

MODELS FOR HADRONIC REACTIONS: DUALITY, ABSORPTION AND QUARKS\*

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ABSTRACT

Elastic and inelastic hadronic two-body reactions are briefly discussed from the points of view of resonance models, exchange models, geometrical-optical models and quark models. All of these approaches are assumed to provide different descriptions of the same hadronic amplitudes. A qualitative dual model involving all of these approaches is discussed. The material is presented in a "picture book" form, based on lectures delivered in the Ettore Majorana school, Erice, July 1970.

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## Introduction

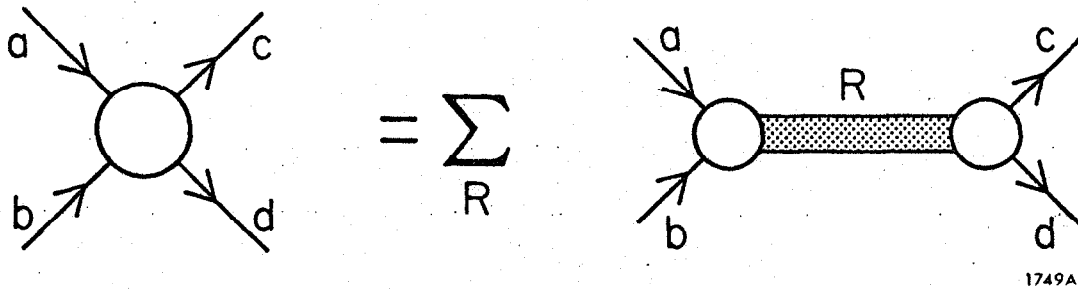
One of the most interesting recent developments in our understanding of the strong interactions is probably the convergence of several different models into a successful qualitative picture of hadronic reactions. The introduction of this picture was triggered by the discovery of duality, but it has recently been extended to include ideas borrowed from the quark model as well as from various optical models for hadron processes.

Several recent reviews<sup>1</sup> have covered the general features of duality. We shall not repeat these here. What we propose to do in these notes is to outline the overall qualitative dual picture which supposedly describes hadronic processes with two-body final states. We then briefly mention some of the tests which are relevant to the various components of this picture and try to detect some of its successes, limitations and open problems.

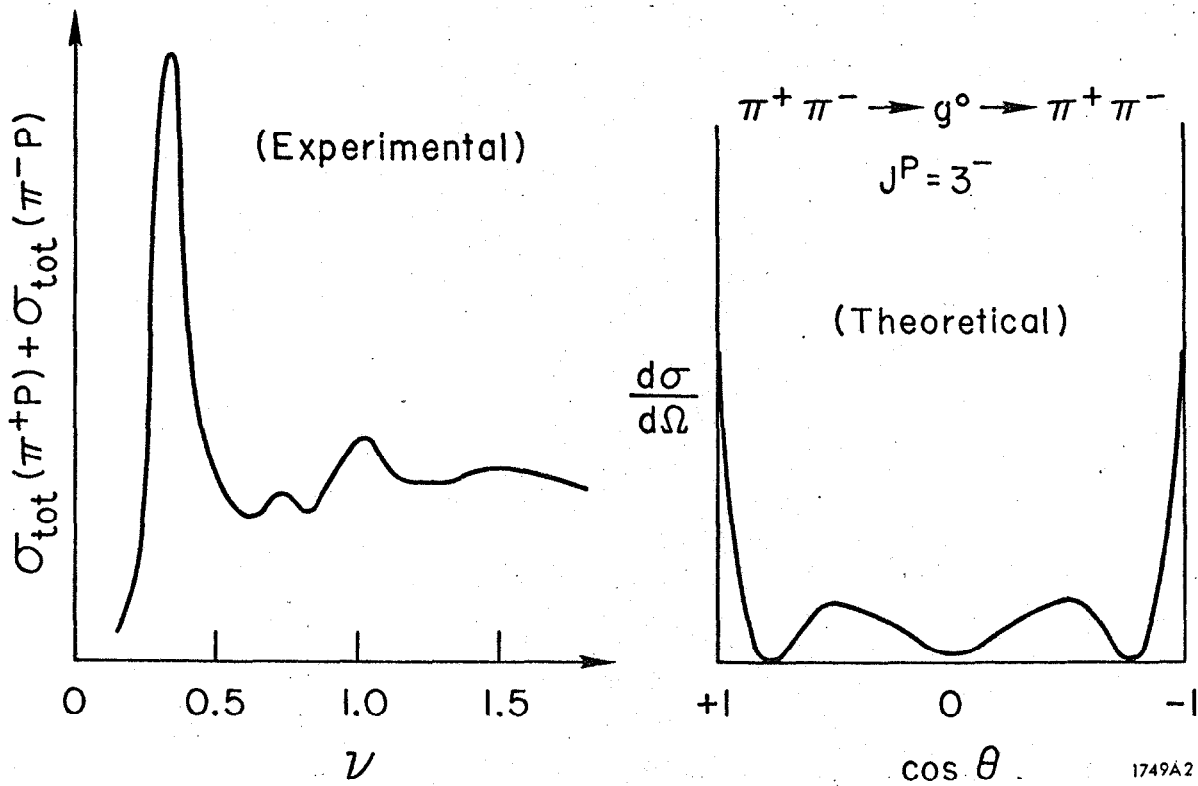
In order to avoid duplicating the many reviews<sup>1</sup> on the subject we use the "picture book" approach and present the material as a collection of relevant figures and figure captions.

In some of the figures (numbers 2, 6, 22, 23 and 24) we have used "smooth lines" through data points. These figures are supposed to indicate trends of the data. They should not be used as sources of data.

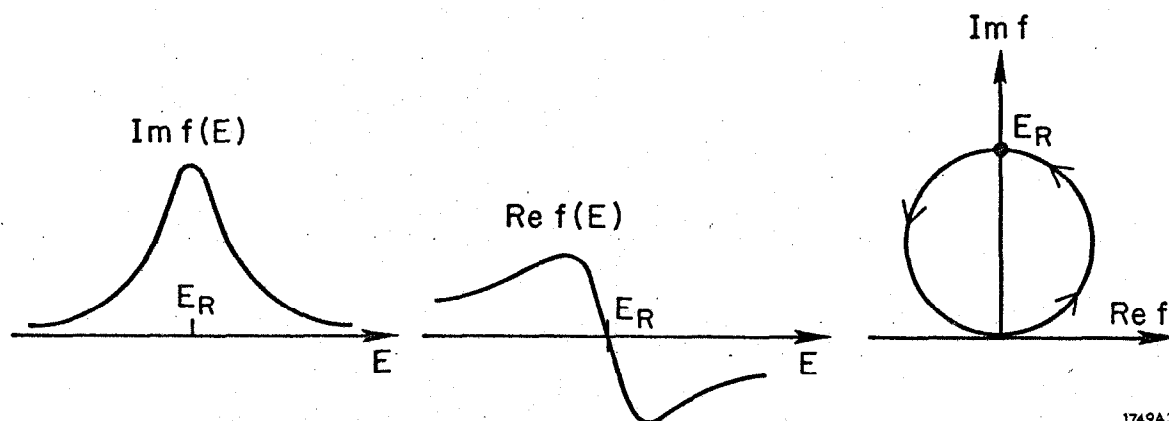
### RESONANCE MODELS



1--We assume that the hadronic amplitude for the process  $a+b \rightarrow c+d$  may be described by a sum of contributions of s-channel resonances.

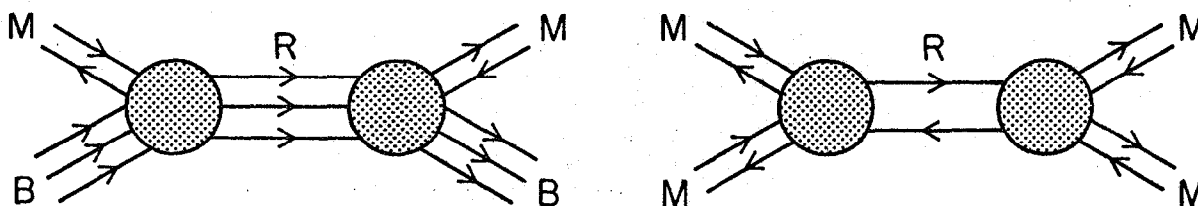


2--If only a few resonances are important — a typical energy dependence of the cross section shows clear resonance bumps (as in  $\sigma_{tot}(\pi p)$  above); a typical angular dependence shows forward and backward peaks (as in the fictitious  $\pi\pi$  distribution above).



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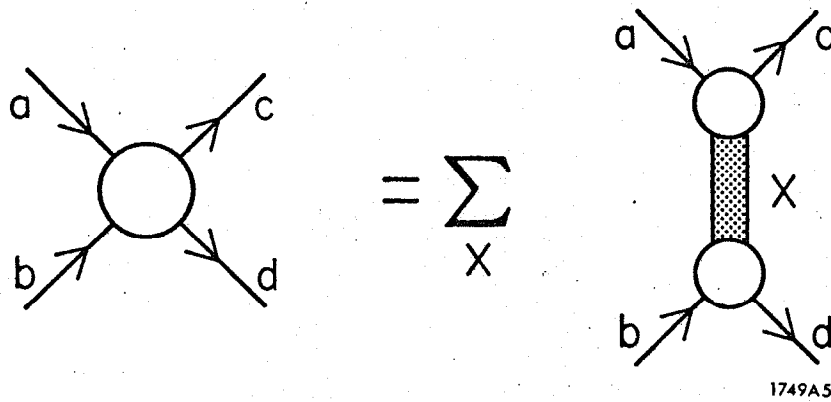
3--The resonance contribution to the imaginary part is concentrated around the resonant energy  $E_R$ . Its contribution to the real part is spread over a much wider energy range. At  $E_R$ , the real part vanishes. The integrated contribution of the real part over a symmetric region around  $E_R$  is usually very small. Hence -- resonance dominance is meaningful only for the imaginary part in a local or semilocal sense. The resonant partial wave amplitude produces a circle in the Argand diagram. The existence of such a circle is the usual criterion for establishing a new resonance.



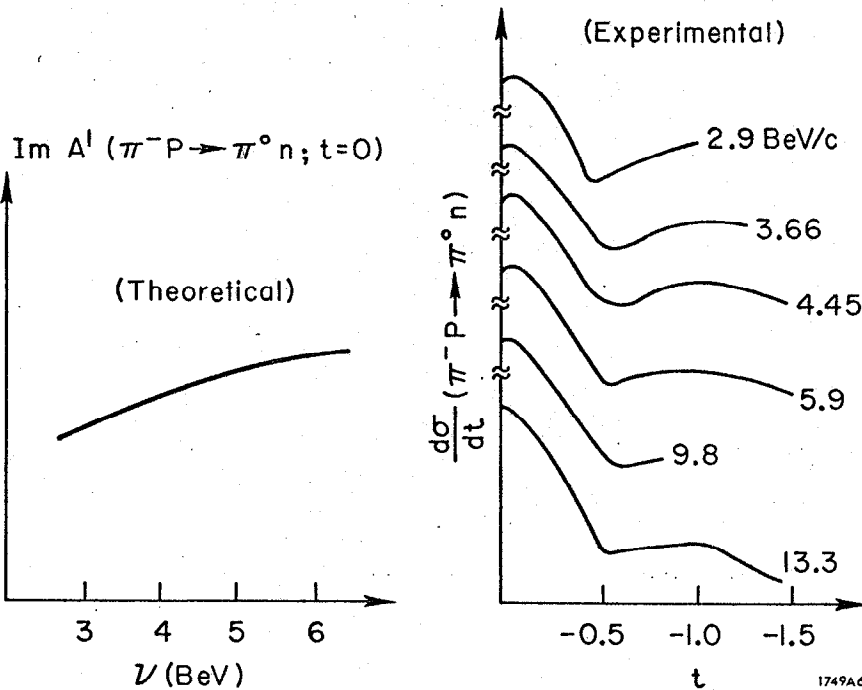
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4--If all s-channel resonances are nonexotic, the quark model description of the resonance-dominance model must follow the diagrams above (for meson-baryon and meson-meson scattering). The details of the shaded "vertices" cannot be determined from resonance dominance alone.

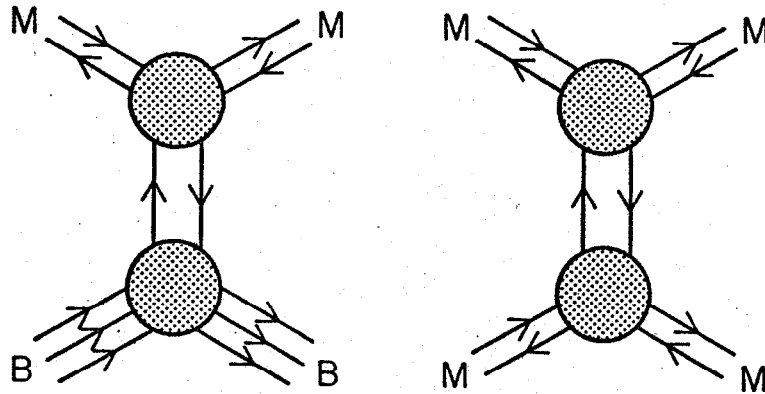
## EXCHANGE MODELS



5--We assume that the hadronic amplitude for the process  $a+b \rightarrow c+d$  may be described by a sum of contributions of  $t$ -channel exchanges (poles, cuts).



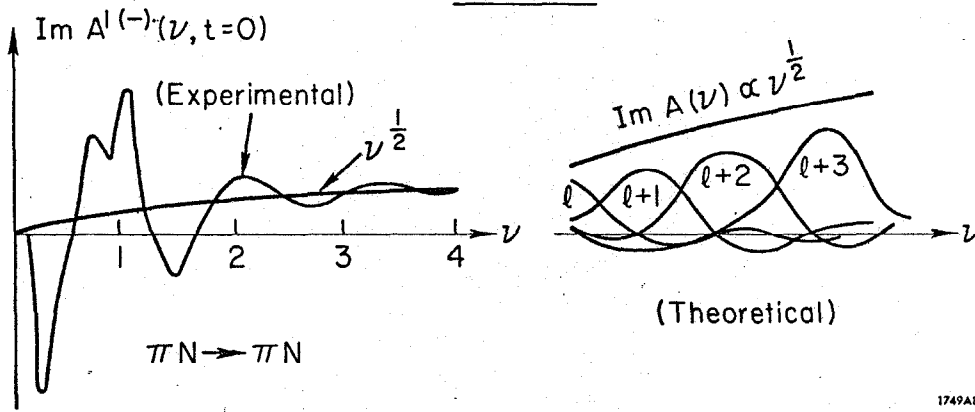
6--The energy dependence of the amplitude is "smooth" (sum of powers  $\sum \beta_i \nu^{\alpha_i}$  or logarithms). The angular distribution is strongly peaked at small  $t$ -values, possibly with some dips (at  $t \sim 0$ ,  $t \sim -0.6$ , etc.). The dips (if any) are supposed to be at fixed  $t$ -values, independent of energy, as shown above in the case of  $\pi^- p \rightarrow \pi^0 n$ .



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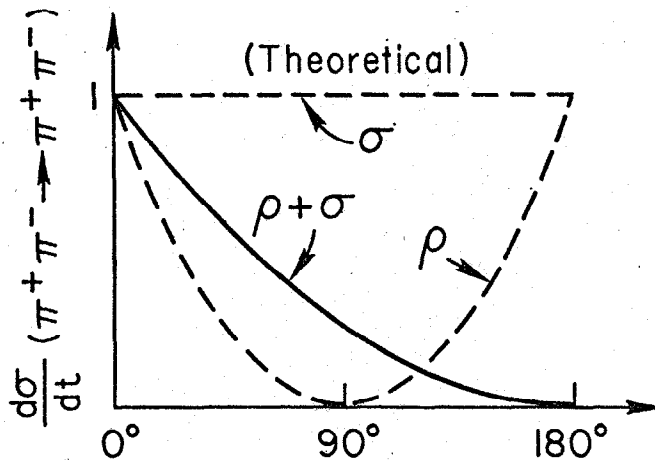
7--If all t-channel exchanges are nonexotic, the quark model description of the exchange model must follow the diagrams above (for meson-baryon and meson-meson scattering). The details of the shaded "vertices" cannot be determined from the exchange model alone.

DUALITY



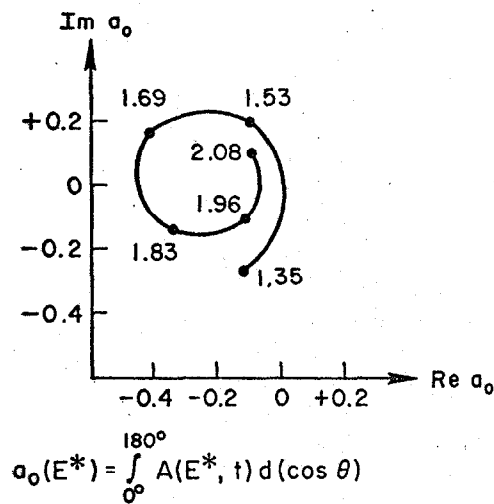
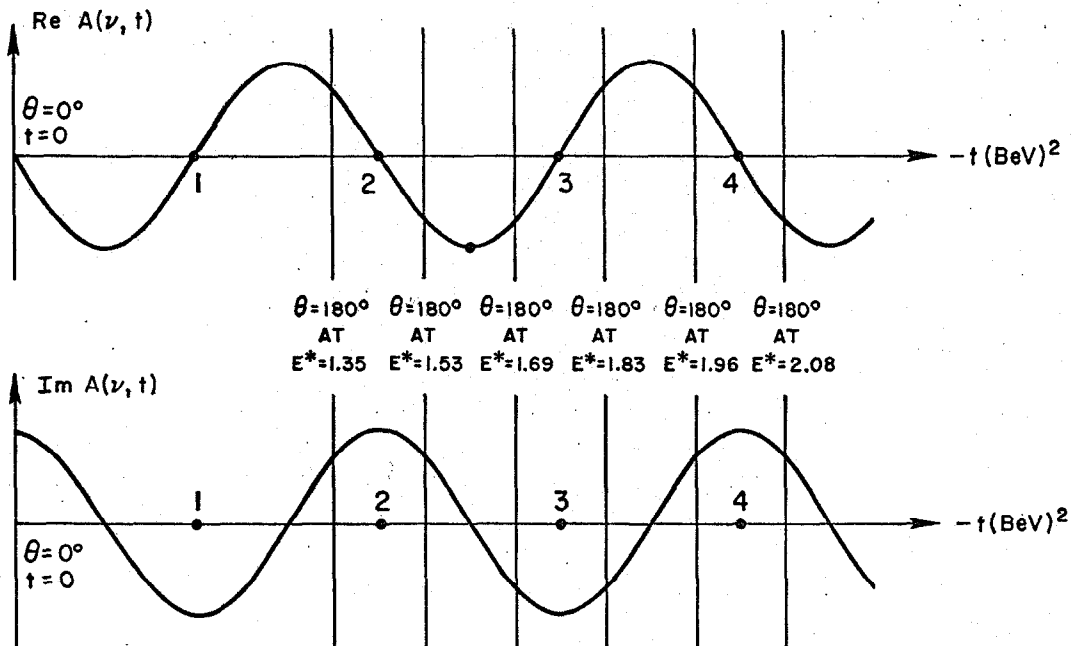
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8--A resonance dominance model and an exchange model may simultaneously describe the imaginary part of the same amplitude in the same kinematic region. If only a few resonances are important, they reproduce the exchange model only when averaged over an energy range<sup>2</sup> (as in the  $\pi N \rightarrow \pi N$  amplitude above). A sum of sufficiently large number of resonances can actually reproduce the smooth energy dependence of an exchange model. Every partial wave would exhibit a bump, but their sum will be smooth (as shown schematically above).



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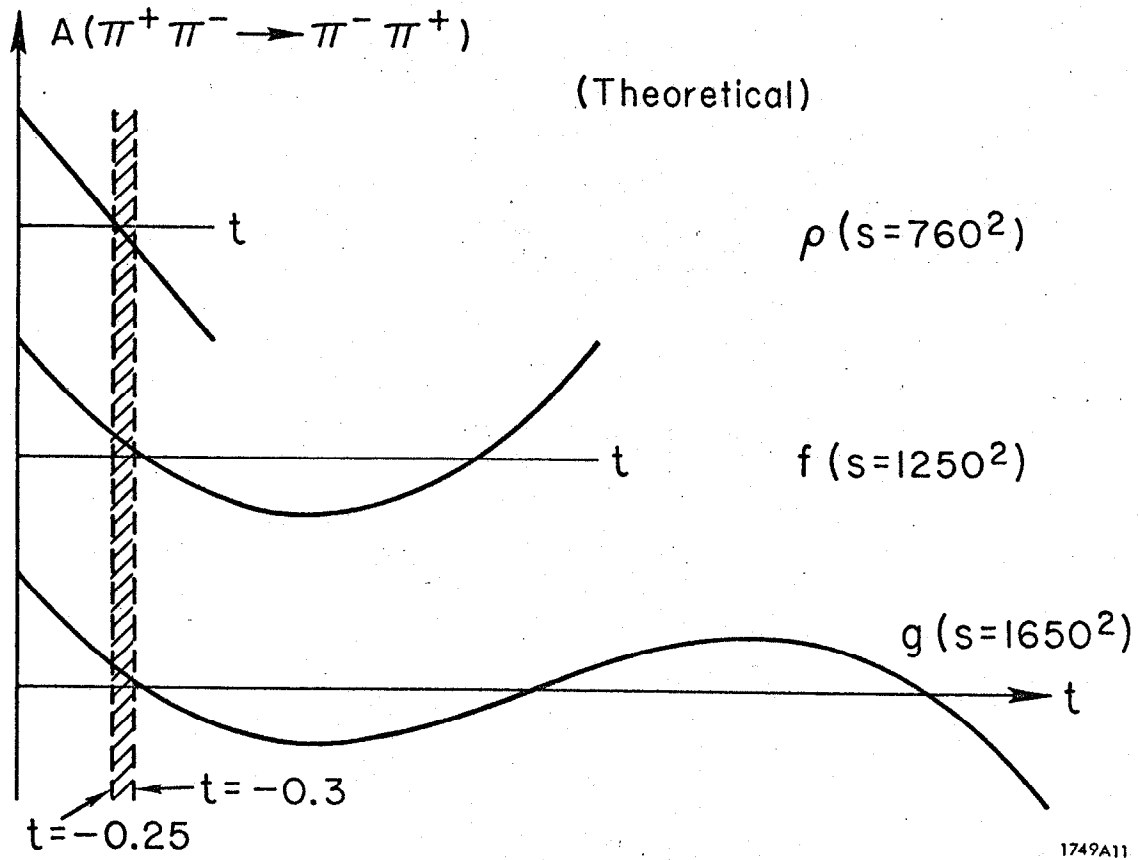
9--A sum of two or more s-channel resonances can easily create a forward peak which is not accompanied by a backward peak, reproducing the typical behavior of a t-channel exchange. The trivial example above shows this for  $\pi^+ \pi^-$  scattering at  $\sqrt{s} = 760$  MeV, assuming  $\rho$ -only,  $\sigma$ -only and equal  $\rho$ - $\sigma$  terms (all normalized to 1 at  $t=0$ ).



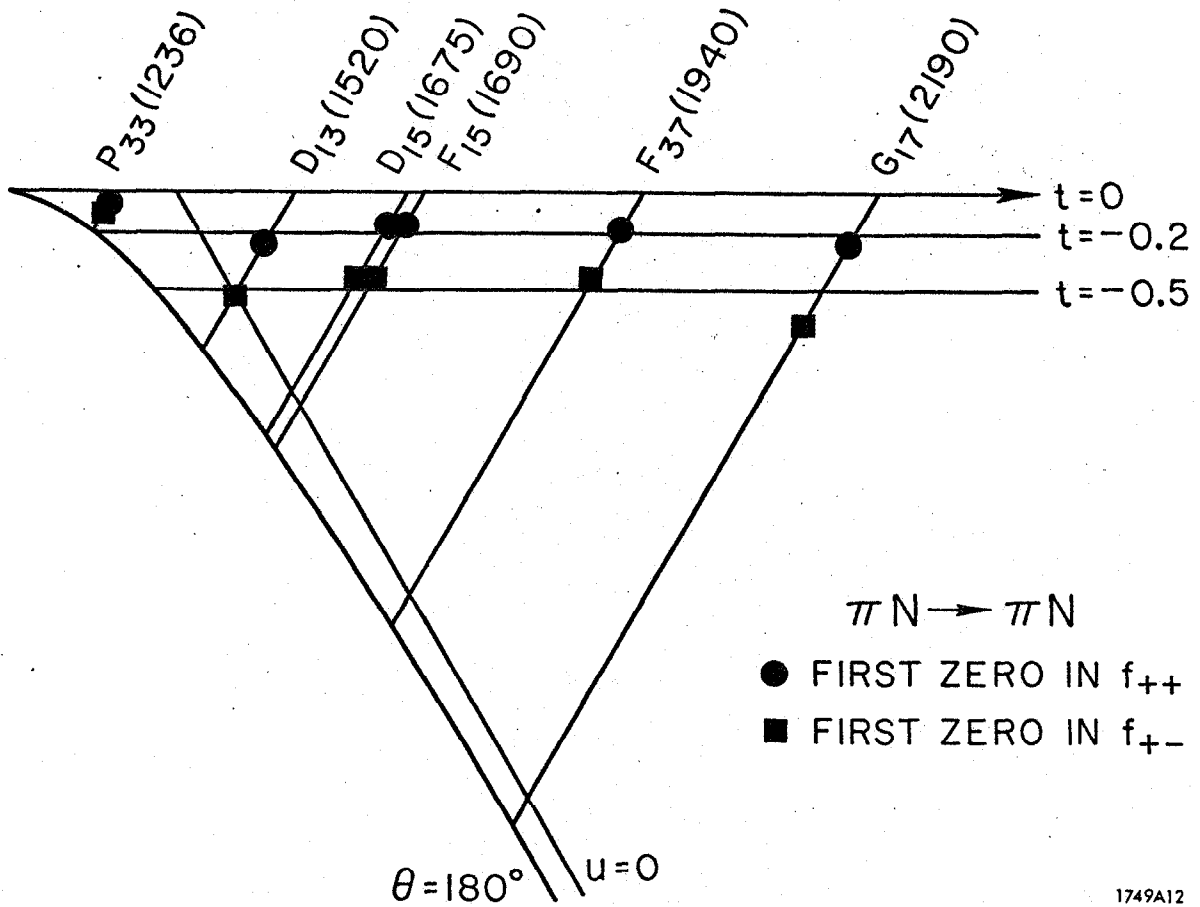
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10--An amplitude of the form  $\left\{ \left( 1 \pm e^{i\pi\alpha(t)} \right) / \left( \sin \pi\alpha(t) \Gamma[\alpha(t)] \right) \right\} \nu^{\alpha(t)}$  is a typical Regge pole exchange amplitude. It has a "smooth"  $\nu$ -dependence at fixed  $t$ . Nevertheless, it can produce resonance-like circles in the Argand plot for its partial waves. The figure shows how this happens for a fictitious  $\pi$ - $\pi$  amplitude  $A(\nu, t) = e^{i\pi\alpha(t)}$  where  $\alpha(t) = 0.5 + t$ .  $\text{Re } A(\nu, t)$  and  $\text{Im } A(\nu, t)$  are plotted versus  $t$ . At different c.m. energies  $E^*$ , different  $t$ -values correspond to  $\theta = 180^\circ$ . The s-wave Argand plot (shown) and all other partial waves exhibit circles. A full Reggeistic amplitude would usually show the same features.<sup>3</sup>



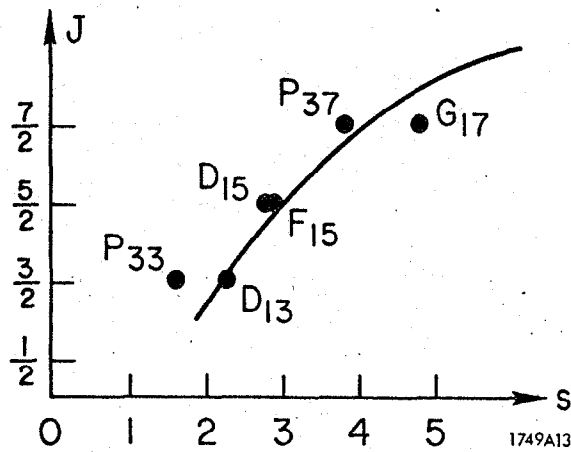


- 11--How can different s-channel resonances at different energies produce dips in angular distributions at the same t-values (as demanded by exchange models)? As an example consider  $\pi^+ \pi^-$  elastic scattering and assume that  $\rho(760)$ ,  $f(1250)$  and  $g(1650)$  dominate the amplitude in their respective energies. The figure shows that in all three energies the first zero is around  $|t| \sim 0.25 - 0.3 \text{ BeV}^2$ . As the energy grows, the t-range covered by the angular distribution becomes larger. At the same time the spin of the dominant resonance becomes larger and the first zero occurs at a smaller value of  $\theta$ . The two effects cancel and produce zeroes at a fixed t-value. This cancellation can occur only if the masses and spins of the resonances are correlated in a specific way.



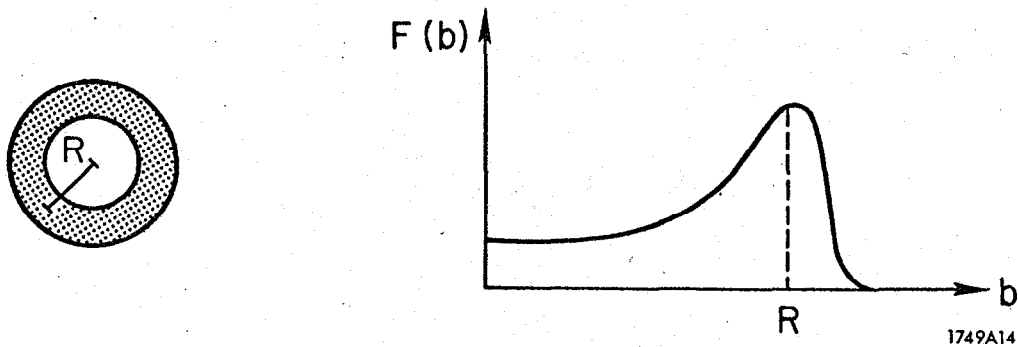
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12--Another striking example of the same phenomenon<sup>2</sup> is observed in  $\pi N$  scattering. The first zeroes in the contributions of the prominent resonances to the s-channel helicity amplitudes  $f_{++}$  and  $f_{+-}$  are approximately at fixed t-values.

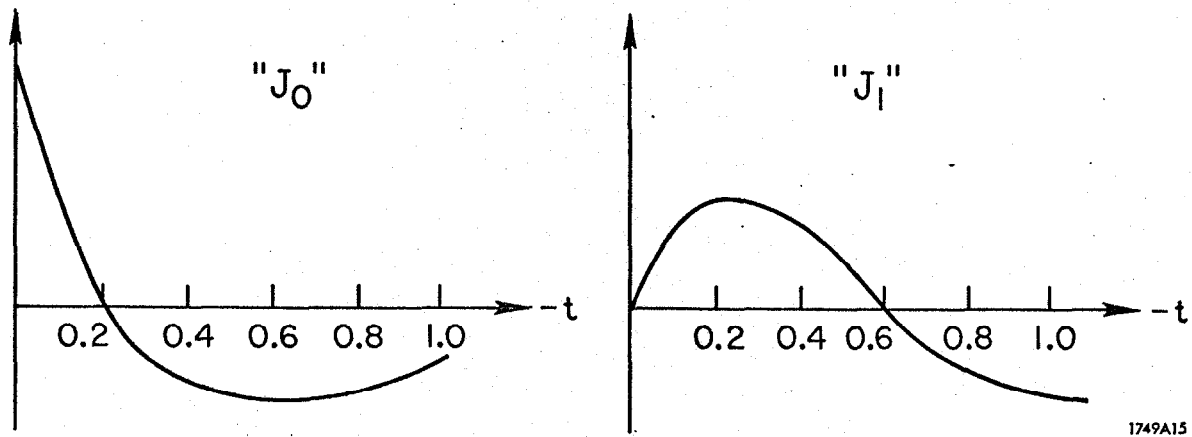


13--The spin-energy relation which all prominent resonances should obey in order to produce dips at the same t-value is, asymptotically  $J \propto \sqrt{s}$ . This does not necessarily imply that the Regge trajectories have such a form.

GEOMETRICAL-OPTICAL MODELS

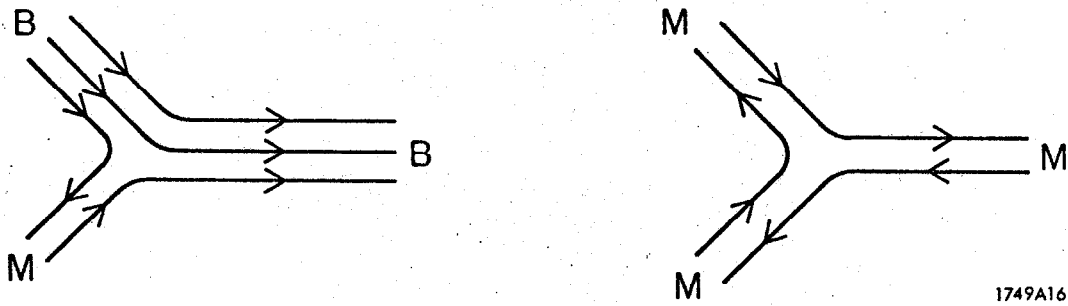


14--Most geometrical and optical models predict that every single inelastic two-body reaction is dominated by the most peripheral partial waves and impact parameters within the range of interaction.  $f_{\Delta\lambda}^S(t) \sim \int F(b) J_{\Delta\lambda}(b\sqrt{-t}) b db$  is the impact parameter representation of an s-channel helicity amplitude involving helicity change  $\Delta\lambda$ . A typical  $F(b)$  for an inelastic process is presumably heavily dominated by the region  $b \sim R$  or  $l \sim kR$  ( $k=c.m.$  momentum). Note that  $l \sim kR$  gives  $J \propto \sqrt{s}$ , the same relation obtained in the previous figure. The parabola there (Fig. 13) is given by  $J+1/2 = kR$  for  $R=1$  fermi.

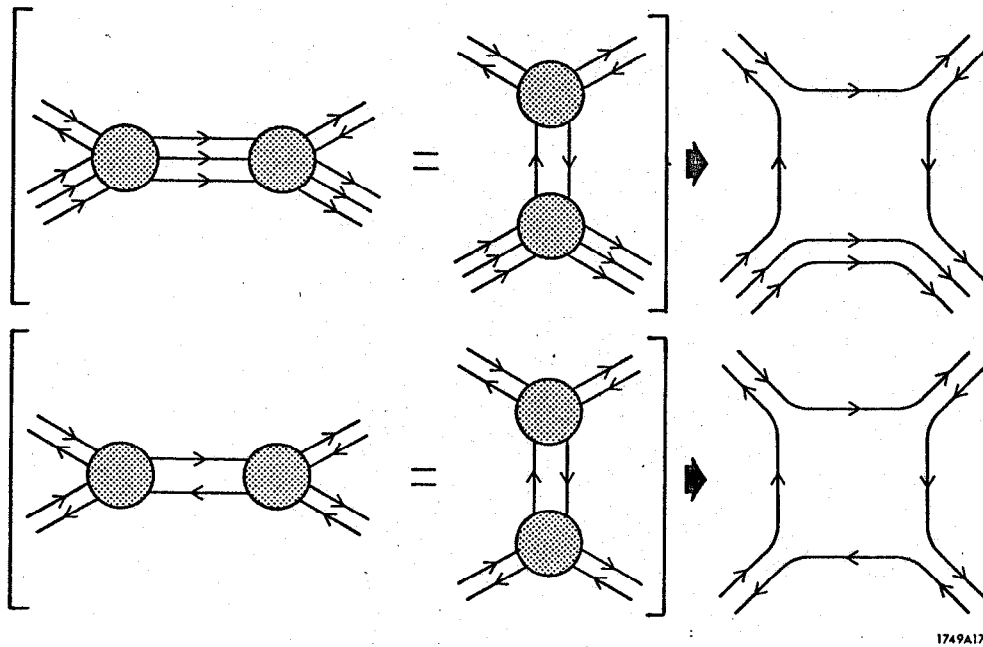


15--The  $t$ -dependence of  $f_{\Delta\lambda}^S$  is qualitatively given by  $J_{\Delta\lambda}(R\sqrt{-t})$  with  $R \sim 1$  fermi. For  $\Delta\lambda=0, 1$  we have functions of the form '"J\_0"' and '"J\_1"' whose dips, bumps and zeroes are the same as those of  $J_0$  and  $J_1$ .

### QUARK MODELS

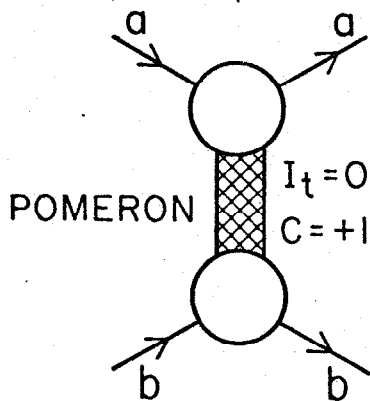


16--In resonance models as well as in exchange models we have BBM and MMM vertices, where B and M are, respectively, nonexotic baryonic and mesonic states. The quark model description of these vertices is given by the diagrams above, if we assume that the two ends of a single quark line cannot belong to a single hadron.

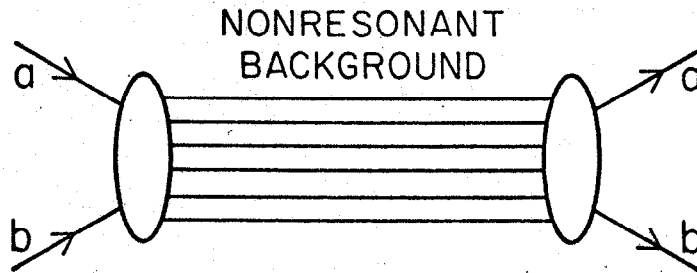


17--The quark descriptions of the resonance dominance model (Fig. 4) and of the exchange model (Fig. 7) become identical if and only if we replace all shaded vertices by the quark diagrams of the BBM and MMM vertices (Fig. 16). This gives us the duality diagrams<sup>5</sup> which describe an amplitude which is simultaneously nonexotic in the s-channel and t-channel.

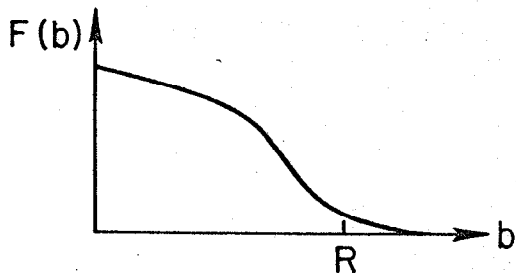
ELASTIC PROCESSES



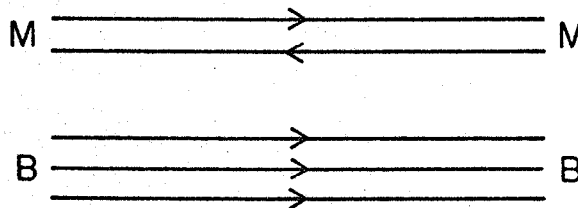
t - CHANNEL PICTURE



s - CHANNEL PICTURE



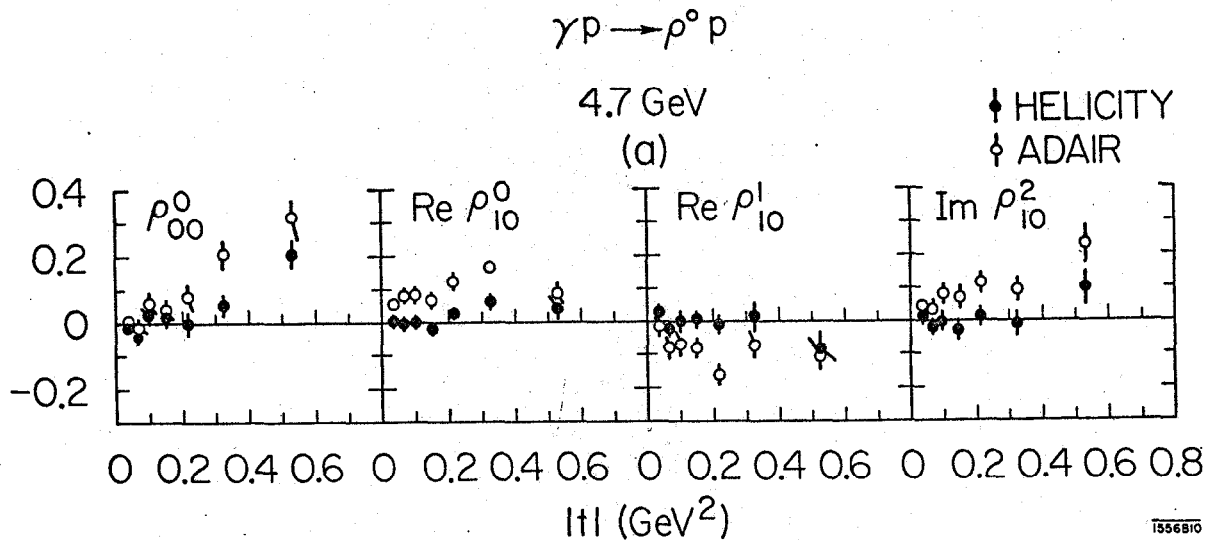
"OPTICAL" DESCRIPTION



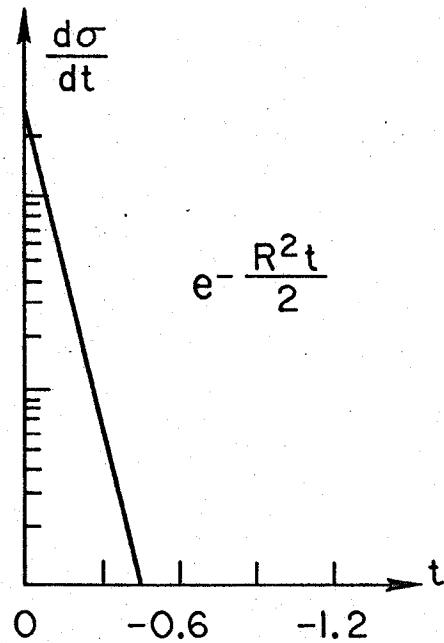
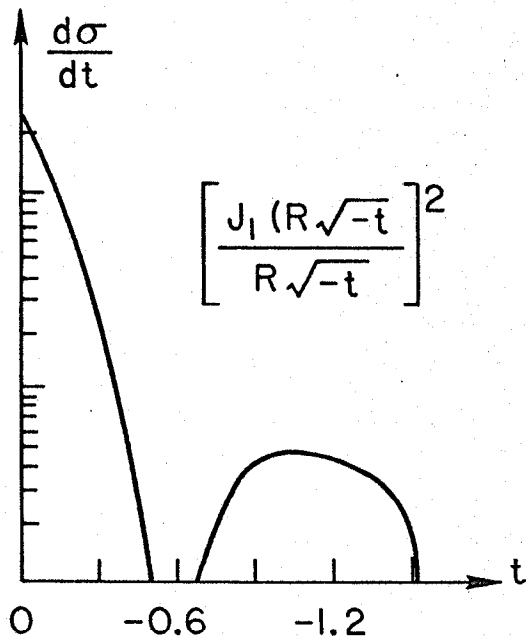
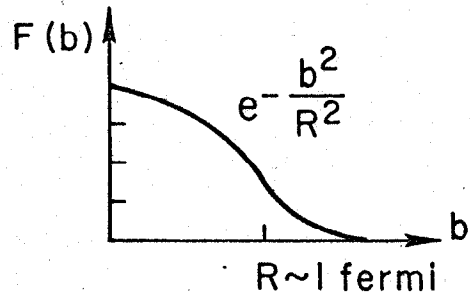
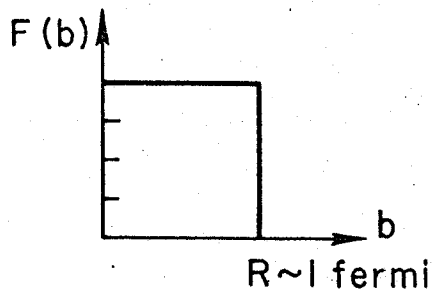
QUARK DESCRIPTION

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18--Elastic amplitudes are dominated at high energies by Pomeron exchange. This term is not dual to s-channel resonances since it contributes equally to exotic and nonexotic elastic processes such as  $K^+p$  and  $K^-p$  scattering.<sup>6</sup> We assume that the Pomeron is dual to the nonresonant background in the s-channel.<sup>6,7</sup> The optical description of the Pomeron term presumably involves significant contributions from all impact parameters  $b \leq R$ . The quark model description does not allow the exchange of any quantum numbers. It should therefore involve only the elastic scattering of  $qq$  and  $q\bar{q}$  without any s-channel annihilations.



19--Several phenomenological considerations indicate that the Pomeron exchange term may conserve the s-channel helicity.<sup>8</sup> These include the helicity frame density matrix elements for  $\gamma p \rightarrow \rho^0 p$  (shown above<sup>9</sup>) and the analysis of  $\pi N$  elastic scattering at high energies. We shall therefore assume that the Pomeron term contribute only to  $\Delta\lambda=0$  amplitudes and has the form  $\int_0^\infty b db F(b) J_0(R\sqrt{-t})$  with the  $F(b)$  of Fig. 18.



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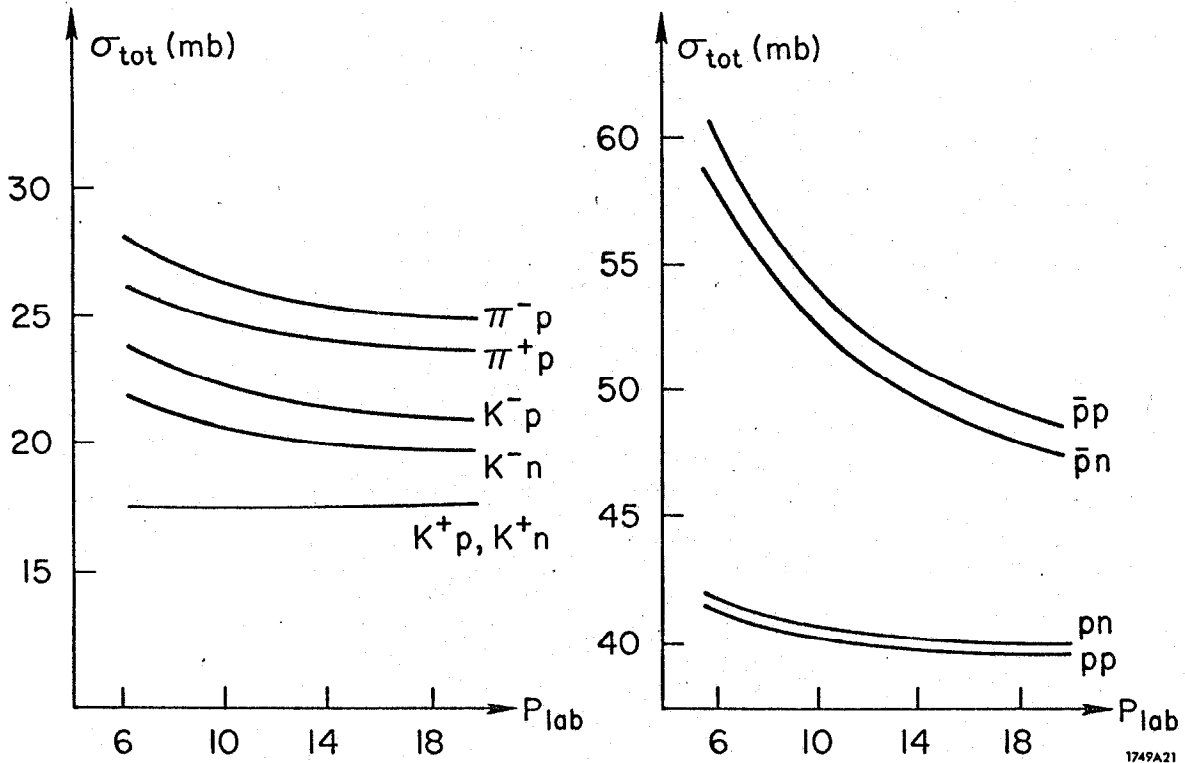
20--Does the Pomeron contribution to a differential cross section exhibit dips? The answer depends on the exact form of  $F(b)$  for the Pomeron contribution. A Gaussian  $F(b)$  gives a pure exponential  $t$ -dependence for  $d\sigma/dt$  and, hence, no dips. A constant  $F(b)$  for  $b \leq R$  would give a dip somewhere around  $t = -0.6$  for  $R \sim 1$  fermi. Only experiment (or additional theoretical assumptions) can distinguish between these possibilities.

Model	Component I	Component II
Resonance Models (s-channel picture)	Sum of nonexotic resonances in the s-channel.	Nonresonant background in the s-channel
Exchange Models (t-channel picture)	Ordinary nonexotic t-channel exchanges, including single particle exchange and particle-Pomeron cuts.	Pomeron exchange (no quantum numbers exchanged)
Geometrical-Optical Models	Dominant contributions from peripheral impact parameters $b \sim R$ . Qualitative features for s-channel helicity flip $\Delta\lambda$ , given by $J_{\Delta\lambda}(R\sqrt{-t})$ .	Substantial contributions from all impact parameters $b \lesssim R$ . s-channel helicity is conserved ( $\Delta\lambda = 0$ ).
Quark Models	Annihilation of a $q\bar{q}$ pair and the creation of another $q\bar{q}$ pair (Duality Diagrams).	Sum of $qq$ and $q\bar{q}$ elastic scattering contributions.

21--We can now formulate the two-component model for elastic and inelastic hadronic amplitudes. The model states that the imaginary part of any hadronic two-body amplitude can be described as the sum of two components. Every one of these components can be described in several ways, within the framework of the different approaches which we have discussed so far. These descriptions are summarized in the table.

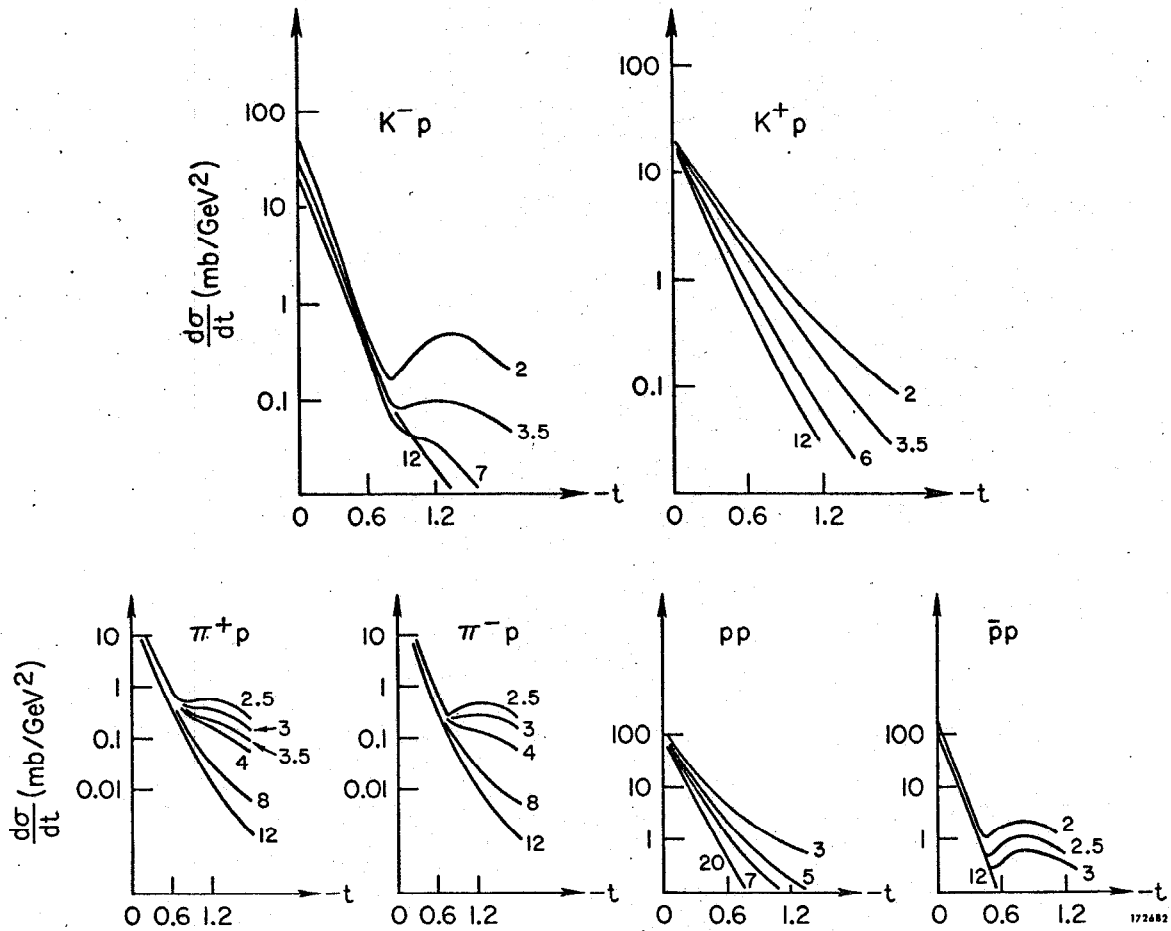


EXPERIMENTAL TESTS: TOTAL CROSS SECTIONS

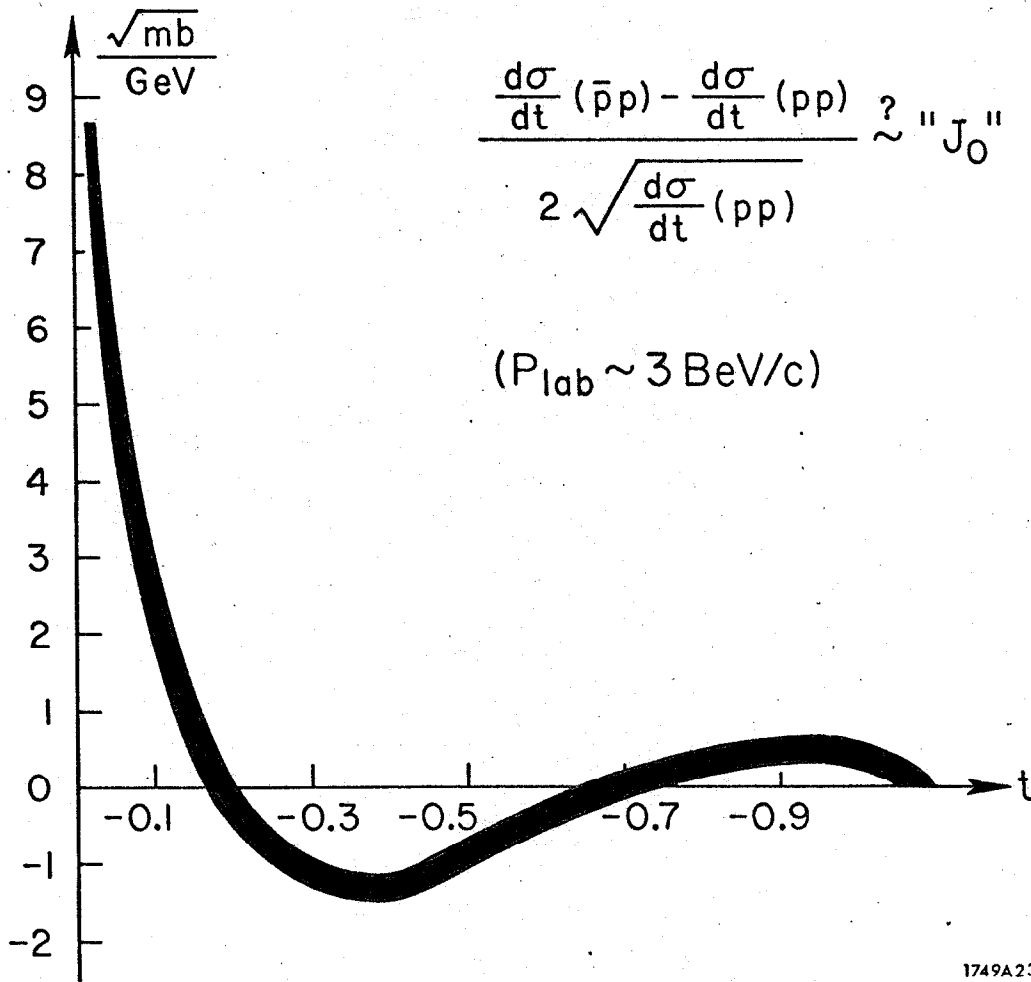


22--If the imaginary part of the forward elastic amplitude can be described as a sum of nonexotic s-channel resonances and a Pomeron exchange term we expect<sup>6</sup>: (i) Exotic total cross sections should be constant in energy ( $K^+ p, K^+ n, \bar{p} p, p n$ ). (ii) Nonexotic total cross sections ( $K^- p, K^- n, \pi^+ p, \pi^- p, \bar{p} p, \bar{p} n$ ) should decrease towards their high energy limits (since the resonance contribution to an elastic process at  $t=0$  is always positive). These predictions are verified by the data.

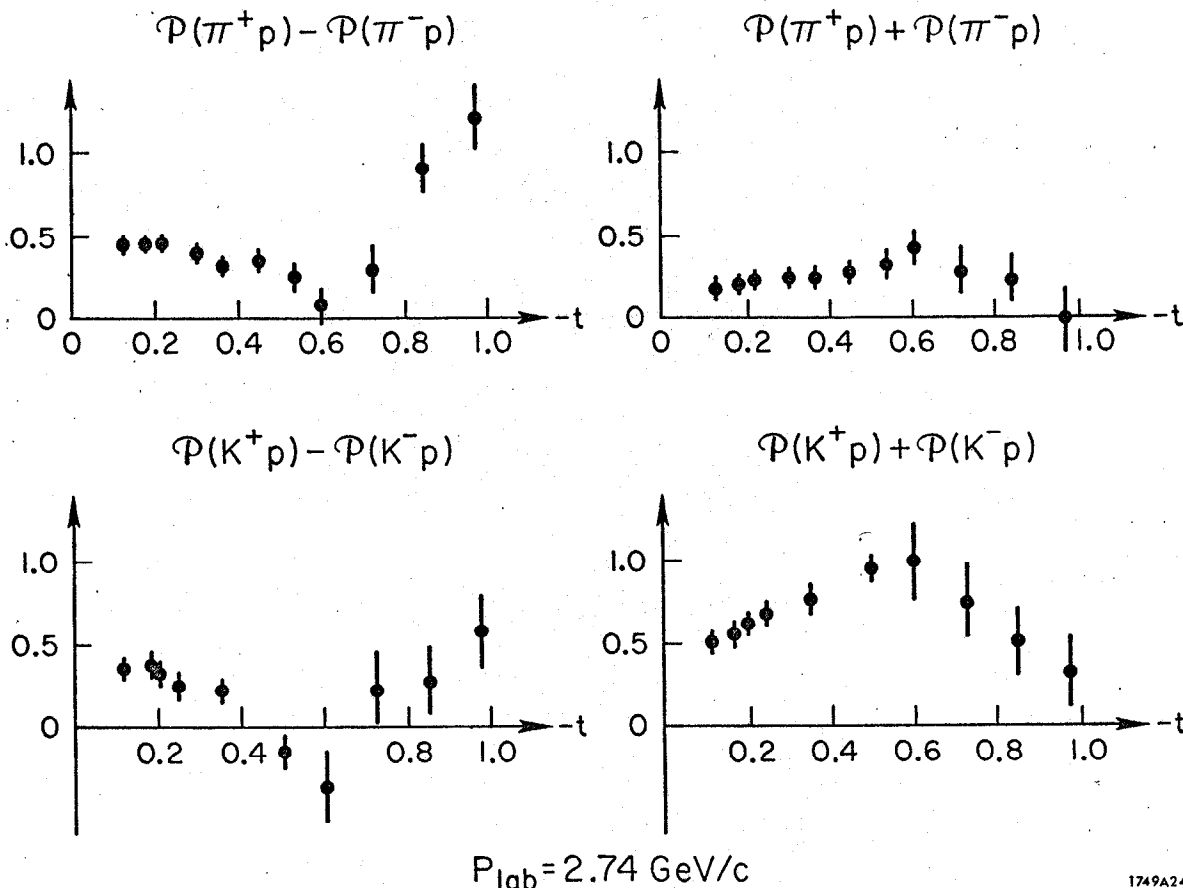
EXPERIMENTAL TESTS; ELASTIC SCATTERING



23--The leading contributions<sup>10</sup> to the elastic differential cross sections are presumably the Pomeron term ("P")<sup>2</sup> and the Pomeron-Resonance interference term which is a pure  $\Delta\lambda=0$  term and has the form ("P")("J<sub>0</sub>"). This second term should be absent in the exotic elastic processes ( $K^+p$ ,  $pp$ ). These processes show no structure for  $|t| < 1 \text{ BeV}^2$ . Hence, the "P" term has no structure (see also Fig. 20). The ("P")("J<sub>0</sub>") term is positive at  $t=0$ , changes sign at  $t \sim -0.2$  and has a minimum at  $t \sim -0.6$  (see Fig. 15). All of these properties are obeyed by the data for the non-exotic and exotic elastic processes.



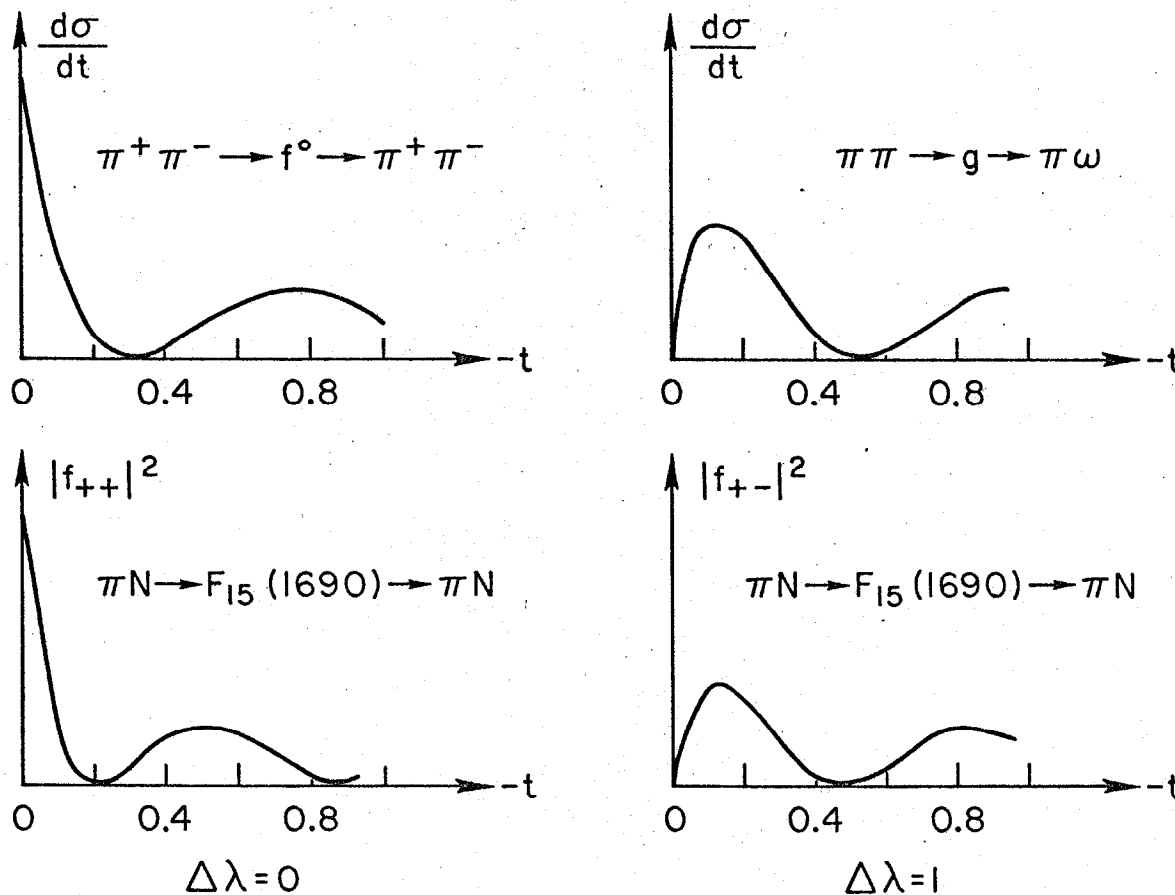
24--The difference between the  $K^-p$  and  $K^+p$  differential elastic cross sections should be given<sup>10</sup> by  $2("P")("J_0")$ . The same holds for the  $\pi^-p-\pi^+p$  and the  $\bar{p}p-pp$  differences. If  $d\sigma(\bar{p}p)/dt \sim ("P")^2 + 2("P")("J_0")$  and  $d\sigma(pp)/dt \sim ("P")^2$ , we can use experimental data in order to extract  $"J_0"$ . The data indicate that, qualitatively, the model works (compare with Fig. 15).



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25--The leading term in the polarization in elastic  $\pi p$  and  $Kp$  scattering comes<sup>10</sup> from the product of the Pomeron term "P" and the real part of the  $\Delta\lambda=1, s$ -channel helicity amplitude  $\text{Re} f_{\Delta\lambda=1}^s$ . We have assumed that  $\text{Im} f_{\Delta\lambda=1}^s \sim J_1(R\sqrt{-t})$ . The real part will have the form<sup>10</sup> " $J_1$ "  $\tan(\pi\alpha(t)/2)$  and " $J_1$ "  $\cot(\pi\alpha(t)/2)$  for crossing-odd and crossing-even amplitudes, respectively, assuming a  $\nu^{\alpha(t)}$  asymptotic energy dependence. " $J_1$ "  $\tan(\pi\alpha(t)/2)$  has a double zero around  $t \sim -0.6$ . This is verified by the experimental<sup>11</sup> polarization differences above. " $J_1$ "  $\cot(\pi\alpha(t)/2)$  should "explode" to infinity at  $t \sim -0.6$  but a logarithmic term in the energy dependence is sufficient to keep it finite. The experimental<sup>11</sup> polarization sums actually show a bump around  $t \sim -0.6$ .

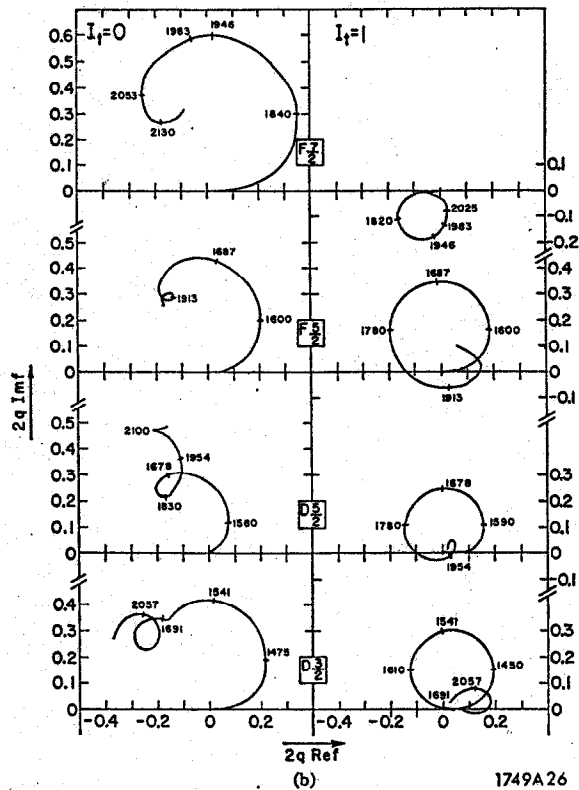
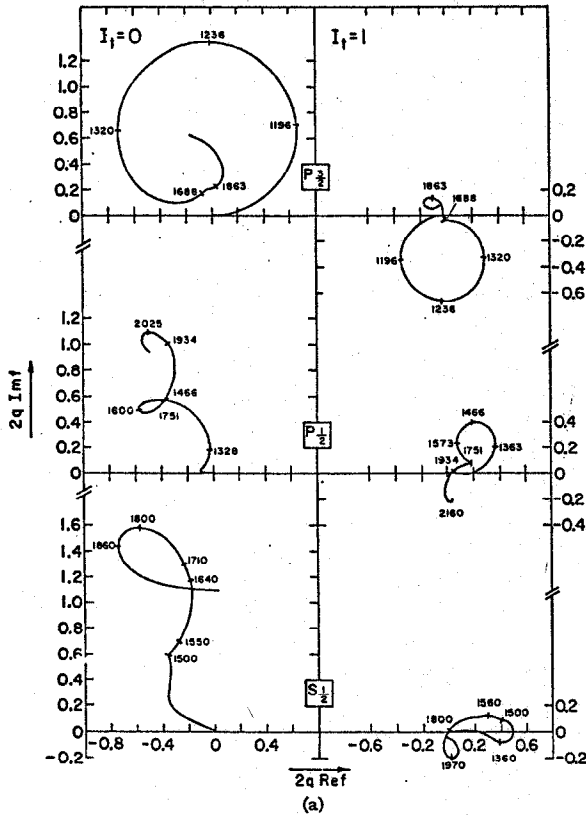
EXPERIMENTAL TESTS: RESONANCE CONTRIBUTION



(THEORETICAL CURVES)

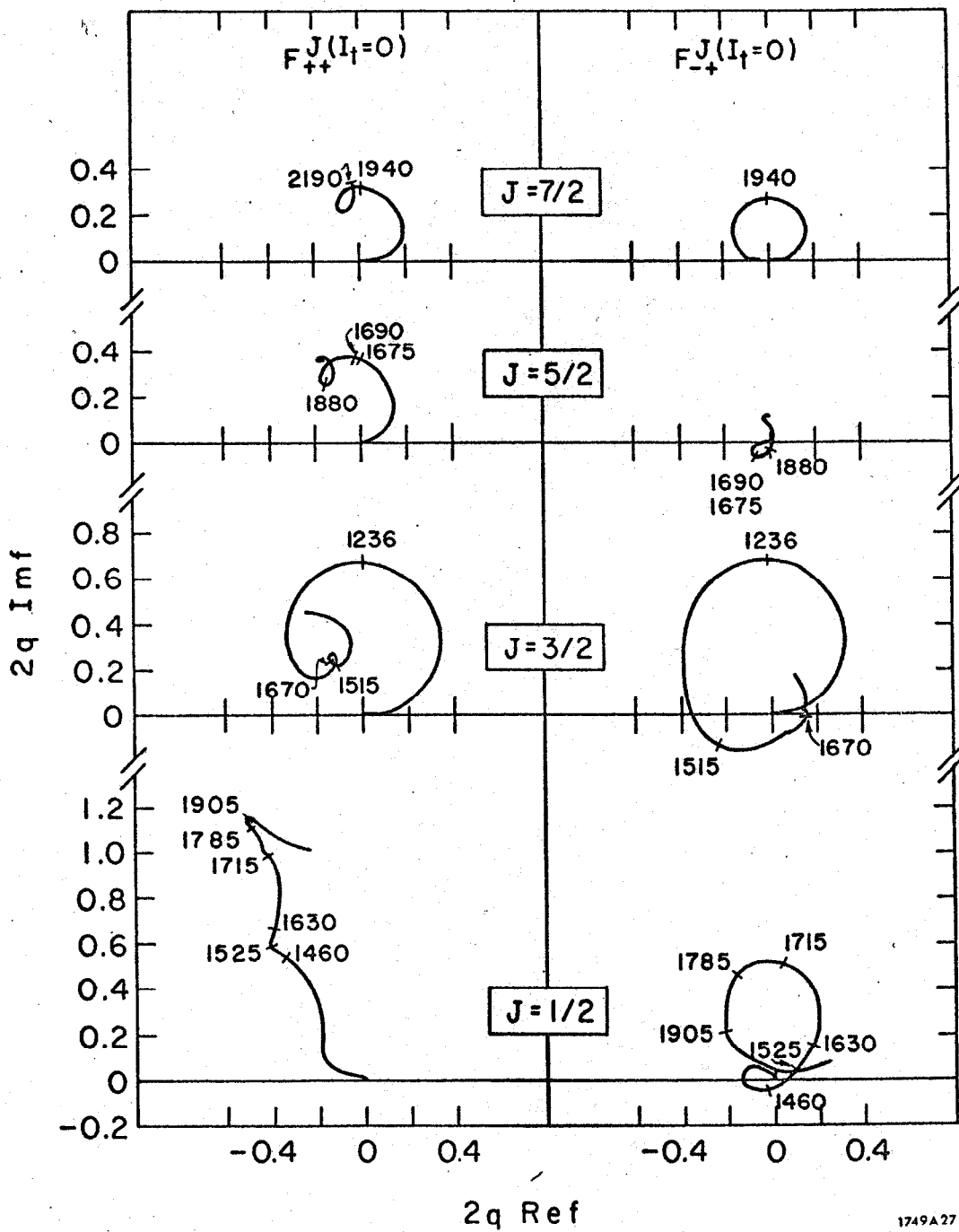
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26--The "first component" of the hadronic amplitudes comes from the resonance contributions. Its contribution to the differential cross section should have the approximate shape of  $[J_0(R\sqrt{-t})]^2$  for  $\Delta\lambda=0$  and  $[J_1(R\sqrt{-t})]^2$  for  $\Delta\lambda=1$ . This should be true for the contribution of the individual prominent resonances, as shown above, as well as for all inelastic amplitudes (where the Pomeron term is absent).



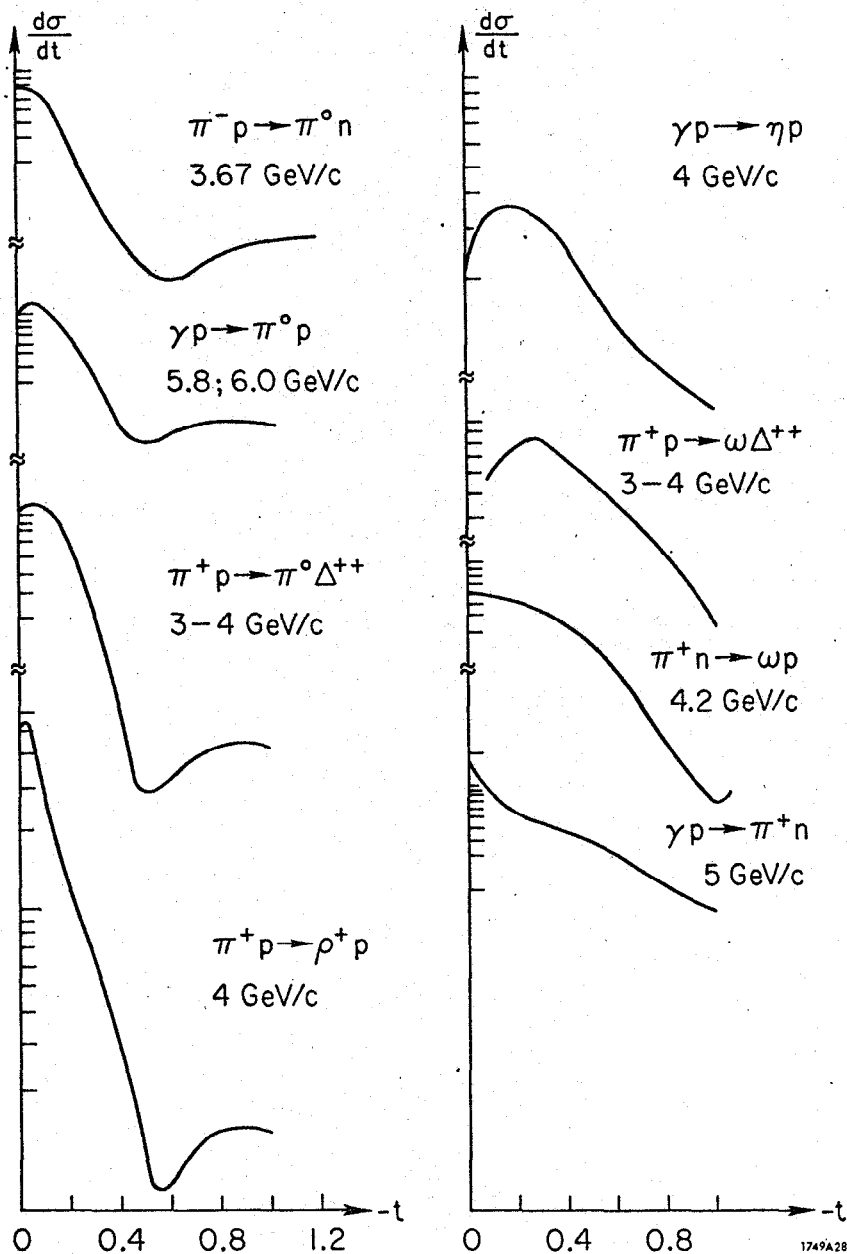
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27--When the Pomeron term is absent, only s-channel resonances should contribute. This should be the case for the  $I_t=1$  component of every  $\pi N$  partial wave amplitude ( $I_t=t$ -channel isospin). The  $I_t=1$  projections should therefore show "clean" circles in the Argand diagram. The  $I_t=0$  components include resonances as well as contributions from Pomeron exchange. Their Argand diagram should show circles superimposed on a predominantly imaginary background. This is confirmed by the  $\pi N$  phase shift in a striking way. <sup>12</sup>



1749A27

28--If we now take the  $I_t=0$   $\pi N$  partial wave amplitude (Fig. 27) and separate the contributions of  $\Delta\lambda=0$  and  $\Delta\lambda=1$ , we should find no Pomeron contributions in the  $\Delta\lambda=1$  part, and hence -- only resonance circles. The  $\Delta\lambda=0$  part should include the Pomeron and resonances. This seems to be verified<sup>13</sup> by the phase shifts.



29--The above eight inelastic processes should all have major contributions from  $\omega$  exchange or  $\rho$  exchange. In a pole-model they should all have clear dips at  $t=-0.6$ . Data indicate that four of them do, but four do not have such dips. In our picture the four processes showing dips should be dominated by the  $\Delta\lambda=1$  s-channel helicity amplitudes while the other four should have comparable contributions from  $\Delta\lambda=1$  and  $\Delta\lambda=0$  (or  $\Delta\lambda=2$ ). Several versions of the absorption model<sup>4, 14</sup> actually possess this property, but only very detailed polarizations and density matrix measurements can verify it.



## Concluding Remarks

1. We have not discussed here the many algebraic predictions of duality, and particularly those results which are related to the quark picture. These aspects of the model have been thoroughly analysed in many recent reviews<sup>1</sup>. We shall only state here that the only outstanding difficulty remains the  $\bar{B}B$  puzzle.<sup>15</sup>

2. Another interesting question which we did not discuss is the possible relation between the parton model for deep inelastic electron scattering and our dual model for hadronic reactions. Experimentally, it seems that if we apply the two component theory to the deep inelastic structure functions both components of the amplitude obey approximate scaling<sup>16</sup>. Various theoretical speculations on the subject have been recently proposed.<sup>17</sup>

3. The assumption of "peripheral dominance" for inelastic processes is, of course, independent of duality. It should be extremely interesting to pursue as many independent tests of this assumption as possible. This assumption, by itself, cannot be considered as a "model" for hadronic reactions. However, if true, it is an extremely strong constraint that all future models will have to obey.

4. Our understanding of the nature of the real part of hadronic amplitudes is limited to conclusions that can be drawn by using fixed- $t$  dispersion relations together with our theoretical assumptions concerning the imaginary part. Except for our remarks about polarizations (figure 25) we have avoided making predictions concerning the real parts of scattering amplitudes. Better understanding of the real part is obviously needed.

I would like to thank my colleagues in the Weizmann Institute for many helpful discussions. I owe special thanks to Y. Avni, M. Kugler, A. Schwimmer and Y. Zarmi.

## References

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