirely over the scaling region if $|q_a^2|$ is large, while the second is over the resonance region.

Bloom and Gilman actually considered the integrals up to a fixed $\omega' = s/q^2$ along a and b, rather than up to a fixed ω (and found that the sum rule is satisfied to $\sim 10\%$ [17]). In the variable ω' scaling seems to begin at very low missing mass so that the range of the integrals in the sum rule is small. The approximate local equality of the integrands in this variable is therefore not very suprising — schematically:



If νW_2 is plotted against ω this "local equality" at finite and infinite q^2 is no longer observed. This is simply a reflection of the fact that the domain where νW_2 is a function of ω' only is much larger than the domain where it is a function of ω only.

Bloom and Gilman assumed that the local equality at fixed ω' and different q^2 could be taken to an extreme and used to calculate $(\nu W_2^{\text{neutron}})/(\nu W_2^{\text{proton}})$ at threshold in terms of the elastic form factors. It is quite remarkable that their result agrees with experiment.

Since the Bloom-Gilman sum rule depends only on scaling in a certain region and analyticity its success is not directly related to whether the resonances survive at large $|q^2|$ relative to some background. Plots of νW_2 against ω' [16] suggest that they do survive but detailed fits to the data are needed to establish this point.

Regge behaviour and FESR

The suggestion that νW_2 and W_1 are Regge behaved for large $\nu > N(q^2)$ at fixed q^2 is quite compatible with scaling[18,19] provided $N(q^2) \sim q^2$ and the residue functions satisfy

$$\beta_i(q^2) \sim (q^2)^{1-\alpha_i} .$$

In this case

$$F_2(\omega) = \sum_i b_i \omega^{\alpha_i - 1}$$

$$\left(\omega > \omega_R = \lim_{-q^2 \to \infty} \frac{2N(q^2)}{-q^2} \right) .$$

The leading trajectories are the $P(\alpha=0)$, P' and A2 ($\alpha = \frac{1}{2}$) and the data can indeed be fitted by[20]

$$F_2^{\gamma p}(\omega) = 0.275 + \frac{0.185}{\sqrt{\omega}}$$

(which falls to $F_2(\infty)$, just as $\sigma_\gamma(\nu)$ does).

Assuming Regge behaviour at large ω, we obtain the FESR

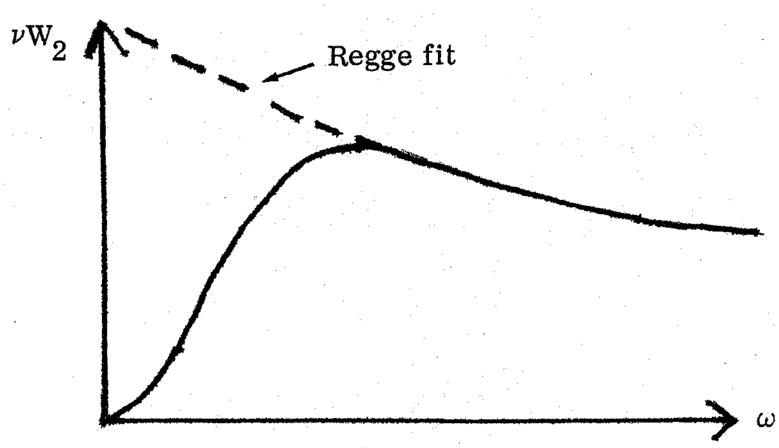
$$\int d\omega \, (F_2(\omega) - F_2^{\mathrm{Regge}}(\omega)) = - \frac{C\pi}{2}$$

where $C \neq 0$ if there is a Regge pole at $\alpha = 0$ (fixed or moving) in the virtual forward Compton amplitude T_2 of which W_2 is the imaginary part

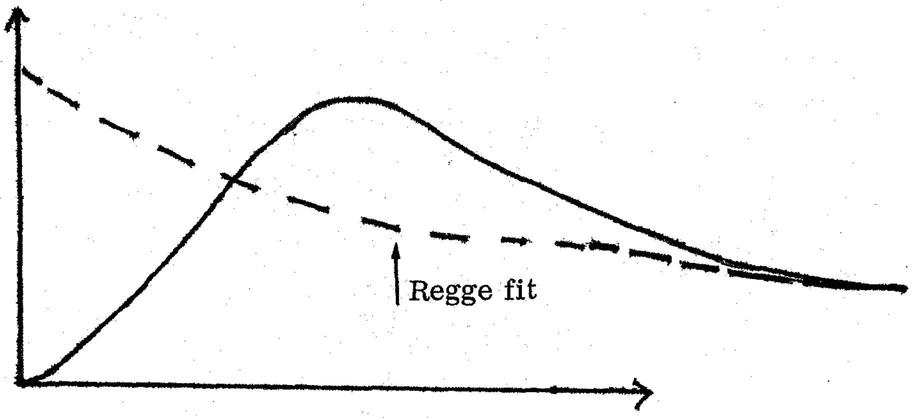
$$T_2(\nu, q^2) \xrightarrow{\nu \to \infty} \frac{\mathcal{C}(q^2)}{\nu^2} + \alpha \neq 0 \text{ Regge poles.}$$

$$C = \lim_{-q^2 \to \infty} \frac{2\,\mathcal{C}(q^2)}{-q^2} .$$

If we make a Regge fit from near the maximum of νW_2 we will clearly find $C > 0$:



If $C \leq 0$, then Regge behaviour must be approached from above



What does theory tell us about C? Cheng and Tung have argued[21] that the residues of fixed poles are polynomials. Accepting this pro tem (and assuming that any $\alpha = 0$ pole is fixed) the only possibility consistent with scaling and the kinematic constraint $T_1 + (\nu^2 T_2)/q^2 \xrightarrow[q^2 \to 0]{} 0$ is

$$T_1 \xrightarrow{\nu \to \infty} a + \alpha \neq 0 \text{ Regge poles}$$

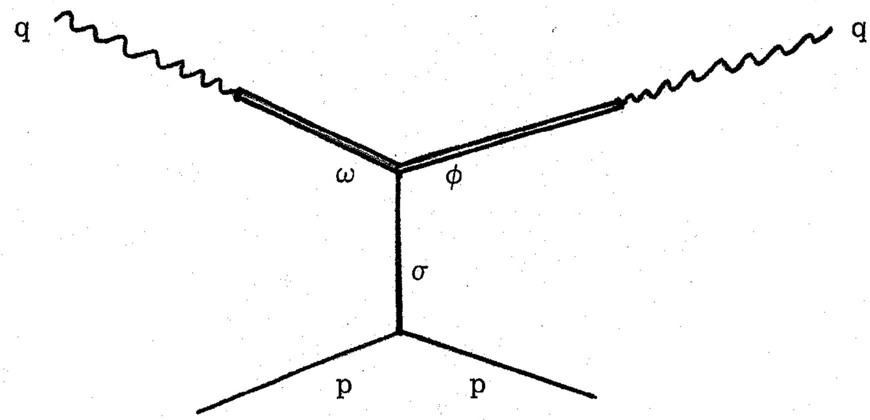
$$T_2 \xrightarrow{\nu \to \infty} \frac{-aq^2}{\nu^2} + \alpha \neq 0 \text{ Regge poles}$$

The constant a can be evaluated by using an FESR for T_1/ν at $q^2 = 0$ giving

$$1 + \frac{1}{2\pi^2 \alpha} \int d\nu \left(\sigma_\gamma(v) - \sigma_\gamma^{\text{Regge}}(v) \right) = \int d\omega \left(F_2(\omega) - F_2^{\text{Regge}}(\omega) \right) = -\frac{C\pi}{2}$$

This is the sum rule of Cornwall, Corrigan and Norton[22] and Rajaraman and Rajesakaran[23]. The left-hand side has been evaluated by Damashek and Gilman[24] and by Dominguez, Ferro Fontan and Suaya[25] and found to be $\sim +1$. A cursory examination of the data suggests that the right hand side is < 0 and therefore the sum rule fails, which would imply that the residue of the $\alpha = 0$ pole is not a polynomial.

At first sight it seems trivial to exhibit models with fixed poles with non-polynomial residues, e.g.



Unless it is somehow cancelled as $q^2 \to M_\omega^2$ and $q'^2 \to M_\phi^2$ this diagram alone is impossible, however, since it implies the existence of a fixed pole in strong interactions. This is essentially the argument of Cheng and Tung[21]: singularities which could induce non-polynomial behaviour of the residues probably give rise to fixed poles in strong interactions and/or photoproduction. The argument is not compelling if one views the spectre of fixed poles in photoproduction with equanimity.

It would obviously be interesting to examine the sum rule for the residue of the $\alpha = 0$ pole at various fixed values of q^2 if sufficiently accurate data exists. This is being investigated by F. Close and R. Suaya at SLAC.

Summary of what has been learned from e^-N

In this section I shall try to summarise something of what we have learned about inelastic electron scattering in the last few months.

1) Models in which the proton and neutron structure functions are the same at large $|q^2|$ are excluded experimentally.[19,26]

2) The DLY model is in trouble because σ_L/σ_T is small at all ω.

3) Configuration mixing ($\epsilon \neq 0$) and the presence of gluons leave the quark parton model with little predictive power which is not possessed by more general models.

4) Contrary to earlier folklore, the contribution of the resonances may survive in the scaling limit.

5) Finite energy sum rules indicate the possible presence of a $J = 0$ Regge pole (fixed or moving) with a non-polynomial residue, if Regge behaviour is assumed at large ω.

6) The problem of the proton neutron mass difference remains obscure.[27]

7) Some models of $[\dot{J}, J]$ give results in reasonable agreement with the deuteron data.[28,29] Definitive tests of these models and models of $[J, J]$ urgently require accurate $\nu/\bar{\nu}$ experiments at large energies.

8) Experiment indicates that the leading light cone singularity of products of operators is given by renormalized perturbation theory, apart from logs (the neighborhood of the light cone is the configuration space region conjugate to the deep inelastic region in momentum space in this process). In other words the logs which break scale invariance in perturbation theory do not add up to a power and totally destroy scaling by changing the leading singularity — giving "anomalous dimensions" in Wilson's language.[30] (For a review of the light cone approach see references 31 and 32.)

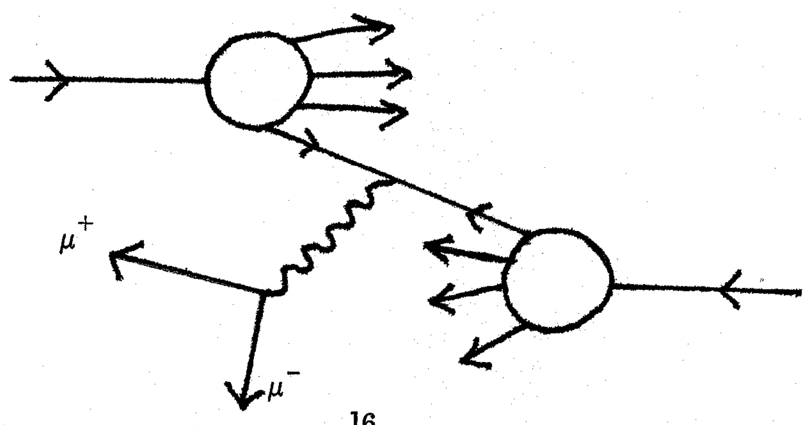
Related processes

Finally I shall list some processes which will shed light on the various models of highly inelastic electron scattering.

1) Inelastic $\nu/\bar{\nu}$ scattering.

2) Coincidence measurements in inelastic electron scattering (some preliminary results are already available[33]). It should be stressed that the scarcity of predictions for these processes reflects the unspohisticated nature of present theories (which have not yet mastered νW_2) and not that the experiments are uninteresting.[34]

3) Experiments on $pp \rightarrow \mu^+\mu^- + \ldots$ at large q^2 and different values of s. This is particularly interesting since several different models have been applied.[6,35,36] In the parton model the dominant diagrams are supposed to be those in which a parton and an antiparton annihilate (which is the only circumstance in which a time like photon can couple two on shell states):



This gives the scale invariant result[16]:

$$\frac{d\sigma}{dq^2} \sim \left(\frac{1}{q^2}\right)^2 f(s/q^2)$$

On the other hand, Altarelli, Brandt and Preparata found[35, 31]

$$\frac{d\sigma}{dq^2} \sim \frac{1}{M_1^4} F_1(s/q^2) + \frac{1}{q^2 M_2^2} F_2(s/q^2)$$

by relating the cross section to the light cone behaviour of operator products and using some Regge assumptions (their calculation seems to involve an implicit assumptions that the limits $s \to \infty$ and $-q^2 \to \infty$ are interchangeable[6] and that the operator expansion is valid in a "strong" sense[32]). They were also able to make a model of the functions F_1 and F_2 which gave a reasonable fit to experiment[37] with two parameters.

This process is particularly interesting because of the connection with $pp \to W + \ldots$. The DLY result implies substantial cross sections at the CERN ISR compared to previous calculations (if W exists with M_W not too enormous); the "light cone" result is gigantic.

4) $\gamma p \to$ "γ"p. This process was studied by Bjorken and Paschos in the framework of the parton model in certain kinematical conditions[9,38]; further work is under way[39].

5) e^+e^- colliding beam experiments. Although in general there is no necessary connection with e^-N scattering, the processes are related in some models. The large annihilation cross sections reported at Frascati, while hardly in the asymptotic region, certainly add credibility to the notion of point like constituents inside the nucleon. Further experimental results are eagerly awaited and we can clearly anticipate a pandemic of theoretical papers on this subject.

Acknowledgement

I am grateful to F. Close, F. J. Gilman and R. Jaffe for comments on the manuscript.

Note added in proof: The theorem on p. 5 ($|\Delta| \leq 1/3$) is incorrect. This was pointed out to me by J. S. Bell who provided a counter example. It is true, however, that if $\Delta \neq 1/3$ the models previously considered are excluded.

Footnotes and References

1. E. D. Bloom, G. Buschorn, R. L. Cottrell, D. H. Coward, H. DeStaebler, J. Drees, C. L. Jordan, G. Miller, L. Mo, H. Piel, R. E. Taylor, M. Breidenbach, W. R. Ditzler, J. I. Friedman, G. C. Hartmann, H. W. Kendall and J. S. Poucher, SLAC-PUB-796, Stanford Linear Accelerator Center (1970) (Report presented at the XV International Conference on High Energy Physics, Kiev, USSR, 1970).

2. Complete references to well known models and results will not be given here. Reference 3 and 4 are fairly recent reviews which contain many references.

3. J. D. Bjorken, Lectures at the Scottish Universities Summer School (to be published).

4. C. H. Llewellyn Smith, "An Introduction to Highly Inelastic Lepton Scattering and Related Processes", CERN TH 1188.

5. G. West, private communication.

6. S. D. Drell and Tung-Mow Yan, SLAC-PUB-808 (to be published in Annals of Physics) and references therein.

7. The spin $\frac{1}{2}$ parton model gives $\sigma_S/\sigma_T = 0$ which is "quite unlikely but not impossible" experimentally. Note, however, that this prediction is for infinite ν and $|q^2|$ ($\nu/|q^2|$ fixed) and is therefore quite compatible with the acceptable fit $R = -q^2/\nu^2$.

8. C. H. Llewellyn Smith, SLAC-PUB-817 and addendum to this SLAC-PUB.

9. J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975 (1969).

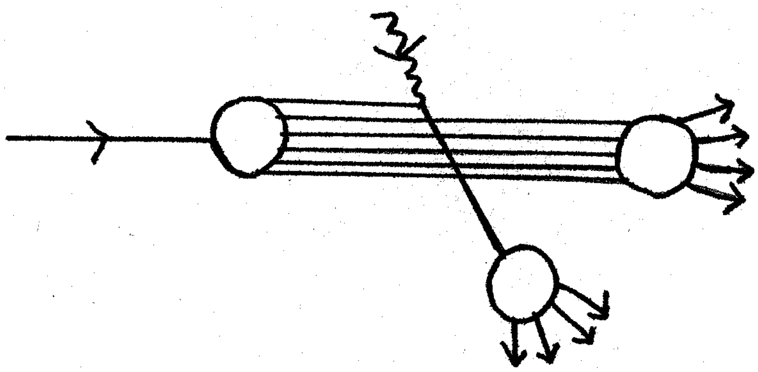
10. For a summary see refs. 8 and 4. See also C. H. Llewellyn Smith, Nucl. Phys. B17, 277 (1970).

11. This result is in the addendum to ref. 8. The lower bound depends on assuming $F_2^{\gamma p} \geq F_2^{\gamma n}$ at unexplored ω. The upper bound involves the assumptions that $< x >_N = 1/N$ and that the nucleon belongs to an SU(3) octet. We have taken the Cabibbo angle to be zero here.

12. For an extreme case see P. Landshoff (CERN TH 1180) who suggests that $\sigma^{\nu p}/\sigma^{\nu n} \simeq 0$. Note that in the quark parton model we expect $\sigma^{\nu n} - \sigma^{\nu p}$ to be near the lower bound (which is reached when $\epsilon = 0$) since ϵ is small.

13. D. Perkins, private communication. This results may be strongly influenced by the production of $I = 3/2$ resonances for which $\sigma^{\nu n}/\sigma^{\nu p} = 1/3$.

14. The work of DLY[6] suggests a cogent objection to the quark parton model. They open up the "black box" of final state interactions and find that the leading terms have the structure:



The two groups of particles are loath to interact because of the cut off in transverse momentum (which ensures that the electron scatters incoherently from the partons). Graphs in which the two groups of particles interact vanish like log s/s in the scaling limit in each order of perturbation theory. This seems to imply that if partons are quarks, then quarks are being produced at SLAC (or else that the model is irrelevant). An escape route is opened by the work of Chang and Yan (SLAC-PUB-793) who studied a model

in which the logs build up to a power when summed to all orders (the non-vanishing terms in which the two groups interact are due to "wee" partons which have little sense of direction and, in their confusion, may affiliate themselves to either group of particles).

15. P. V. Landshoff, J. C. Polkinghorne, Nucl. Phys. B19, 432 (1970).

16. E. Bloom and F. J. Gilman, Phys. Rev. Letters 25, 1140 (1970).

17. E. Bloom and F. J. Gilman, private communication.

18. H. D. I. Abarbanel, M. L. Goldberger and S. B. Treiman, Phys. Rev. Letters 22, 500 (1969).

19. H. Harari, Phys. Rev. Letters 22, 1078 (1969).

20. H. Pagels, Rockefeller University report NYO-4204-6.

21. T. P. Cheng and Wu-Ki Tung, Phys. Rev. Letters 24, 851 (1970).

22. J. M. Cornwall, D. Corrigan and R. E. Norton, Phys. Rev. Letters 24, 1141, (1970) and UCLA preprint "Scaling, Fixed Poles and Electroproduction Sum Rules".

23. R. Rajaraman and G. Rajesakaran, Delhi University Preprint TIRF/TH/70-31.

24. M. Damashek and F. J. Gilman, Phys. Rev. D1, 1319 (1970).

25. C. A. Dominguez, C. Ferro Fontan and R. Suaya, Phys. Letters 31B, 365 (1970).

26. S. Ciccariello, R. Gatto and G. Sartori, Phys. Letters 30B, 546 (1969). G. Mack, Phys. Rev. Letters 25, 400 (1970). These authors assume that the only independent local operators with dimensions ≤ 4 are j_μ and $\theta_{\mu\nu}$. (The present author knows of no Lagrangian model which could represent the real world in which this is true.)

27. H. Pagels (Rockefeller preprint "Electromagnetic Corrections to Nucleon Transitions of V and A Currents") has recently pointed out that if

$$\frac{\sigma_L^p - \sigma_L^n}{\sigma_T^p - \sigma_T^n} << 1$$

then the Cottingham formula for $M_n - M_p$ requires either that $\nu W_2^n > \nu W_2^p$ in the unexplored region or that there is an $\alpha = 0$ pole. (If the $\alpha = 0$ pole has a polynomial residue, we find that it would not contribute to $M_n - M_p$; i.e. the second alternative is that there is an $\alpha = 0$ pole with non-polynomial residue).

28. R. A. Brandt and G. Preparata, Rockefeller preprint "Lepton Hadron Scattering, Gluon Model and Reggeized Symmetry Breaking".

29. J. M. Cornwall, UCLA preprint "Current Commutators and the 2^+ Nonet in Electroproduction and Neutrino Production".

30. K. Wilson, SLAC-PUB-737 and references therein.

31. R. Brandt, Erice lectures (to be published) CERN TH 1218.

32. Y. Frishman, Weizmann preprint "Operator Products at Almost Light Like Distances" (to be published in Annals of Physics).

33. K. Berkleman et al., Cornell preprint CLNS-21 — presented at the Kiev Conference.

34. Some predictions have been made for these processes. See ref. 3 and S. D. Drell and Tung-Mow Yan, Phys. Rev. Letters 26, 181 (1970).

35. G. Altarelli, R. A. Brandt and G. Preparata, Rockefeller preprint "Massive μ Pair Production".

36. S. M. Berman, D. J. Levy and T. L. Neff, Phys. Rev. Letters 23, 1363 (1969) (essentially the same calculation was done by O. Nachtmann, unpublished). V. A. Matveev et al., JINR preprint E2.4768.

37. J. H. Christenson, G. S. Hicks, L. M. Lederman, P. J. Limon, B. G. Pope and E. Zavattini, paper submitted to the Kiev Conference.

38. J. D. Bjorken and E. A. Paschos, Phys. Rev. D1, 1450 (1970).

39. S. D. Drell and R. Jaffe, private communication.