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QUARK TYPE MODELS AND DEEP INELASTIC e-p SCATTERING*

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ABSTRACT

Brandt and Preparata calculated the right-hand side of the Callan-Gross sum rule by evaluating the expectation value between baryons of the equal time commutator $\left[J_i^a, J_j^b\right]$ of U(3) ⊗ U(3) currents on the gluon model which carries a triplet of quarks. They use a symmetry breaking theory based on Regge pole dominance. The agreement of the sum rule with e-p deep inelastic scattering data was good. Here we carry out a similar calculation based on another type of gluon model which carries three triplets of quarks introduced by Han-Nambu. We show that it gives different results due to the anomalous form of the Gell-Mann-Nishijima (G-N) formulae. The free field limit is also discussed and found to give the usual parton model results.

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I. INTRODUCTION

In this note, the object of our interest is the numerical value of the r.h.s. of the following Callan-Gross sum rule.[1]

$$2\int_0^2 d\omega\, F^{ab}(\omega) = \lim_{p_0 \to \infty} E^{ab}(\underset{\sim}{p}) \tag{1}$$

In (1), $F(\omega)$ is defined to be the scaling limit of the maximum helicity flip amplitude of the forward current-proton scattering. That is, if we define,[2]

$$\frac{p_0}{2\pi}\int dx\, e^{iq\cdot x} \langle p|\left[J_\mu^a(x),\, J_\nu^b(0)\right]|p\rangle = p_\mu p_\nu\, W_2(q^2,\nu) + \ldots \tag{2}$$

with $\nu = q\cdot p$, then

$$F(\omega) = \lim_{\nu\to\infty,\ \omega=\frac{\nu}{q^2}} \nu W_2^{ab}(q^2,\nu) \tag{3}$$

and $E^{ab}(\underset{\sim}{p})$ is given by

$$-p_0\int dx\,\delta(x_0)\,\langle p|\left[J_i^a(x),\, J_j^b(0)\right]|p\rangle = iE^{ab}p_ip_j + R^{ab}\delta_{ij} + G^{ab}\epsilon_{ijk}p_k \tag{4}$$

The form (4) is gotten from the fact that the l.h.s. is a second rank tensor in three-space.

The Callan-Gross sum rule (1) can be derived by applying Bjorken limit analysis[3,4] to (2). The number $E^{ab}(\underset{\sim}{p})$, as can be seen from (4), is in general model dependent. Recently, Brandt and Preparata[5] calculated E^a in the framework of the gluon model in which the neutral vector meson B_μ binds a triplet of quarks. Their result was in good agreement with e-p deep inelastic scattering experiment.[6]

The above mentioned gluon model belongs to a class of models in which the SU(3) × SU(3) currents are constructed from bilinear combinations of spin 1/2 fields and the fermions interact via neutral vector mesons coupled to a conserved vector current and the chiral SU(3) × SU(3) symmetry is broken only by mass terms. It was shown that for this class of models, the second order radiative correction to relative rates of general semileptonic processes are finite.[11] If one wants to render the corrections for each processes finite, so that we may calculate corrections to the lepton-hadron universality, one must further impose special relations for the equal time commutators of the spatial components of the hadronic currents. The model of Cabbibo *et al.*,[10] in which the SU(3) ⊗ SU(3) currents are constructed from the three triplets of quarks first introduced by Han and Nambu, satisfies this relation. Their model was also shown to reproduce most of the results of the usual quark models when applied to static problems.[10] Therefore it might be of interest to know what these other models give for the value of E^{ab}. In this note, we shall show that the above mentioned model of Cabbibo *et al.* gives E^{ab} which is not in agreement with experiment. This result will be seen to be caused by an extra quantum number, i.e., the charm number of the theory which enters as an additional term in the usual G-N formulae.

In Section II, we will describe briefly the Brandt-Preparata method for the sake of completeness. We also discuss the free field limit. In Section III, we essentially use the method described in Section II to calculate E^{ab} for the model of Cabbibo *et al.* We shall see that the result doesn't agree with experiment, although for the free field limit both models give the same results. Finally discussion of our result and some concluding remarks are given in Section IV.

II. B-p METHOD

In order to calculate E^{ab}, B-p takes a quark model with Lagrangian density

$$\mathscr{L} = \overline{\psi}(x)\ (i\gamma_1\gamma - M)\ \psi(x) + g\,B_\mu(x)\ \overline{\psi}(x)\ \gamma_\mu\,\psi + \text{Boson terms} \tag{5}$$

with

$$M = \alpha_0\lambda_0 + \alpha_8\lambda_8 \tag{6}$$

$B_\mu(x)$ is SU(3) singlet vector meson (gluon) field, so that the only SU(3)⊗SU(3) symmetry breaking effect is due to quark mass term assumed to have the form (4).

The SU(3) vector current is taken to be

$$j_\mu^a(x) = \frac{1}{2}\,\overline{\psi}(x)\,\gamma_\mu\lambda^a\,\psi(x) \tag{7}$$

and the electromagnetic current is given by the Gell-Mann-Nishijima formulae

$$j_\mu^{e,m}(x) = j_\mu^{(3)}(x) + \frac{1}{\sqrt{3}}\,j_\mu^{(8)}(x) \tag{8}$$

Since we know the explicit form of the e-m current by (7) and (8), we can calculate E^{aa} by means of Eq. (4) and compare with electroproduction data. For this purpose, B-p proceeds as follows.

By means of the equation of the motion inferred from Lagrangian (5), and using canonical equal time commutation relation (ETCR), we can reduce (4) into the form

$$p_0 \quad p\; \overline{\psi}(x)\, V_\mu\gamma_\nu\, \lambda^a\, \psi(x)\; p\rangle = E^a p_\mu p_\nu - g_{\mu\nu}R^a \tag{9}$$

where $E^{ab}=d^{abc}E^c$ and $V_\mu = -i\gamma_\mu + gB_\mu(x)$. Notice that the tensor structure of (9) says that E^a doesn't depend on $\underset{\sim}{p}$. By taking a trace of (9), and noting that the energy momentum tensor $\theta_{\mu\nu}(x)$ satisfies (2)

$$m^2 = p_\mu p^\mu = p_0\,\langle p|\theta^\mu_\mu(x)|p\rangle = p_0\,\langle p_0|M^0 + \alpha_8\,\overline{\psi}(x)\,\lambda_8\,\psi(x)|p\rangle \tag{10}$$

We can further reduce (9) into the form

$$\alpha_0 d^{a0b}\,Se^b + \alpha_8 d^{a8b}\,Se^b = E_e^a\left(M_e^0 + \alpha_8\,Se^8\right) - 4R^a\,, \tag{11}$$

where we have introduced the definition

$$S_e^a(x) = p_0\,\langle e|\,\frac{1}{2}\,\overline{\psi}(x)\,\lambda^a\,\psi(x)\,|\,e\rangle \tag{12}$$

for baryon octet $\{e'$.

It is important to observe that M_0 in (10) and (11) is SU(3) singlet up to first order in α_8 due to Ademollo-Gatto theorem (12).

Equation (11) is the master equation which is to be solved to evaluate E_e^a. To do this, it is necessary to determine how the quantities S_e^a, E_e^a and R_e^a deviate from exact SU(3) symmetric limit in the real world. To this end, B-p employs their Reggeized symmetry breaking theory. We refer to their paper for details. Here we will only record their result. By Gell-Mann-Nishijima formulae (8), we have $e = (3) + 1/\sqrt{3}\ (8)$ and we have for electromagnetic currents,

$$L^e = \int_0^a d\omega\, F^{ee}(\omega) = \frac{d^{eec}}{2} E^c = \frac{1}{3} E^a + \left(\frac{2}{3}\right)^{3/2} E^0 \tag{13}$$

B-p calculation gives $\frac{1}{3}E^a \sim .1$ and $\left(\frac{2}{3}\right)^{3/2} E^0 \sim .21$ so we have $L_p^a \sim .31$. For neutron scattering, similar calculation gives $L_N^e \sim .24$. These values agree very well with recent SLAC experiment which gives $L_p = .34 \pm .04$ and $L_N = .26 \pm .03$.[6]

Before going to the next section, we will now show that the free field solution $g=0$, $R=0$ and $M_0 = \alpha_0 S_e^0$, so (11) becomes

$$\alpha_0 d^{a0b}\, S_e^b + \alpha_8 d^{a8b}\, S_e^b = E_e^a \left(\alpha_0\, S_e^0 + \alpha_8\, S_e^0\right) \tag{14}$$

Two limits of (14) is interesting:

a. SU(3) limit, $\alpha_8/\alpha_0 \to 0$

In this case, we have

$$E_e^a = \frac{d^{a0b}\, S_e^b}{S_e^0} + \frac{\alpha_8 \left(d^{a8b}\, S_e^b - S_e^8\right)}{\alpha_0\, S_e^b} + 0\left(\frac{\alpha_8^2}{\alpha_0^2}\right) \tag{15}$$

$$\begin{aligned} L^e &= \frac{d^{eea}}{2} E^a = \frac{d^{eea}}{2} \frac{\langle p|\overline{\psi}\lambda^a\psi|p\rangle}{\langle p|\overline{\psi}\,\psi\,|p\rangle} + 0\left(\frac{\alpha_8}{\alpha_0}\right) \\ &= \frac{\langle p|\overline{\psi}\lambda^e\lambda^e\psi|p\rangle}{2\langle p|\overline{\psi}\psi|p\rangle} + 0\left(\frac{\alpha_8}{\alpha_0}\right) = \frac{2}{N}\left(\Sigma Q_0^2\right) + 0\left(\frac{\alpha_8}{\alpha_0}\right) \end{aligned} \tag{16}$$

This is the parton model result.[13] The order α_8/α_0 correction to the proton scattering turns out to vanish.

b. SU(2) ⊗ SU(2) limit

This symmetry limit is given by[14]

$$\sqrt{2}\ \alpha_0 + \alpha_8 = \epsilon_2 \to 0 \tag{17}$$

therefore

$$E_e^a = \frac{d^{a0b}\, S_e^b - \sqrt{2}\, d^{a8b}\, S_e^b}{S_e^0\,(1-\sqrt{2})} \ . \tag{18}$$

$$L^e = \frac{d^{eea}}{2}\, E^a = \frac{1}{3}\, E^e + \left(\frac{2}{3}\right)^{3/2} E^0$$

$$= \frac{1}{2}\left(\frac{2}{3}\right)^2 \frac{\left(S_e^0 - \sqrt{2}\, S_e^8\right)}{S_e^0\,(1-\sqrt{2})} \tag{19}$$

An immediate consequence of (19) is that for nonstrange particle scattering, i.e., for proton and neutron

$$L^e = 0 \tag{20}$$

in the chiral SU(2) ⊗ SU(2) symmetric limit.

III. HAN-NAMBU MODEL

We now proceed with the three triplet model modified by Cabbibo, Maiani and Preparata.[10] Following them, we name the triplets as S triplet, U triplet and B triplet and we will put suffix S, U and B for the quantities associated with them. The Lagrangian of the system is given by

$$\sum_i \overline{\psi}_i(x)\left(i\gamma\cdot\delta - \alpha_0\lambda^0 - \alpha_8\lambda_8\right)\psi_i(x) + gB_\mu(x)\,\overline{\psi}_i(x)\,\gamma_\mu\,\psi_i(x) + \text{Boson term} \tag{21}$$

Notice that (21) preserves the SU"(3) symmetry of Han-Nambu.[8] This is symmetry under the exchange of S, U and B multiplets. In general, we will require that this symmetry is good for strong interactions.

The unitary current in this model is given by,

$$j_\mu^a(x) = \frac{1}{2}\left(\sum_i \overline{\psi}_i(x)\,\lambda^a\gamma_\mu\,\psi_i(x)\right) \tag{22}$$

Now because of the charm quantum number, the Gell-Mann-Nishi ima formula is not given by (8) but is of the form,

$$j_\mu^{em}(x) = j_\mu^{(3)}(x) + \frac{1}{\sqrt{3}}\,j_\mu^8(x) + \frac{1}{3}\,j_\mu^c(x) \tag{23}$$

where $j_\mu^c(x)$ is charm current given by

$$j_\mu^c(x) = i\left(\overline{\psi}_S(x)\,\gamma_\mu\,\psi(x) + \overline{\psi}_U(x)\,\gamma_\mu\,\psi_\mu(x) - 2\,\overline{\psi}_0(x)\,\gamma_\mu\,\psi_B(x)\right) \tag{24}$$

Since $j_\mu^c(x)$ is not a component of the unitary current, we need to modify slightly the procedure of Section II, to get the value of E^{ab} for the electromagnetic current.

We first define for i=S, U, B,

$$j_{(i)\mu}^a(x) = \frac{1}{2}\,\overline{\psi}_i(x)\,\lambda^a\gamma_\mu\,\psi_i(x) \tag{25}$$

Then considering (24), (23) can be written

$$\begin{aligned} j_\mu^{e,m}(x) = {} & j_{\mu(S)}^3(x) + \frac{1}{\sqrt{3}}\,j_\mu^8(S) + \sqrt{\frac{2}{3}}\,j_\mu^0(s) \\ & + j_\mu^3(U) + \frac{1}{\sqrt{3}}\,j_{\mu(U)}^8 + \sqrt{\frac{2}{3}}\,j_{\mu(U)}^0 \\ & + j_{\mu(B)}^3 + \frac{1}{\sqrt{3}}\,j_{\mu(B)}^8 - 2\sqrt{\frac{2}{3}}\,j_{\mu(B)}^0 \end{aligned} \tag{26}$$

So if we know $E^a_{(i)}$ defined by

$$p_0 \langle p | \overline{\psi}_i(x) \, V_n \gamma_\nu \lambda^a \, \psi_i(x) | p \rangle = E^a_i \, p_\mu p_\nu - g_{\mu\nu} \, R^a_i \tag{27}$$

then L_Q is given by

$$L_Q = \frac{1}{2}\left(d^{Q_S Q_S c} \, E^c_S + d^{Q_U Q_U c} \, E^c_{(U)} + d^{Q_B Q_B c} \, E^c_B \right) \tag{28}$$

where

$$\begin{aligned} Q_S &= (3) + \frac{1}{\sqrt{3}} \, (8) + \sqrt{\frac{2}{3}} \, (0) \\ Q_U &= (3) + \frac{1}{\sqrt{3}} \, (8) + \sqrt{\frac{2}{3}} \, (0) \\ Q_B &= (3) + \frac{1}{\sqrt{3}} \, (8) - 2\sqrt{\frac{2}{3}} \, (0) \end{aligned} \tag{29}$$

To calculate E_i we derive from (27) the analogue of (11).

$$\alpha_0 \, d^{a0b} \, S^b_{(i)e} + \alpha_8 \, d^{a8b} \, S^b_{e(i)} = E^a_{e(i)} \left(M^0_e + \alpha_8 \, S^8_e \right) - 4 \, R^a_{e(i)} . \tag{30}$$

We now use SU"(3) symmetry to infer that

$$S^b_{(i)e} = S'^b_e \quad \text{and} \quad S^b_e = 3 \, S'^b_e$$

also

$$M^0_e = 3 \, M^0_{(i)e} = 3 \, M'^0_e \tag{31}$$

Using (31), (30) is written

$$\alpha_0 \, d^{a0b} \, S'^b_e + \alpha_8 \, d^{a8b} \, S'^b_e = 3 \, E'^a_e \left(M'^0_e + \alpha_8 \, S'^8_e \right) - 4 \, R'^a_e \tag{32}$$

Now (32) is identical to (11) if we identify E_e and S_e in (11) with $3E'_e$ and S'_e in (32) respectively. So $3E^a_e$ should be the same as E^a_e found in Section II. Putting this value into (28), we get, for the proton case,

$$L_Q = .31 + \frac{2}{3} \sim 1 \tag{33}$$

In (33), 2/3 is due to the last terms of (29), which are the result of the unusual form of the Gell-Mann-Nishijima formulae. Since the experimental value of L_Q is $\sim .3$, (33) certainly contradicts the experiment.

We remark here that for the free field case, we get similar results as (16) (for the SU(3) limit);

$$L^Q = \frac{\times 2 \langle p | \Sigma Q_i^2 \bar{\psi}_i \psi_i | p \rangle}{\langle p | \Sigma \bar{\psi}_i \psi_i | p \rangle} \tag{34}$$

Here i runs for each particle of the three triplets.

IV. DISCUSSIONS

The procedure of Sections II and III rests upon several assumptions which could not be justified very easily, if at all. One such assumption is that we can play with field products at the same space time points. The other related assumption is the validity of the Bjorken limit, which is essential for the derivation of the Callan-Gross sum rule. The former difficulty is well-known and discussed extensively in literatures.[15] The latter point is also well-known and is sully discussed by several authors.[16, 17] In these circumstances, it is rather surprising that we obtained so good an agreement with experiment using our naive gluon model. Also, in view of the rather speculative nature of the symmetry breaking scheme, the success of the model is remarkable and might indeed be fortuitous.

Without further discussing these highly nontrivial points, we notice that the discrepancy of Sections II and III is entirely due to the different form of the G-N formulae (8) and (23). The additional charm number term in (23) is essential to have correct spatial commutation relation specified by the condition of finite radiative correction to the β decay processes.[10]

Finally the free field result of Section II is of some interest from the point of view of the recently proposed weak PCAC scheme. If one can say that 2/3 is nearer to 1/3 than 0 is, then the free field result is in favor of the SU(3) symmetric weak PCAC scheme.

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