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ELASTIC HADRONIC PROCESSES, DUALITY AND ABSORPTION*

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ABSTRACT

A simple qualitative model for hadronic two-body amplitudes is proposed. The model combines the two-component theory of duality with the main idea of the absorption model. The qualitative predictions of the model for elastic differential cross sections and polarizations correctly reproduce several interesting regularities of the data. In particular, the presence of dips in nonexotic elastic processes and their absence in exotic reactions is explained.

Dedicated to the memory of Amos de Shalit, teacher and friend.

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I. Introduction

Prior to the introduction of duality (1) into hadron dynamics, high energy hadronic reactions were usually analysed only in terms of the allowed t-channel exchanges. On the basis of such an analysis, one would expect processes such as K^+p and K^-p elastic scattering to show a similar qualitative behavior at high energies, since both processes allow the same exchange mechanisms.

Duality tells us, however, that the s-channel quantum numbers are also relevant at high energies and that the t-channel exchanges should obey constraints which are imposed by the s-channel picture. Since K^+p and K^-p scattering have completely different s-channel quantum numbers, we then expect the s-channel picture to lead to a different qualitative behavior of these two processes at high energies. This different behavior is actually observed in the energy dependence of the total cross sections and in the angular distribution of the elastic cross sections.

In fact, it has been known for some time (2) that several empirical regularities relate the s-channel quantum numbers of hadronic elastic processes and the properties of the corresponding total and elastic cross sections at high energies. These regularities are summarized in Table 1.

The first regularity listed in the table is the approximate constancy of exotic total cross sections as opposed to the energy variation of nonexotic total cross sections. This correlation has been explained (3) in a very simple way within the framework of the "two-component theory" of duality in which the imaginary part of any hadronic amplitude is described as a sum of a Pomeron exchange term and a contribution of all s-channel resonances (3, 4).

The second regularity listed in the table has not been explained, so far. Empirically, the angular distributions for all nonexotic elastic hadronic processes exhibit dips somewhere between $t = -0.4$ and $t = -0.8$ BeV^2. The exotic processes do not show such dips. This behavior is schematically displayed in Figure 1, which should be considered as a rough guide rather than an accurate presentation of data. The dips appear at fairly low energies and slowly disappear as the energy is increased. One can always make the trivial statement that these dips are "caused" by the s-channel resonances, and that they should therefore appear only in nonexotic processes. This statement (which is probably correct) does not explain, however, why the dips are produced, why they appear around $t \sim -0.6$ BeV^2 and how they are related to the resonances.

In this paper we present simple qualitative answers to these questions within the framework of a simple dual theory involving Regge cuts or absorption corrections (depending on one's favorite language).

We will briefly touch on the third regularity of Table 1 — the shrinking of exotic elastic distributions and the absence of shrinking in nonexotic elastic processes.

II. A simple dual model including absorption

Following ideas that have been proposed several years ago (5) in the context of various optical or geometrical models, we construct a qualitative dual model, based on these assumptions:

(i) The imaginary part of any hadronic two-body amplitude is a sum of two components: The first component represents the total contribution of all s-channel resonances or the contribution of all "ordinary" (Non-Pomeron) t-channel

exchanges. The second represents Pomeron exchange or a nonresonant s-channel background. All s-channel resonances and all "ordinary" t-channel exchanges are assumed to be nonexotic. The "ordinary" exchanges include the exchange of a single trajectory or particle, as well as the exchange of a cut produced by a Pomeron and a single trajectory. Such cuts are also nonexotic.

(ii) The first component (resonances = ordinary exchanges) is dominated by the most peripheral partial waves within the radius of interaction ($R \sim 1f$). At a c.m. momentum k, the dominant contributions correspond to angular momenta $\ell \sim kR$. Qualitatively, this means that the first component of any s-channel helicity amplitude obeys[1]:

$$\mathrm{Im}\, f^{s}_{\lambda_1 \lambda_2, \lambda_3 \lambda_4} \propto "J"_{\Delta\lambda}\,(R\sqrt{-t})$$

where $\Delta\lambda$ is the absolute magnitude of the total s-channel helicity change and $J_{\Delta\lambda}$ is the ordinary Bessel function of order $\Delta\lambda$. $"J"_{\Delta\lambda}$ refers to a function which has the qualitative features of $J_{\Delta\lambda}$, namely — it has zeroes, minima and maxima at the same t-values. $"J"_{\Delta\lambda}$ could be, for example, $e^{At} J_{\Delta\lambda}(R\sqrt{-t})$ or $f(s)\, J_{\Delta\lambda}(R\sqrt{-t})$, etc. The details of $"J"_{\Delta\lambda}$ cannot be determined without additional assumptions concerning the energy dependence of the amplitude at fixed t-values or the precise relative strength of the various contributing partial waves. However, its general features are common to the many geometrical and optical models that assume the dominance of peripheral contributions (5).

(iii) The second component (Pomeron = background) involves significant contributions from all partial waves $\ell \leq kR$. The t-dependence of this component may or may not show structure (dips, peaks, zeroes). For example — equal contributions from all $\ell \leq kR$ partial waves would yield a dip around $t \sim -0.6$ BeV^2

(for $R \sim 1$ f) but a Gaussian distribution over the relevant partial waves would yield a structureless exponential t-dependence. In the absence of convincing theoretical arguments for one possiblity or the other, we shall appeal to experiment and show in the next section that the t-dependence of this term is probabily structureless.

(iv) The Pomeron (= background) term conserves the s-channel helicities (6). It therefore contributes only to $\Delta\lambda = 0$ amplitudes.

(v) The real parts of all amplitudes can, in principle, be determined from the corresponding imaginary parts (except for possible real polynomials). This can be done either by using fixed-t dispersion relations, or by applying theorems which relate the high energy behavior of an amplitude to its phase (7). We return to this point in section IV.

The qualitative picture which is outlined by these assumptions can be applied to elastic as well as inelastic hadronic reactions. Parts of it have been applied to inelastic processes in various forms by many people. Our assumptions here are consistent with most versions of the absorption model as well as with the basic ideas of duality. Note, however, that we apply the geometrical-optical ideas only to the imaginary part of the amplitude, insisting that the correct relation between energy dependence and phase is obeyed. This relations is often ignored in absorption models (5). Our motivation for restricting assumption (ii) to the imaginary part is based on the belief that the peripheral resonances are actually the dominant contributors. It is well known that resonance dominance assumptions can apply locally only to the imaginary part of the amplitude.

III. Elastic hadron reactions

The imaginary part of any elastic scattering amplitude should, in principle, include the resonance contribution as well as the Pomeron term. Using the assumptions of the previous section we learn that:

$$\mathrm{Im} f^{s}_{\Delta\lambda=0} = \text{"P"} + \text{"}J_0\text{"}$$

$$\mathrm{Im} f^{s}_{\Delta\lambda=1} = \text{"}J_1\text{"}$$

where "P" is the Pomeron contribution and "J_0", "J_1" have the features outlined in Figure 2.

At extremely high energies, the Pomeron term presumably dominates. At lower energies, we can define a "hierarchy" of contributions to the differential cross sections:

(A) Contributions which are entirely due to the Pomeron term — $(\text{"P"})^2$.

(B) Interference terms between the Pomeron and the resonances or the ordinary exchanges — ("P") ("J").

(C) Contributions which are entirely due to non-Pomeron terms — $(\text{"J"})^2$.

In the zeroth approximation the Pomeron term (A) determines gross features such as the approximate constancy of elastic cross sections or the small ratio between the real and the imaginary parts of the amplitude.

We propose here that the next approximation (which is presumably valid at moderate energies of several BeV's) is given by considering only contributions of the types (A) and (B). In that case the differential cross section of an elastic process will be roughly described by an expression of the form:

$$\frac{d\sigma}{dt} \sim ("P")^2 + 2("P")("J_0") .$$

where all contributions of type (C) are neglected and the real part of the Pomeron term is assumed to be small. In elastic processes with exotic s-channel quantum numbers, the "J_0"-term should vanish, since it represents the sum of all s-channel resonances. In such cases, if we neglect terms of type (C), we find:

$$\frac{d\sigma}{dt} \sim ("P")^2 .$$

The observed t-dependence of $\frac{d\sigma}{dt}(K^+p)$ and $\frac{d\sigma}{dt}(pp)$ indicates no dips or bumps for $|t| < 1$ BeV^2 (Figure 1). We therefore conclude that "P" has no structure[2], as already hinted in section II. The structure in K^-p, π^+p, π^-p and $\bar{p}p$ elastic scattering must therefore come from the ("P")("J_0") term which is present in these processes and absent in K^+p and pp scattering. Since "P" is positive and structureless, the entire dip-bump structure must come from "J_0".

What can we learn from the "J_0"-term (Figure 2a)?

(i) At $t = 0$, "J_0" must be positive since it is a sum of s-channel resonances in an elastic amplitude. Hence —

$$\frac{d\sigma}{dt}(K^-p)_{t=0} > \frac{d\sigma}{dt}(K^+p)_{t=0} ; \quad \frac{d\sigma}{dt}(\bar{p}p)_{t=0} > \frac{d\sigma}{dt}(pp)_{t=0}$$

(ii) At $t \sim -0.2$ BeV^2, "J_0" changes sign. Consequently,

$$\left[\frac{d\sigma}{dt}(K^-p) - \frac{d\sigma}{dt}(K^+p)\right], \left[\frac{d\sigma}{dt}(\pi^-p) - \frac{d\sigma}{dt}(\pi^+p)\right] \text{ and } \left[\frac{d\sigma}{dt}(\bar{p}p) - \frac{d\sigma}{dt}(pp)\right]$$

should all change sign around this t-value.[3]

(iii) At $t \sim -0.6$ BeV^2, "J_0" has a minimum (a maximum in absolute magnitude). Hence the elastic differential cross sections for K^-p, π^+p, π^-p and $\bar{p}p$ scattering should all have minima around this t-value. Furthermore, since at $t = 0$, $\frac{d\sigma}{dt}(\pi^-p) > \frac{d\sigma}{dt}(\pi^+p)$, the $t = -0.6$ dip in π^-p scattering should be deeper than that in π^+p scattering. The difference $\frac{d\sigma}{dt}(\bar{x}p) - \frac{d\sigma}{dt}(xp)$ should have a maximum in absolute magnitude at $t \sim -0.6$ (for $x = \pi^+, K^+, p$).

(iv) Since the energy dependence of the ("P")("J_0") term is different than that of the ("P")^2 term, all the above features should slowly disappear as the energy increases. $\frac{d\sigma}{dt}(\bar{x}p)$ should approach $\frac{d\sigma}{dt}(xp)$ and the $t \sim -0.6$ dip should disappear.

All of these qualitative predictions are obeyed by the data (Figure 1). The observed difference between the K^-p and K^+p cross sections, for example, resembles the schematic "J_0" of Figure 2a.

It is clear that our crude model is very far from being a quantitative theory. At the same time it is gratifying that a simple set of reasonable assumptions (none of which were specifically designed to explain the elastic data) is so successful. The presence of dips, their approximate positions, the crossover phenomenon and the relative sizes of the cross sections are all correctly predicted by this qualitative picture.[4]

IV. Polarization in elastic scattering

The polarization in elastic $\pi^\pm$ and $K^\pm p$ scattering is given by cross terms between the $\Delta\lambda = 0$ and $\Delta\lambda = 1$ amplitudes. The Pomeron by itself gives no polarization since it presumably contributes only to $\Delta\lambda = 0$. If we use the approximation of the previous section and neglect type (C) terms we find that the leading contribution to the polarization is given by:

$$\mathcal{P} \sim \frac{("P")(\mathrm{Re}\, f^{s}_{\Delta\lambda=1})}{("P")^{2}}$$

Any structure in the polarization will therefore be determined by $\mathrm{Re}\, f^{s}_{\Delta\lambda=1}$.

We can determine the real part of an amplitude at high energy from its imaginary part, if the asymptotic energy dependence is known. Assume that $\mathrm{Im}\, f \to \nu^{\alpha}$ as $\nu \to \infty$ for fixed t. We then know (7) that for a crossing-even amplitude, $(\mathrm{Re}\, f)/(\mathrm{Im}\, f) \to \cot \frac{\pi\alpha}{2}$ and for a crossing-odd amplitude $(\mathrm{Re}\, f)/(\mathrm{Im}\, f) \to \tan \frac{\pi\alpha}{2}$. Logarithmic terms in the energy dependence will not change this result, except for cases in which the ratio is infinity. In such a case a logarithm can make it finite and the ratio is ambiguous[5]. If we know the imaginary part and the energy dependence we can therefore predict the real part, for $-1 < \alpha < 1$ in the case of a crossing-odd amplitude and for $\alpha \neq 0$ in the case of a crossing-even amplitude. If our $f^{s}_{\Delta\lambda=1}$ is crossing-odd it will have the form:

$$\mathrm{Re}\, f^{s}_{\Delta\lambda=1} \sim ("J_1") \tan \frac{\pi\alpha(t)}{2}$$

where $\alpha(t)$ is the "effective" trajectory determined by the energy dependence of the crossing-odd contribution. We do not know much about the real part of the crossing-even amplitude since $\alpha = 0$ presumably occurs somewhere around $t \sim 0.5$ and logarithmic terms can have important influence[5]. The only polarization we can predict is therefore the <u>difference</u> between the $\pi^{+}p$ and $\pi^{-}p$ or $K^{+}p$ and $K^{-}p$ polarizations. We predict[6]:

$$\mathcal{P}(\pi^{+}p) - \mathcal{P}(\pi^{-}p) \sim \frac{("J_1") \tan \frac{\pi\alpha(t)}{2}}{("P")}$$

where $\alpha(t) \sim 0.5 + t$ is taken, say, from the data on $\pi^- p \to \pi^0 n$. A similar behavior is predicted for $\mathscr{P}(K^+p) - \mathscr{P}(K^-p)$.

"J_1" changes sign around $|t| \sim 0.6$ BeV^2 (Figure 2b). $\tan \frac{\pi\alpha(t)}{2}$ changes sign roughly at the same place. The polarization difference should therefore have a double zero[2,7] around $|t| \sim 0.6$ BeV^2. Figure 3 shows that the data for both πp and Kp polarization is not inconsistent[8] with this prediction.

V. Concluding remarks

We conclude with several remarks related to possible further applications of our naive model.

(a) The third regularity in table I together with our interpretation of the elastic cross sections indicate that the ("P")^2-term definitely shrinks at present accelerator energies. This is seen in the K^+p and pp elastic cross sections where, according to our approximation, only ("P")^2 contributes. The fact that nonexotic elastic distributions do not shrink is easily explained in the following way: The pure Pomeron term, ("P")^2, has a certain slope. The addition of the ("P")("J_0") term increases this slope (the slope in K^-p scattering is larger than the K^+p slope, etc.) As the energy increases, the slope of the ("P")^2-term increases, but the relative importance of the ("P")("J_0") term decreases. The two effects presumably cancel each other, thereby producing an approximately constant slope for $\frac{d\sigma}{dt}$.

(b) If the Pomeron term shrinks indefinitely, it should have a non-negligible real part at large t-values. If the slope of the Pomeron trajectory is 0.5 BeV^{-2}, the real part of its contribution could be about 40% of the imaginary part at $|t| \sim 0.5$ BeV^2. However, at least for $|t| < 1$ BeV^2, this real part is also structureless and the real part of the ("P")("J_0") term would probably be fairly small (if anything, it would tend to deepen the $t \sim -0.6$ dip).

(c) There are several interesting questions which we cannot answer without adding explicit quantitative assumptions to our model. These include the possible energy dependence of the positions of the dips or the crossover points, the explicit separation of the various contributions to $\frac{d\sigma}{dt}$, etc. These interesting questions are closely related to the possible energy dependence[9] of the interaction radius R. However, such an energy dependence is presumably at most logarithmic and its effects should be negligible as long as we maintain our present primitive level of discussion.

(d) Important structure is observed in pp elastic scattering at larger t-values ($|t| > 1$ BeV^2). These effects do not disappear at high energies. They may be associated with the Pomeron term. Our model cannot say much about this term. In fact — the only information about the t-dependence of "P" that we have used (the absence of structure at $|t| < 1$) was taken from the data and not from any theoretical consideration.

(e) Our picture can be translated into a quark language and the "J_0" term in $\frac{d\sigma}{dt}$ should be described by the duality diagrams (9). A crude quark counting assumption would then predict that the relative depths of the dips in K^-p, π^-p and π^+p elastic scattering obey a 2:2:1 ratio, respectively. We do not know how to isolate the "depth of the dip" but it seems that this prediction is, at least, not completely wrong (Figure 1). We do not dare to use quark assumptions in a dual model for $\bar{p}p$ scattering, in view of the famous difficulty of duality in baryon-antibaryon processes (10).

The author acknowledges many helpful discussions on the general features of absorption models with many people. Special thanks go to M. Kugler and Y. Zarmi.

Process	K^+p, pp	K^-p, $\bar{p}p$, π^+p, π^-p
s-channel quantum numbers	EXOTIC	NON-EXOTIC
s-channel resonances	NO	YES
High energy properties:		
I. "Flat" total cross-section	YES	NO
II. Dip in $\frac{d\sigma}{dt}$	NO	YES
III. Shrinkage of $\frac{d\sigma}{dt}$	YES	NO

Table I

Qualitative behavior of total and elastic cross sections.

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FOOTNOTES

1. This expression can be derived in several ways. See e.g. A. Dar, reference (5). Basically, the idea is that in the spinless case the contribution of angular momentum ℓ is given by $P_\ell(\cos\theta)$ which, at high energy and small θ is approximated by $J_0(R\sqrt{-t})$ where $\ell = kR$. In the case of spins P_ℓ is replaced by $d_{\lambda\mu}$ but the argument is essentially the same.

2. This contradicts the assumptions of G. Berlad, A. Dar and G. Eilam, reference (8). Their approach would encounter difficulties in K^+p and pp scattering. It would also contradict the energy dependence of the dip at $t = -0.6$ in πp scattering, since a dip caused by the Pomeron would not disappear at high energies.

3. This is the famous "crossover" effect.

4. Note that the dips are not caused by the vanishing of any term. They are actually produced as maxima in the absolute magnitude of the difference between the elastic particle and antiparticle cross sections.

5. These statements are true in spite of the fact that they are formulated in an extremely careless way from a mathematical point of view. If we start from a known imaginary part and multiply it by $\tan\frac{\pi\alpha}{2}$, the product must vanish arount $\alpha = 0$. Logarithmic terms can slightly change the position of the zero but not much else. However, when $\alpha = \pm 1$ we would get an infinite real part. Logarithmic terms may make it finite and the transition from a positive value of $\tan\frac{\pi\alpha}{2}$ to a negative value, which would normally occur via infinity,

could happen via zero. A predicted infinity could thus be changed by a logarithm into a predicted zero in the real part.

6. I thank V. F. Weisskopf for asking the right question that led to this prediction, during a discussion of polarization effects in absorption models.

7. Note that the double zero comes from the $\Delta\lambda = 1$ amplitude and not by multiplying two zeroes belonging to different $\Delta\lambda$'s.

8. I thank C. Michael for pointing out to me that the Kp polarization obeys this prediction.

9. The only absorption model which explicitely assumed an energy dependent radius is the Dar-Weisskopf model (reference 5). One can give several intuitive arguments for and against such an effect, and it seems that we shall have to wait for experiments at a few hundred BeV, before we can settle this issue.

FIGURE CAPTIONS

Figure 1: Schematic representation of the experimental differential cross-sections for elastic hadron scattering. The incident energies are marked for each curve. For a detailed display of the actual data see e.g. the compilation by Morrison, reference (2). The curves in the figure are free-hand drawings through data points and they are supposed to display only the main qualitative features. (See table I.)

Figure 2: Typical "J_0" and "J_1" functions. The relevant features are the zeroes, minima and maxima which are determined by the functions $J_0(R\sqrt{-t})$ and $J_1(R\sqrt{-t})$ with $R \sim 1$ f. "J_0" changes sign at $|t| \sim 0.2$ and has a minimum around $|t| \sim 0.6$. "J_1" vanishes at $t = 0$ and $|t| \sim 0.6$ and has a maximum around $|t| \sim 0.2$.

Figure 3: Data for the polarization differences, $\mathscr{P}(\pi^+p) - \mathscr{P}(\pi^-p)$ and $\mathscr{P}(K^+p) - \mathscr{P}(K^-p)$. The 2.74 GeV/c and the 6 GeV/c points are taken, respectively, from the first and second references in (11).

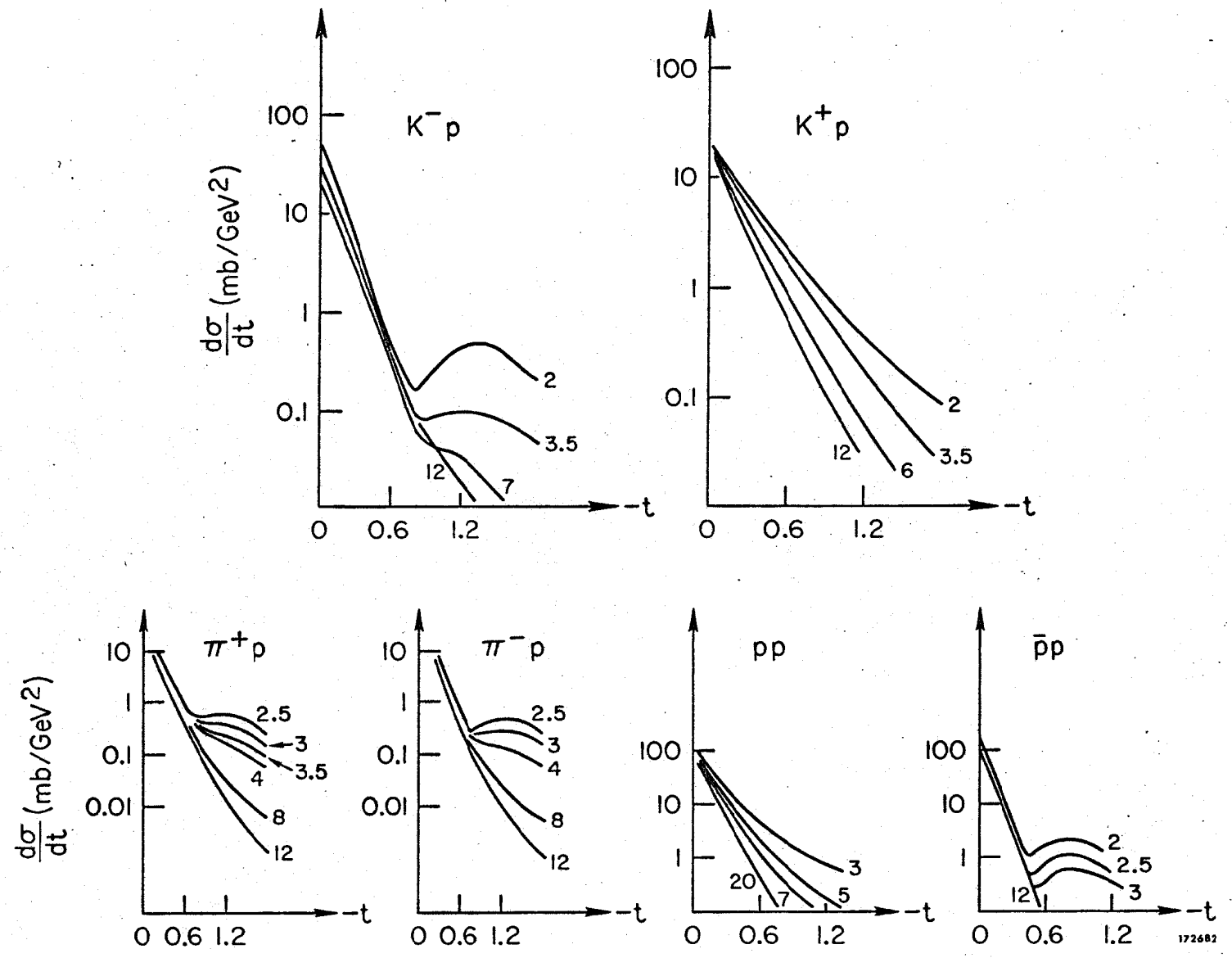
K^-p
K^+p
π^+p
π^-p
pp
$\bar{p}p$
$\frac{d\sigma}{dt}$ (mb/GeV^2)
-t
0
0.6
1.2
100
10
1
0.1
0.01
2
3.5
12
7
6
2.5
3
4
8
20
5
172682

Fig. 1

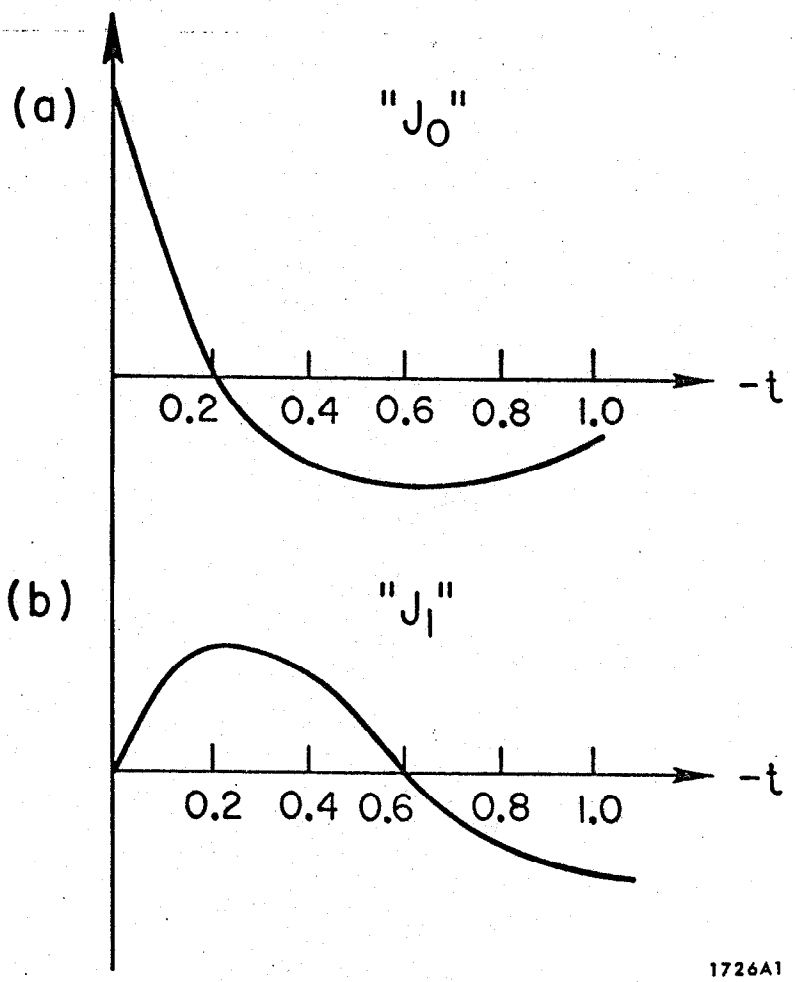

Fig. 2

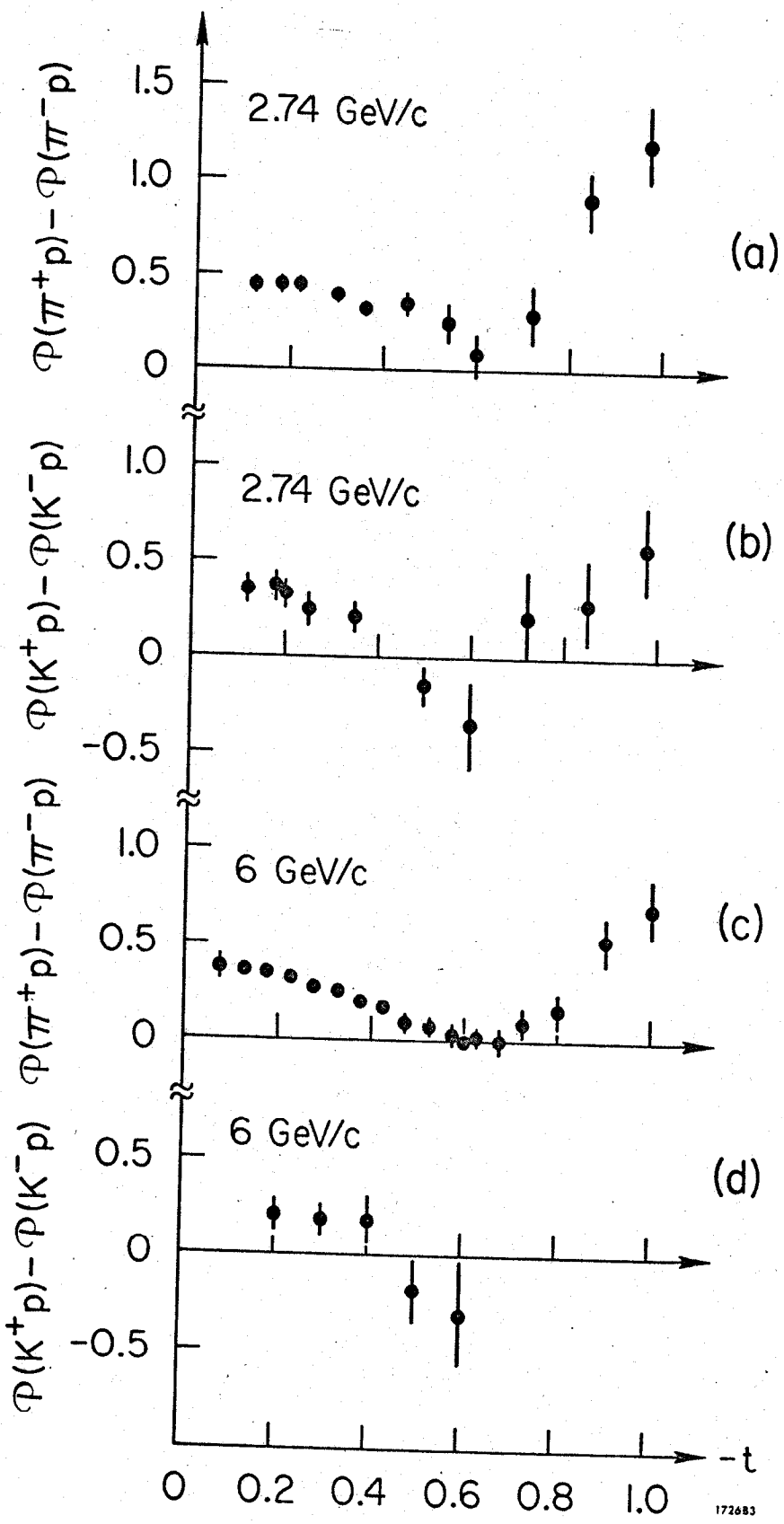
2.74 GeV/c
(a)
$\mathcal{P}(\pi^+p)-\mathcal{P}(\pi^-p)$
1.5
1.0
0.5
0
2.74 GeV/c
(b)
$\mathcal{P}(K^+p)-\mathcal{P}(K^-p)$
1.0
0.5
0
-0.5
6 GeV/c
(c)
$\mathcal{P}(\pi^+p)-\mathcal{P}(\pi^-p)$
1.0
0.5
0
6 GeV/c
(d)
$\mathcal{P}(K^+p)-\mathcal{P}(K^-p)$
0.5
0
-0.5
0
0.2
0.4
0.6
0.8
1.0
-t
172683

Fig. 3