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A METHOD FOR MEASURING THE PHOTON-PHOTON TOTAL CROSS SECTION*

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ABSTRACT

In the interaction of a high energy particle with a proton or nucleus, the recoil spectrum of the target at very low momentum transfer is related simply to the total cross section of the incident particle with the photon. Estimates of the size of the effect plus recent developments in low recoil techniques suggest that measurement of the photon-photon, as well as the π and K-photon cross sections, may be possible.

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The interaction of one photon with another remains one of the fundamental but unmeasured quantities in particle physics. There has been a considerable discussion, both theoretical and experimental on photon-photon elastic scattering,[1] $\gamma + \gamma \rightarrow \gamma + \gamma$, sometimes involving such Faustian methods as (two) simultaneous nuclear explosions.[2] A measurement, of the photon-photon total cross section, however, may in fact be within the scope of more mundane contemporary laboratory techniques and accelerators. The imaginary part of the forward photon elastic scattering would then follow from the optical theorem, of course, leaving only the real part to be found from dispersion relations or other extreme means.

Such a measurement may be possible by exploiting the phenomena of Coulomb production (Fig. 1) in which an incoming beam particle (k) strikes a virtual photon (q) in the Coulomb field of a charged target particle Z and makes a final state f. If the beam is highly relativistic and the energy of k is large compared to the mass of k and the mass M* of the system f, then the reaction takes place with a very small momentum transfer and the virtual photon q may be thought of as a real photon. The process has a characteristic peak at very small q^2, reflecting the very long range of the interaction. At high energy the very small value of q^2 in the photon propagator may overcome the factor $\alpha = 1/137$ in the matrix element relative to processes taking place in the nuclear matter of the target. For the case where f is a single particle ("Primakoff Effect")[3] the rate is related to the widths $\Gamma(f - k + \gamma)$[4] and with incident photons has been used to find the widths $\Gamma(\pi^0 \rightarrow \gamma\gamma)$ and $\Gamma(\eta^0 \rightarrow \gamma\gamma)$.[5] Generalizations of the effect to an arbitrary system f may be envisaged, such as that recently proposed[6] where f is chosen to be the π-π system in order to examine the π-π interaction outside the range of other hadrons.

A basic result of the general analysis[4, 6, 7] is the following: If the process $f \rightarrow k + \gamma$ exists for a real, $q^2 = 0$ photon, then the Coulomb production process at high energy always has the form in the lab system

$$\frac{d\sigma}{dq^2} \sim \frac{q_T^2}{q^4} \simeq \frac{q_T^2}{\left[q_T^2 + q_L^2\right]^2} , \tag{1}$$

regardless of the nature of the state f.

The q_T and q_L are the momentum transfer transverse and parallel to the beam direction and we have used the fact that the energy transfer in the lab is negligible.[8] Equation (1) is a reflection of the fact that the reaction may be viewed from a frame, such as the rest frame of f, in which the field of the then fast moving target Z appears as a superposition of almost real transverse photons; a relatively simple direct proof is given in Ref. 6 .

The significance of this result is that it leads us to expect that if we sum over all states f of a given mass M*, the resulting summed cross section also has the form Eq. (1), with a coefficient given essentially by $\sum_f \sigma(k + \gamma \to f) = \sigma^{\gamma k}$, at an M* given by $M^{*2} = (q + k)^2$. Thus an observation of the target recoil spectrum at small momentum transfers can give $\sigma(\gamma k)$.

This expectation is verified by a formal calculation as follows: Just as for the more familiar case of electroproduction, structure functions[9] may be defined for transitions induced by a photon on k to final states f. Summing over f and averaging over any polarization variable of the incident particle, we have in terms of nonsingular functions F_0 and F_1

$$\begin{aligned} M_{\mu\nu} &= \overline{\sum_k} \sum_f M_\mu^{f*} \, M_\nu^f \\ &= F_0 \left[\delta_{\mu\nu} - \frac{k_\mu q_\nu + k_\nu q_\mu}{k \cdot q} + q^2 \, \frac{k_\mu k_\nu}{(k \cdot q)^2}\right] \\ &\quad + F_1 \left[O(q^4)\right] \end{aligned} \tag{2}$$

The F_1 term is negligible since q^2 is small. For a free photon on k we have

$$\frac{(2\pi)^4}{4k \cdot q} \frac{1}{2} \sum_{i=1}^{2} \epsilon_\mu^i \epsilon_\nu^i M_{\mu\nu} = \sigma^{\gamma k} \tag{3}$$

which fixes F^0. When the same photon is exchange we have Coulomb production with a cross section in terms of lab quantities

$$\frac{d\sigma}{d^3P} = \frac{\alpha Z^2}{\pi^2} \frac{(q \cdot k)}{k\, 4M^2} \sigma^{\gamma k} \frac{1}{q^4} \left[(P_1 + P_2)_\mu (P_1 + P_2)_\nu T_{\mu\nu}\right] \tag{4}$$

P is the three vector momentum of the recoiling target P_2 in the lab while M is the mass of the target Z and $P_1 + P_2$ is the current (magnetic terms being neglected with q^2 small) and $T_{\mu\nu}$ the tensor multiplying F_0 in Eq. (3). Since $T_{\mu\nu}$ obeys current conservation we can replace $P_1 + P_2 \longrightarrow 2P_1$, and in the lab the quantity in brackets becomes $4M^2\ T_{44}$. The simplification of the expressions is greatly aided by the high energy approximations,[8] applicable in the lab:

$$q^2 \cong \underline{q}^2 = q_T^2 + q_L^2$$

$$q_L \cong \frac{M^{*2} - M_k^2}{2k}$$

$$(q \cdot k) = \frac{1}{2}\left(M^{*2} - q^2 - M_k^2\right) \cong \frac{M^{*2} - M_k^2}{2} . \tag{5}$$

Then the quantity in brackets is $4M^2 (q_T^2/q_L^2)$ and Eq. (4) is

$$\frac{d\sigma}{d^3P} = \frac{\alpha Z^2}{\pi^2} \frac{1}{q_L^3} \sigma^{\gamma k} \left[\frac{q_T/q_L}{1 + (q_T/q_L)^2}\right]^2 \tag{6}$$

or with θ the lab angle of the target recoil

$$\frac{d\sigma}{d^3P} = \frac{\alpha Z^2}{4\pi^2} \frac{1}{q_L^3} \sigma^{\gamma k}(M^*) \sin^2 2\theta \tag{6'}$$

The expression (6) shows the characteristic form referred to above and can be considered a proof of the statement concerning Eq. (1). Equation (6') indicates that a counter at fixed θ essentially observes a simple momentum spectrum: $d\sigma/dP \sim 1/P$.

The very small value of q_L obtainable at high energy is what makes it possible that the effect is observable.

Estimation of Eq. (6) for incident photons involves a guess for the total photon-photon cross sections at the energy M*. At low M* this cross section should be dominated by $\gamma + \gamma \rightarrow e^+ + e^-$ with[10] $\sigma(\gamma\gamma \rightarrow e^+e^-) \approx \frac{4\pi\,\alpha^2}{M^{*2}}\left[2\ln(M^*/m_e) - 1\right]$ and Eq. (6) amounts to a description of the Bethe-Heitler process. The cross section here will be quite large, but with unmeasurably tiny recoils, and is presumably not interesting since the electron pair production process is well understood. At large values of M*, however, we enter totally new territory and number of interesting questions arise. Does the purely electrodynamic cross section innocuously go down as M* goes up or do we enter a novel domain of "high energy electrodynamics?" When $M^* > 2M_\pi$ and multi-pion states are produced in the photon-photon collision does the cross section approach some kind of "diffractive" limit? If so, with what cross section and at what energy? On dimensional grounds we might expect the $\gamma\gamma \rightarrow$hadrons cross section to be characterized by $\alpha^2/M_\pi^2 \approx 1\mu b$. If there is a diffractive regime the "ρ-photon analogy"[11] suggests an estimate of γ-γ scattering at high energy as ρ-ρ scattering times the "γ-ρ coupling" taken twice. This gives $\sigma_{TOTAL}(\gamma\gamma) \approx g_{\gamma\rho}^2\, \sigma_{TOTAL}(\rho\rho) \approx 0.2\,\mu b$, assuming $\sigma_{TOTAL}(\rho\rho) \approx 20$ mb. This estimate presumably only applies at high energy, $M^* > 2M_\rho$, where the process $\gamma + \gamma \rightarrow \rho + \rho$ kinematically resembles the elastic scattering $\rho + \rho \rightarrow \rho + \rho$.

If we take an example of a k = 20 GeV photon making an M* at 500 MeV, (where electrodynamic and hadronic contributions to $\sigma^{\gamma\gamma}$ may be comparable) then $q_L \approx 6$ MeV/c and Eq. (6) gives

$$\frac{d\sigma}{dq^2 dM^*} = \frac{\alpha Z^2}{4\pi}\,\frac{M^*}{k}\,\frac{1}{q_L^3}\,\sigma(M^*)\sin^2 2\theta$$

$$\approx 0.075\, Z^2 \sigma^{\gamma\gamma}(M^*) \sin^2 2\theta\ /\text{GeV}^2\ \text{MeV} \qquad (7)$$

We can compare this with photoproduction of M* = 500 MeV pion pairs, the dominant competing process in this region. A bubble chamber result[12] at 4.7 GeV on hydrogen gives

$$\frac{d\sigma}{dq^2 dM^*} \approx 0.1\ \mu b/\ \text{GeV}^2\ \text{MeV},$$

a number which is probably decreasing somewhat with energy[13].

Even on hydrogen then, it appears that the Coulomb production signal may not compare unfavorably with potential backgrounds and that a measurement of $\sigma(\gamma\gamma)$ may be possible by this method. The main experimental problem would appear to be the observation of very small target recoils, which must be accomplished with a resolution $\lesssim q_L$. Results[14] reported from Serpukhov at the Kiev Conference involved using a gas jet in vacuum as a target with recoil proton resolution on the order of magnitude required here; perhaps a similar technique would be applicable.

Should experiments prove feasible with polarized photons, it would be possible to investigate the polarization dependence of the photon-photon cross section, since the virtual exchanged photon is polarized in the reaction plane. A polarization dependence would show up in the form of a term in $\sigma(\gamma\gamma)$ varying as $\left(\epsilon_\mu(k)(P_1+P_2)_\mu\right)^2$ that is, as $(\underline{\epsilon}(k)\cdot \underline{q}_T)^2$ since $\epsilon(k)$ is purely transverse. A simple diffractive model would predict the absence of such a term, but it is entirely possible, particularly at low M*, that there is a strong polarization dependence.

We may also take k to be a pion or kaon to study $\sigma(\gamma K)$ or $\sigma(\gamma\pi)$. For an incident high energy pion, for example, using the same conditions as for Eq. (7) the kinematics are the same as for the photon, so taking $\sigma(\gamma\pi) \approx \alpha/M_\pi^2 = 140\ \mu b$ gives at M* = 500 MeV and k = 20 GeV

$$\frac{d\sigma}{dq^2 dM^*} \approx 10\ \mu b\ Z^2 \sin^2 2\theta\ /\ \text{GeV}^2\ \text{MeV} \tag{8}$$

Since the rate goes as k^2 in $d\sigma/dq^2dM^*$, while the hadronic background must be constant or go down if total cross sections are not to grow, we can look forward to quite substantial effects at Batavia energies if the necessary resolution is available to exploit the decrease in q_L.

If these problems of observing a heavy slow moving recoil are resolved, the use of a high Z material as the target will, of course, increase the rates involved and enhance the Coulombic process relative to nuclear production. This is particularly true for pions (or kaons) producing relatively low mass M*'s since a coherent production selection rule[8,15] strongly suppresses the production of $\pi\pi$ (or $K\pi$) states. In the interaction of 15 GeV π^- with Freon, for example, the coherent production at low M* is almost exclusively through the 3π channel. For M* between 500 to 800 MeV this cross section[16] is roughly

$$d\sigma/dq^2dM^* \sim 1.8\ \mu b/GeV^2\ MeV,$$

which should also be constant or slowly dropping with energy. Comparing with Eq. (8) and considering that the effective Z of the material was like that of Fl, $Z \approx 9$, it would appear that the Coulomb processes is dominant at presently available energies.

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