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CHARGE DISTRIBUTIONS AND MULTIPERIPHERALISM*

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ABSTRACT

A new distribution of charged and neutral pions is proposed on the basis of a resonance dominated Amati-Fubini-Stanghellini multiperipheral model. The model is in good agreement with the present experimental data.

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The multiperipheral model (MPM) provides a unifying scheme for the classification of multiparticle reactions. Such a scheme is badly required by the growing amount of experimental information on the subject and by the difficulty inherent in the study of functions of many kinematical variables.

The MPM has recently received convincing support by the experimental study of the energy dependence of the average multiplicity.[1] However, the detailed analysis of particular $2 \rightarrow n$ body reactions have not yet given compelling evidence for (or against) the dominance of a multiperipheral mechanism: because of the poor statistics and the large amount of freedom in parameterization it is impossible to reach definite conclusions. It looks more meaningful to focus our attention on the main features of particle production reactions, common to all processes and to derive general predictions that can discriminate the various models.

Among such general features are the different charge distributions to which considerable attention has been recently devoted.[2,3,4] These distributions are hopefully independent of the details of the underlying dynamics and there is good experimental evidence that they are actually independent of the particular reactions examined.[2]

The most striking features of the data are, in our opinion, the following:

a. The charged particles are distributed in a Poisson-like distribution (Fig. 2).

b. The (rather preliminary) data on neutral particles indicate that the average number of neutral particles is strongly correlated with the number of charged particles (Fig. 4).

Feature (a) is consistent with a MPM production mechanism since the MPM gives for the probability of producing n-identical bosons at a definite center-of-mass energy $\sqrt{s}$, the distribution[5]:

$$P(n, s) = \frac{(g^2 \log s)^n}{n!} s^{-g^2} \qquad (1)$$

where g is the coupling constant of the boson to the multiperipheral chain.

In the physically relevant situation in which most of the produced particles are pions, a single Poisson distribution of the type (1) cannot hold due to charge and isospin conservation. These constraints can be taken into account within the MPM by assuming a definite isospin structure of the multiperipheral chain. The introduction of such a structure does not change the general Poisson-like character of the charged-particle distribution.[8] However, since the average subenergy of a pion in the final state is of the order of $.5 - .7\ \mathrm{GeV}^2$, it is hard to accept that the direct emission of the pions from the multiperipheral chain is the dominant mechanism. Actually, feature (b) contradicts the direct emission of pions: Multiperipheral chains with different isospin properties give at best no correlation between the charged and neutral pions and usually a trend opposite to the experimental one.[6] A strong correlation between the charged and neutral pions can appear, however, if the pions are produced in pairs with well-defined "s-channel" isospin (e.g., in a charged I=1 state, for every charged π we have a neutral one).

Therefore in order to reproduce features (a) and (b) the dominant mechanism should be <u>a multiperipheral production of pairs of pions in well-defined isospin states</u>. A model which fulfills the above requirements is the Amati-Fubini-Stanghellini (AFS) MPM with the $\pi\pi$ scattering approximated by the production of an I=1 (ρ) and an I=0 (σ,f) resonances (Fig. 1). Since the subenergy between two neighboring resonances is of the order of 1.5 GeV the assumption of the resonances being multiperipherally produced is now more acceptable (at least in an average sense).

Before calculating the distribution corresponding to the AFS model we want to stress that although the calculations were made for this specific model the general features would be kept as long as the above-mentioned requirements are met.

From Fig. 1 we see that in order to produce n-pions we should produce n/2 resonances through a multiperipheral chain containing pions (we will actually use only the fact that the exchanged particles have I=1). Each possible ordering of the final resonances gives a contribution of the form (1) and we assume that the contribution is sizeable only in the phase space region in which the longitudinal momenta (in the lab frame) of the produced resonances are ordered in increasing magnitude. Therefore interference terms between diagrams with different orderings can be neglected and the diagrams add incoherently. Then we obtain the probability of producing r-σ's, m-ρ^0's and n-ρ^+'s (together with n-ρ^-'s because of charge conservation):

$$P(r,n,m,s) = \frac{\left(g_\sigma^2 \ln s\right)^r}{r!} P_1(n,m,s)/S(s) \tag{2}$$

with

$$P_1(n,m,s) = \frac{\left(g_\rho^2 \ln s\right)^{2n+m}}{(2n+m)!} 2^n \binom{n+m-1}{n-1} \tag{3}$$

and

$$S(s) = \sum_{r,n,m} P(r,n,m,s) \tag{4}$$

where g_σ and g_ρ are the $\sigma\pi^+\pi^-$ and $\rho^0\pi^+\pi^-$ coupling constants respectively. The two coupling constants can be related using the condition that no I=2 amplitude appears in the crossed channel of the π-π scattering amplitude: this condition gives $g_\sigma = g_\rho \equiv g$. We have checked that our results do not depend critically on the relation between g_σ and g_ρ. The function S(s) can be put in a convenient form

$$S(s) = \sqrt{2}\ s^{g^2} g^2 \ln s \int_0^1 du\ s^{g^2 u} \sqrt{\frac{1-u}{u}}\ I_1\left(2\sqrt{2}\, g^2 \ln s \sqrt{u(1-u)}\right) \tag{5}$$

where I_1 is the Bessel function of imaginary argument. From (5) the asymptotic behavior of s can be estimated: $S(s) \underset{s\to\infty}{\longrightarrow} s^{3g^2}$.

In an analogous way the asymptotic values of the average number of σ's and charged and neutral ρ's can be calculated:

$$\langle n(\sigma)\rangle = \sum_{r,n,m} P(r,n,m)\, r = g^2 \ln s \tag{6a}$$

$$\langle n(\rho^+)\rangle = \sum_{r,n,m} P(r,n,m)\, n = \frac{2(g^2 \ln s)^2}{S(s)} \int_0^1 du\, s^{g^2 u}\, I_0\left(2\sqrt{2}\, g^2 \ln s \sqrt{u(1-u)}\right) (1-u)$$

$$\xrightarrow[s\to\infty]{} \frac{2}{3} g^2 \ln s \tag{6b}$$

$$\langle n(\rho^0)\rangle = \sum_{r,n,m} P(r,n,m)\, m = \frac{\sqrt{2}(g^2 \ln s)^2}{S(s)} \int_0^1 du\, s^{g^2 u}\, I_1\left(2\sqrt{2}\, g^2 \ln s \sqrt{u(1-u)}\right) \cdot \sqrt{u(1-u)}$$

$$\longrightarrow \frac{2}{3} g^2 \ln s \tag{6c}$$

Therefore the average total pion multiplicity is given asymptotically by $\bar{n}_\pi = 6\, g^2 \ln s$ and for each charge

$$\bar{n}_{\pi^+} = \bar{n}_{\pi^0} = \bar{n}_{\pi^-} = 2\, g^2 \ln s \tag{7}$$

From (2) it is easy to obtain the probability distribution $\Pi(n_+, n_0, s)$ for the emission of n_+-π^+'s (and the same number of π^-'s) and n_0-π^0's:

$$\Pi(n_+, n_0, s) = \sum_{m_1=0}^{\min\left(n_+, \frac{n_0}{2}\right)} \sum_{m_2=0}^{n_+-m_1} P\left(n_+ + \frac{n_0}{2} - 2m_1 - m_2, m_1, m_2, s\right) \cdot$$
$$\cdot \left(\frac{1}{3}\right)^{n_+ + \frac{n_0}{2} - 2m_1 - m_2} 2^{n_+ - m_1 - m_2} \begin{pmatrix} n_+ + \frac{n_0}{2} - 2m_1 - m_2 \\ n_+ - m_1 - m_2 \end{pmatrix} \tag{8}$$

We compare now the predictions of (8) with the existing data:

a. Charged particle distributions at fixed energy. We plot in Fig. 2 the quantity:

$$\Pi^{+}(n_{+}, s_{+}) \equiv \sum_{n_0} \Pi(n_{+}, n_0, s) \tag{9}$$

for s=50 and compare it with the data of Ref. 5. As we see the shape of the curve is Poisson-like and fits the data very well. Essentially every model having a Poisson-type distribution succeeds in fitting this kind of data.

b. Energy dependence of the charged particle distribution. In Fig. 3 we plot $\Pi^{+}(n_{+}, s)$ as a function of s for different fixed values of n_{+} and compare it with proton-proton cross sections for different number of prongs. The agreement with the data is again good. The general trend of the curves is what is expected in the MPM, i.e., the constant p-p total cross section is built at each energy mainly by a few partial cross sections which decrease after reaching their maximum, while other partial cross sections take over. We remark that the "2^{-k} rule" for the ratios of the partial cross sections proposed in Ref. 4 on the basis of the apparent leveling-off of the cross sections is not fulfilled in the present model. However, the model fits the data well (Fig. 3) on the basis of which the above mentioned rule was proposed.

c. Correlation between π^{0}'s and π^{+}'s. As we mentioned one of the motivations for considering the present distribution was the inability of other distributions to reproduce correctly the dependence of the average number of π^{0}'s on the number of prongs. This quantity is expressed by

$$A(n_{+}, s) = \sum_{n_0} n_0 \, \Pi(n_{+}, n_0, s) \Big/ \sum_{n_0} \Pi(n_{+}, n_0, s) \tag{10}$$

As shown in Fig. 4 the experimental data suggest that A rises with n_+ for low values of n_+ and this trend is well reproduced by (8) whereas a constant for $A(n_+, s)$ is predicted by all models in which π^0 and π^+ are independently produced. In most "classical" MPM the trend is actually to give a decrease of $A(n_+, s)$ with n_+ as, e.g., in the Chew-Pignotti MPM with an alternate I=0, I=1 chain. This is due to the fact that when n_+ increases less and less phase space is left for the production of extra neutral particles. In our model on the contrary, an increase in the number of π^+ or π^- in the final state is likely to increase the number of ρ^+ and ρ^- multiperipherally produced and this automatically increases the number of the final π^0. The production of the σ provides a constant background as in the model considered by Horn and Silver.[3]

Let us conclude with some short remarks:

a. The introduction of a definite isospin mechanism in the MPM is likely to increase the probability of the subsequent emission of several neutral pions, providing a possible multiperipheral explanation for the occurrence of "fireball events".[7,8]

b. As we do not use the model for any dynamical purpose, the crudeness of the parameterization of the π-π cross section as the sum of two resonances should not be harmful: what really matters is the fact that this cross section has definite isospin properties.

c. We remark that in this model all fixed multiplicity cross sections decrease to zero asymptotically: this is of course due to the fact that we do not introduce any diffractive mechanism in the production amplitude. Hopefully this shortcoming is not too serious for the charge distribution problem, because we expect diffractive processes to correspond to low final multiplicities, and therefore not to have too much weight in global properties of the kind that we studied.

d. From the relation $\bar{n}_\pi \xrightarrow[s\to\infty]{} 6g^2 \ln s$ we can estimate $g^2 \approx .18$, corresponding to a width of 180 MeV in the I=1 channel. However, due to the fact that the function S(s) increases only as s^{3g^2}, we expect the imaginary part of the elastic amplitude obtained from this model through unitarity to behave at most as $s^{.5}$. The introduction of a diffractive mechanism of the type discussed in (c) could hopefully boost this behavior without too much affecting the average multiplicity results. Unfortunately the detailed dynamical calculations on essentially the same model performed in Refs. 9, 10 and especially 11 seem to indicate that this goal is not easy to accomplish.

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FIGURE CAPTIONS

1. The multiperipheral diagrams assumed to be dominant.

2. Comparison of the charged particles distributions predicted by the model with the data of Ref. 6.

3. Energy dependence of the various multiplicities of charged particles in pp scattering. The data is taken from the compilation of Ref. 4.

4. Dependence of $\bar{n}_0$ on n_+. The data are taken from Ref. 6. For comparison we show the prediction of a multiperipheral model with alternate I=0 and I=1 exchange (Ref. 5).

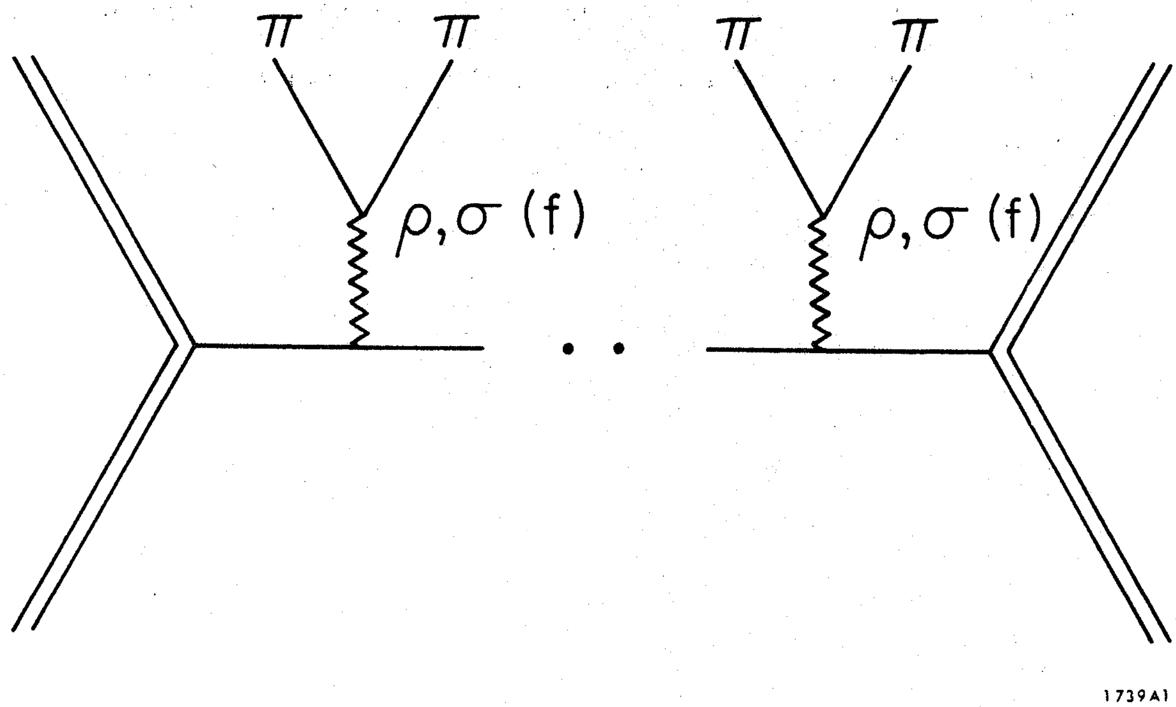


Fig. 1

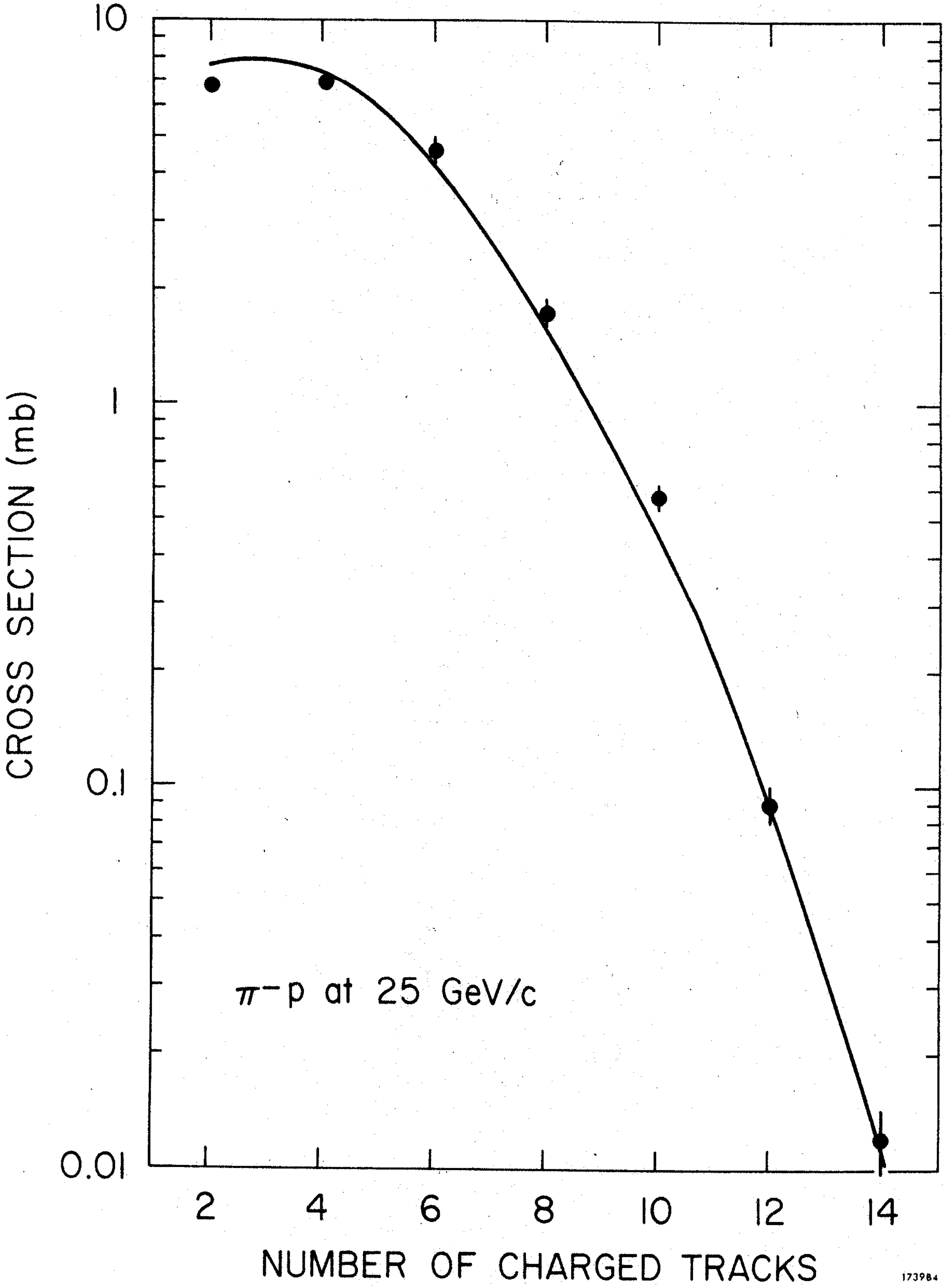
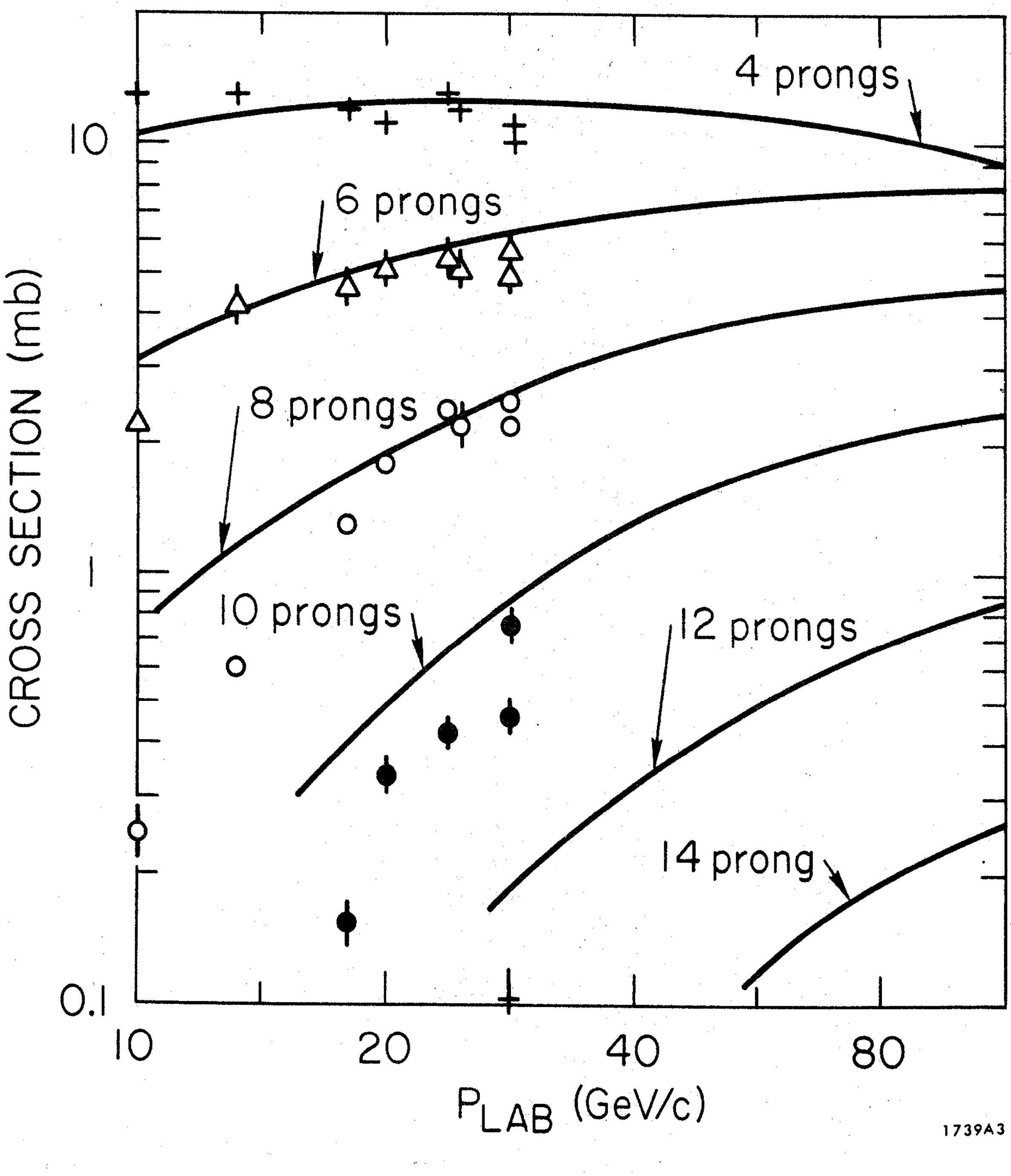
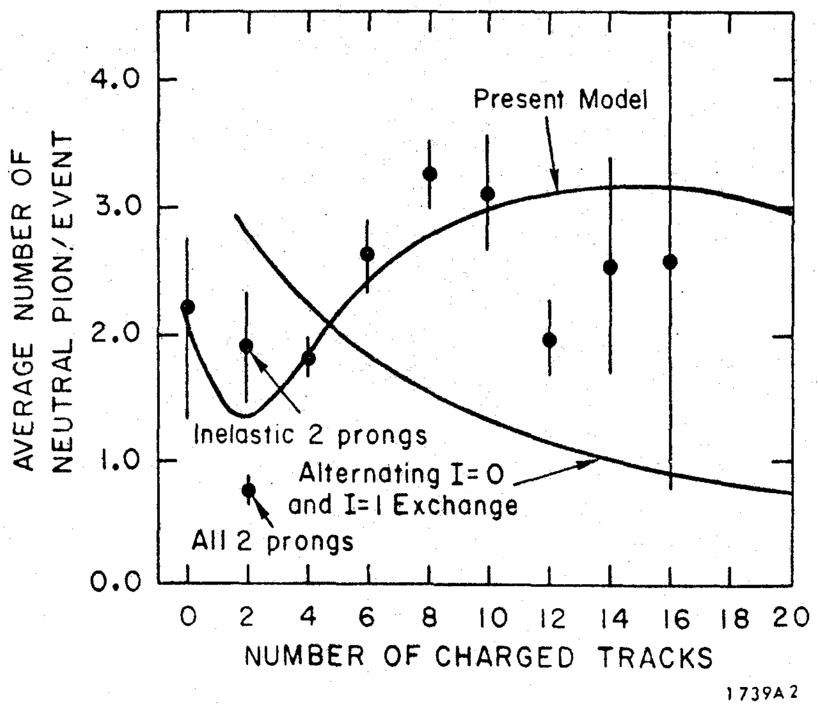

10
1
0.1
0.01
CROSS SECTION (mb)
π⁻p at 25 GeV/c
2
4
6
8
10
12
14
NUMBER OF CHARGED TRACKS
173984

Fig. 2



4 prongs
6 prongs
8 prongs
10 prongs
12 prongs
14 prong
CROSS SECTION (mb)
10
1
0.1
10
20
40
80
P_{LAB} (GeV/c)
1739A3

Fig. 3



AVERAGE NUMBER OF NEUTRAL PION/EVENT
4.0
3.0
2.0
1.0
0.0
Present Model
Inelastic 2 prongs
Alternating I=0 and I=1 Exchange
All 2 prongs
0
2
4
6
8
10
12
14
16
18
20
NUMBER OF CHARGED TRACKS
1739A2

Fig. 4