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A POSSIBLE EXPLANATION FOR THE RAPID APPROACH TO "UNIVERSALITY" OF THE INELASTIC ELECTRON SCATTERING STRUCTURE FUNCTIONS*

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ABSTRACT

We have determined the single variable analyticity in the complex x-plane of the inelastic electron scattering structure functions, with s kept fixed and real, to all orders of Feynman perturbation theory. We find that its Landau singularities, which move as a function of s, rapidly approach their asymptotic s-independent position once s is large. We discuss how this observation offers a possible explanation for a rapid approach to "universality" of the inelastic electron scattering structure functions and shows that $sW_2(s,q^2/s)$ should "scale" faster than $\nu W_2(s,x)$.

Recent experimental data[1] on inelastic electron scattering indicate that for fixed x the structure functions W_1 and νW_2 become approximately independent[2] of s once s is far above the resonance production region ($s \geq 4$ GeV^2). We will call this region the "deep inelastic region". This fact has been referred to as "scaling" of the structure functions.[3,4] We consider two forms of "scaling". The first is the s independence of the magnitude of the structure functions for fixed $x=q^2/(2P\cdot q)$, which we call "universality of magnitude". The second is the s-independence of the shape of the curve of the structure functions versus x, which we call the "universality of shape".

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We propose that a rapid approach to a universal shape for the νW_2 (or W_1) curve for $s \gg 4$ GeV^2 can be understood as a consequence of the rapid approach of its physical x-sheet singularities to their s-independent asymptotic position once s is large enough. This is provided the "strengths" of these singularities (i.e.: residues of poles and discontinuities across cuts) are slowly varying functions of s for large s. We ignore spin and other quantum numbers since they affect only the "strengths" of these singularities and not their position. We analyze the Landau singularities of the Feynman integrals contributing to the non-Born term part of the inelastic structure function $W_i(s,x)$ for fixed real s. In a more detailed publication we will show to all orders of Feynman perturbation theory that the only Landau singularities on the physical sheet of the complex x-plane are the s independent normal q^2 threshold branch points (for real time like q^2) and the set of anomalous singularities like $x_\pm(s)$, which move with s, and correspond to the single loop box or triangle reduced graphs shown in Fig. 1. Their equation is given by[5]

$$q^2_\pm(s) \equiv \tau + \kappa - (M^2-\beta-\kappa)(s-\tau-\beta)/(2\beta) \pm (1/2\beta)[\lambda(M^2,\beta,\kappa)\ \lambda(s,\tau,\beta)]^{1/2} \qquad (1)$$

$$\lambda(x,y,z) \equiv [x-(\sqrt{y}+\sqrt{z})^2][x-(\sqrt{y}-\sqrt{z})^2] = \lambda(z,x,y),\ \text{etc.} \qquad (2)$$

$$x_\pm(s) = \eta_{M^2\pm}\Big/\left(1-\eta_{M^2\pm}\right) = -1/\omega_\pm(s) \quad \text{where} \quad \eta_\alpha(s) \equiv q^2/(s-\alpha) \qquad (3)$$

We define the class of variables η_α for various values of the real parameter α to show, later, that in general the quantity $(s-\alpha)\ W_2(s,\eta_\alpha)$ "scales" at much lower s than $\nu W_2(s,$ x or $\omega)$.

Since for a nonzero absorptive part $s > (\sqrt{z}+\sqrt{\beta})^2$ therefore $\lambda(s,\tau,\beta) > 0$. So in case (Euclidean) the lower vertex is internally and externally stable, then $\lambda(M^2,\beta,\kappa) < 0$ and $x_\pm$ represent a pair of complex conjugate ordinary anomalous Landau singularities. On the other hand, in case (pseudo-Euclidean) $\lambda(M^2,\beta;\kappa) > 0$

then $x_\pm$ represent a pair of virtual anomalous singularities on the time-like part of the real x-axis.

To understand the origin of these singularities we consider the contribution to W_i from a single peripheral graph leading to a two particle final state

$$W_i(s,q^2) \sim (\pi/2)\{\lambda(s,q^2,M^2)\}^{-1/2} \int db^2\, \delta^{(+)}(b^2-\beta) \int dt^2\, \delta^{(+)}(t^2-\tau) \int_{k^2_{min}(t^2,b^2)}^{k^2_{max}(t^2,b^2)} dk^2 (k^2-\kappa)^{-2} \tag{4}$$

The first integration of the double pole leads to a pair of simple poles $\left[\kappa - k^2\binom{max,}{min}\right]^{-1}$ which survive the remaining integration to give a pair of poles in the final answer. If the final state was not two particles or there were form factors at the vertices, then the $\delta^{(+)}$ functions would be replaced by less singular functions and the poles $\left[\kappa - k^2\binom{max,}{min}\right]^{-1}$ would be "smoothed" into a cut on successive integrations. The "nature" of the final singularity, therefore, will depend on the nature of the final state and form factors. However, its position always depends on the sum of the masses of various "legs" shown in Fig. 1.

From Eqs. (1), (2), and (3) we see that when s is large compared to a suitable combination of the internal masses the singularities $x_\pm(s)$ (or $\eta_{\alpha\pm}(s)$) approach their asymptotic positions $x_\pm(\infty)$ (or $\eta_{\alpha\pm}(\infty)$) in the complex x (or η_α) plane. The rapidity of this approach to "asymptopia" can be deduced from these equations.

Since no resonances are observed in the deep inelastic region we ignore the Born terms which give nonanalytic (delta function) contributions to the inelastic structure functions. If we assume[4] (a) that the "strengths" of these singularities and the asumptotic values $\nu W_2(s, |x| = \infty, 0)$ are finite and sufficiently slowly varying functions of s for such large s, then we may expect the $\nu W_2(s,x)$ (or $W_1(s,x)$) versus x curve to assume a universal shape rather rapidly. This is because one can see from the Cauchy's theorem that under our assumptions the significant variations in the shape of such curves are only caused by the motion of the dominant singularities.

To study the rapidity of approach to "universality" we note that

$$\left\{ \left| W_i(s,x) - W_i(\infty,x) \right| / \left| W_i(\infty,x) \right| \right\} \sim \sum_G \left| W_i^{(G)}(s,x) / W_i(s,x) \right| \theta(s - s_{th}(G)) \left| W_i^{(G)}(s,x) - W_i^{(G)}(\infty,x) \right| / \left| W_i^{(G)}(\infty,x) \right| \tag{5}$$

Here $W_i^{(G)}$ represents the contribution to the inelastic structure function from a given discontinuity diagram G. This corresponds to the production of an inelastic final state with threshold mass $\sqrt{s_{th}(G)}$. This shows that to approach universality at relatively small values of s, the discontinuity diagrams G with the smallest relative departure from universality must give the largest fractional contribution to the total inelastic structure function, at that value of s. We <u>assume</u> (b) that such a situation occurs.

It should be obvious that our two main assumptions have a lot of physical content. One may find partial justification for them in the asymptotic analyses of Refs. 4 and in the expectation of relatively low multiplicities of the final states discussed in Refs. 6. But we believe that justifying them in a realistic model at finite s is still an unsolved problem. Predominance of low threshold (or low multiplicity) final states at a given s would provide the experimental justification for the second assumption.

For further analysis of Eq. (5) we need to know the "strengths" and the "nature" of these singularities. We cannot determine these in general from our analysis since they depend on the nature of the couplings and final states. We do, however, show in Eq. (4) that the most singular situation (only as regards the "nature" of the singularity and not its "strength") occurs for the single loop Feynman graph for the virtual forward Compton (VFC) scattering amplitude.

Such graphs correspond to a peripheral production of two particle (or resonance) final state. For such graphs these singularities are a pair of simple poles for the spinless case. Incorporation of spin turns them into a pole plus a logarithm but the "strength" of the logarithmic singularity vanishes asymptotically. We can also show that Feynman graphs corresponding to single peripheral production (i.e., κ in Fig. 1 is a single particle or resonance) of final states with more than two particles lead to more singular situations than the other graphs producing the same final state. However, they need not always give an infinite singularity. Such theoretical questions and models for deep inelastic electron scattering based on analyticity will be discussed elsewhere.

To illustrate our mode of analysis we consider a specific example of peripheral production of two particle final state (ignoring all quantum numbers). For such graphs

$$\nu W_2^{(G)}(s,x) = (1+x)\ f(x)\ \left\{\lambda(s,\tau,\beta)/(s-M^2)^2\right\}^{1/2} \left\{(x-x_+(s))(x-x_-(s))\right\}^{-1} \tag{6}$$

where f(x) depends on the nature of the couplings. Gauge invariance and finiteness of photoproduction would demand $f(x) \sim x$ near $x=0$ for fixed finite s. Using physical values for the various possible exchanged and final state masses (like π, N, ρ, ω) it is very easy to obtain a rather good fit to the latest experimental data[1] with a sum of such terms with $f(x) = Cx(1+x)^2$. One of such (four parameter) slide rule fits for $s \rightarrow \infty$, $-1 \leq x \leq 0$ is:

$$\nu W_2(x) = -x(1+x)^3 \left\{(0.1)\left[x^2+0.022\right]^{-1} + (0.1)\left[(x+1)^2 + 0.022\right]^{-1} + \left[(x+0.337)^2 + 0.550\right]^{-1}\right\} \tag{7}$$

This is shown in Fig. 2. The first, second and third terms correspond to the contribution of π, N and ρ graphs, respectively, shown in Fig. 2. Such a fit cannot be expected to be realistic, unless one believes that the final inelastic state is dominated by peripherally[6] produced pairs of particles or resonances (which may decay to give more particles). But it does indicate the utility of our approach. It also shows that different exchanged masses (dashed curves) contribute differently to W_i for different x at a given s. Thus one pion exchange dominates near photoproduction (x or $q^2 \simeq 0$) while nucleon exchange dominates in the Bjorken region ($x \simeq -1$). Rho exchange dominates in the intermediate regions. For these reasons one cannot expect a single type of particle exchange to work in all of the inelastic domain. In interpreting this fit one should remember that at $s \to \infty$ the position of the singularity is independent of the mass $\sqrt{\tau}$. τ however does affect the rate of approach to universality. Also the singularities closest to the real axis have the three momentum $\left((2M)^{-1}\sqrt{\lambda(M^2,\beta,\kappa)}\right)$ of the exchanged particle (κ) in the laboratory frame (M at rest) very much smaller than β/M. This fact and the reduced diagrams have a rather striking resemblence to the "Parton picture".[4] In fact we believe that our approach may provide an invariant formulation of the "Parton picture".[4]

To study the rapidly of approach to universality for the "pole model" of Eq. (6) we consider the ρ-exchange term. This is the slowest case for our model. In Fig. 3 we plot for this term the relative departures from the asymptotic value ($\Delta\phi(s)/\phi \equiv [\phi(s)-\phi(\infty)]/\phi(\infty)$) of the following quantities

$$\sigma_\alpha(s) = \eta_{\alpha+}(s) + \eta_{\alpha-}(s); \quad d_\alpha(s) = \eta_{\alpha+} - \eta_{\alpha-}; \quad p_\alpha(s) = \eta_{\alpha+}\eta_{\alpha-} \tag{8}$$

These are useful for studying the rate of approach to scaling of W_i, which are of the form

$$F(s, \eta_\alpha) = \psi(\eta_\alpha)\, d_\alpha(s)/\{\eta_\alpha^2 - \eta_\alpha\, \sigma_\alpha(s) + p_\alpha(s)\} \tag{9}$$

This is because

$$\Delta F(s,\eta_\alpha)/F = \Delta d_\alpha(s)/d_\alpha$$

$$+\left\{\eta_\alpha^2 - \eta_\alpha\,\sigma_\alpha(\infty) + p_\alpha(\infty)\right\}^{-1}\left\{\eta_\alpha\,\sigma_\alpha(\infty)(\Delta\sigma_\alpha/\sigma_\alpha) - p_\alpha(\infty)(\Delta p_\alpha/p_\alpha)\right\}\left\{F(s)/F(\infty)\right\} \quad (10)$$

Similar expressions and quantities f_σ, f_d and f_p are defined for the variables x. The first term in Eq. (10) represents the "error" in the "strength" of the singularity and the second term the "error" in its "position".

These equations and Fig. 3 show that the rapidity of approach to universality depends on the variable chosen. From them we can see that the class of functions $(s-\alpha)\,W_2(s,\eta_\alpha)$ become universal at much lower value of s than the class $\nu W_2(s,\,x \text{ or } \omega)$. In general one can choose an optimum value of α which minimizes the overall error in $(s-\alpha)\,W_2(s,\eta_\alpha)$. This value of α will depend on a combination of masses and in general will be nonnegative. For our particular example of the ρ exchange term, $\alpha=0$ seems to give the most rapid approach to scaling. In fact for this case we find from Eq. (10) that

$$\Delta\left\{sW_2\right\}(s,\eta_0)/[\left\{sW_2\right\}(\infty,\eta_0)] \simeq \Delta d_0(s)/d_0 \quad \text{for} \quad \eta_0 \approx -1.5 \quad (11)$$

indicating that in terms of the variable $\eta_0 = q^2/s$ the "error" in $sW_2(s,\eta_0)$ just reflects the "error" in the residue which is given by the curve C_2 in Fig. 3. It shows that this "error" varies from about 29% at $s = 4.84\ \text{GeV}^2$ to about 7% at $s = 20\ \text{GeV}^2$. On the other hand if we use the variable x then at $x = -0.6$ the "error" at $s = 4.84\ \text{GeV}^2$ is 65% and at $s = 20\ \text{GeV}^2$ it is 16.5%. These are a factor of two larger than the "errors" in terms of the variable η_0. To compare with the latest experimental data we calculate the percentage variation in $\nu W_2(s,x=-0.6)$ from $s = 4.84$ to $11.44\ \text{GeV}^2$. We find this to be 50% which is a factor of two larger than the variation shown by the fit to experimental data shown in Fig. 14 of Refs. 1, and it is of the opposite sign. The sign difference could be due to the "tails" of the resonance or due to a different functional form for W_i.

From all this we conclude that it is possible for the inelastic electron scattering structure functions to become universal at relative low values of s. Our equations show no single critical mass which determines the relative departure from universality as $s_{critical}/s$. We find $s_{critical}$ to be a combination of the squares of the masses $\sqrt{\tau}$, $\sqrt{\beta}$, $\sqrt{\kappa}$, M. We also find that the class of variables $(s-\alpha)\ W_2(s, \eta_\alpha)$ can become universal at much lower s than $\nu W_2(s, x \text{ or } \omega)$. $\alpha=0$ seems to give the best case. This in fact could be the possible reason why "scaling" occurs sooner[7] in terms of the variable $\omega' = \omega + M^2/q^2 = 1 + s/q^2$ than ω. Because of the ad hoc nature of our assumptions and the disparity in the values of the predicted and observed departures from universality we cannot claim to have explained the observed rapid approach to universality. But we do feel that our analysis offers a possible mechanism for a rapid, rather than asymptotic, approach to universality by the inelastic electron scattering structure functions.

We wish to thank Professors Sidney Drell, Michael Nauenberg and Dr. Nikola Jurisic for helpful discussions.

REFERENCES AND FOOTNOTES

1. E. D. Bloom et al., Report No. SLAC-PUB-796 (1970).

2. Virtual photon momentum is q and target momentum is P. Our metric is (1,-1,-1,-1). $\nu \equiv (P\cdot q)/M$, $s = (P+q)^2$, $t = (P-P)^2 = 0$, $u = (P-q)^2$, $x = -1/\omega = q^2/(2M\nu)$.

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7. E. D. Bloom and F. J. Gilman; Report No. SLAC-PUB-779 (1970); M. N. Nauenberg, 1970 Kiev conference talk.

FIGURE CAPTIONS

1. Typical reduced Feynman graphs leading to the anomalous box or triangle singularity at t=0. $\sqrt{\tau}$, $\sqrt{\kappa}$, $\sqrt{\beta}$ represent the sum of the masses of the internal lines shown in the reduced graph.

2. The "pole model" fit for $\nu W_2(x)$ at $s=\infty$. The experimental points are from Table III of Ref. 1.

3. Curves showing the s dependence of the parameters which determine the relative departure from universality.

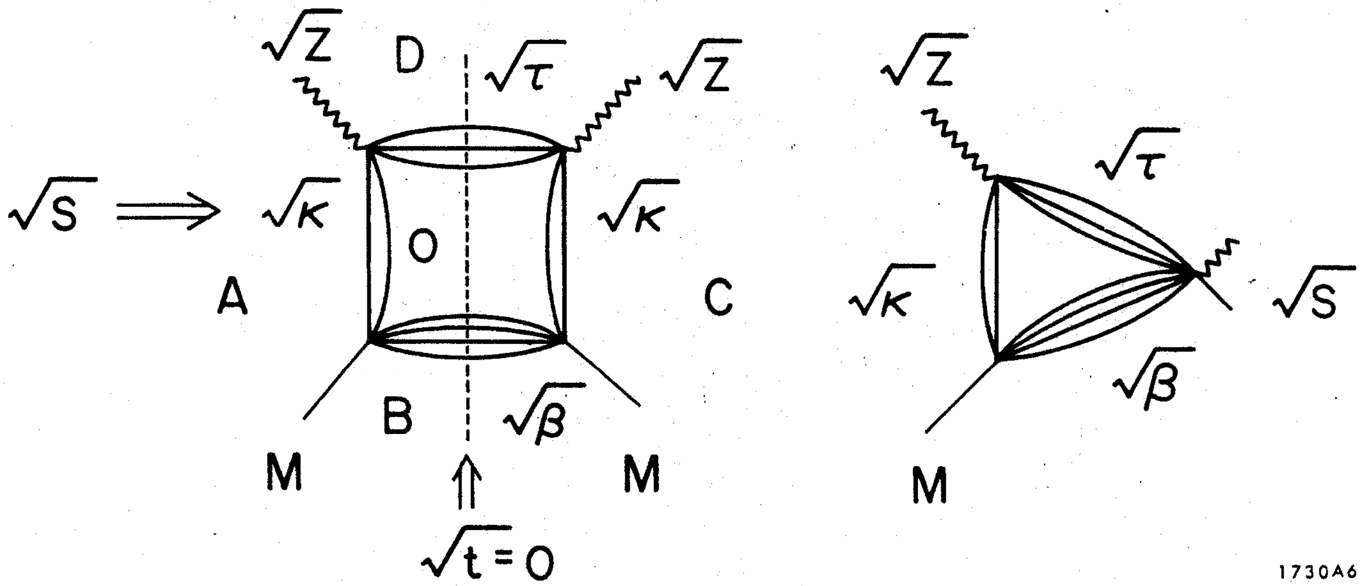
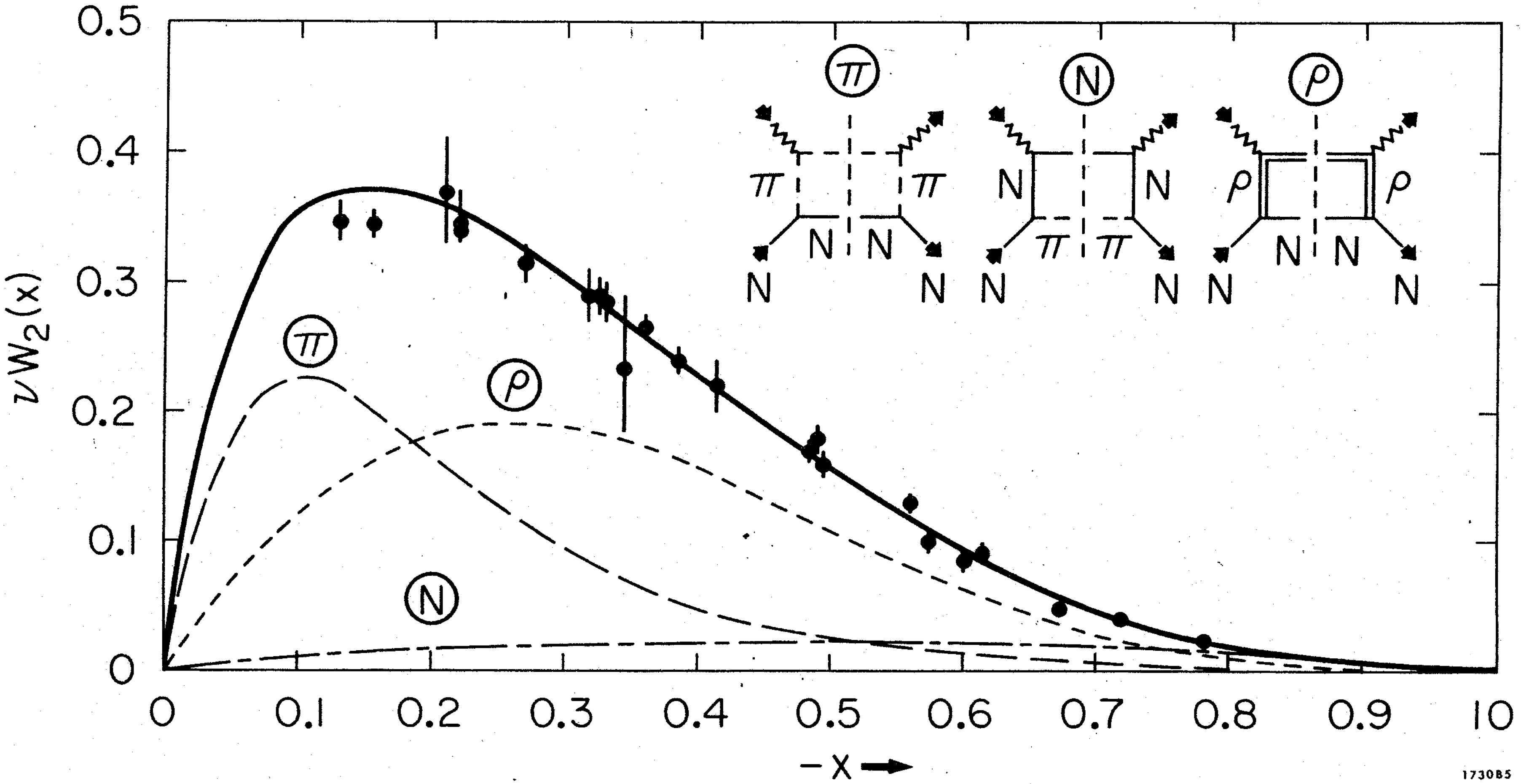


Fig. 1



0.5
0.4
0.3
0.2
0.1
0
$\nu W_2(x)$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
— X →
π
ρ
N
π
π
π
N
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π
π
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ρ
ρ
ρ
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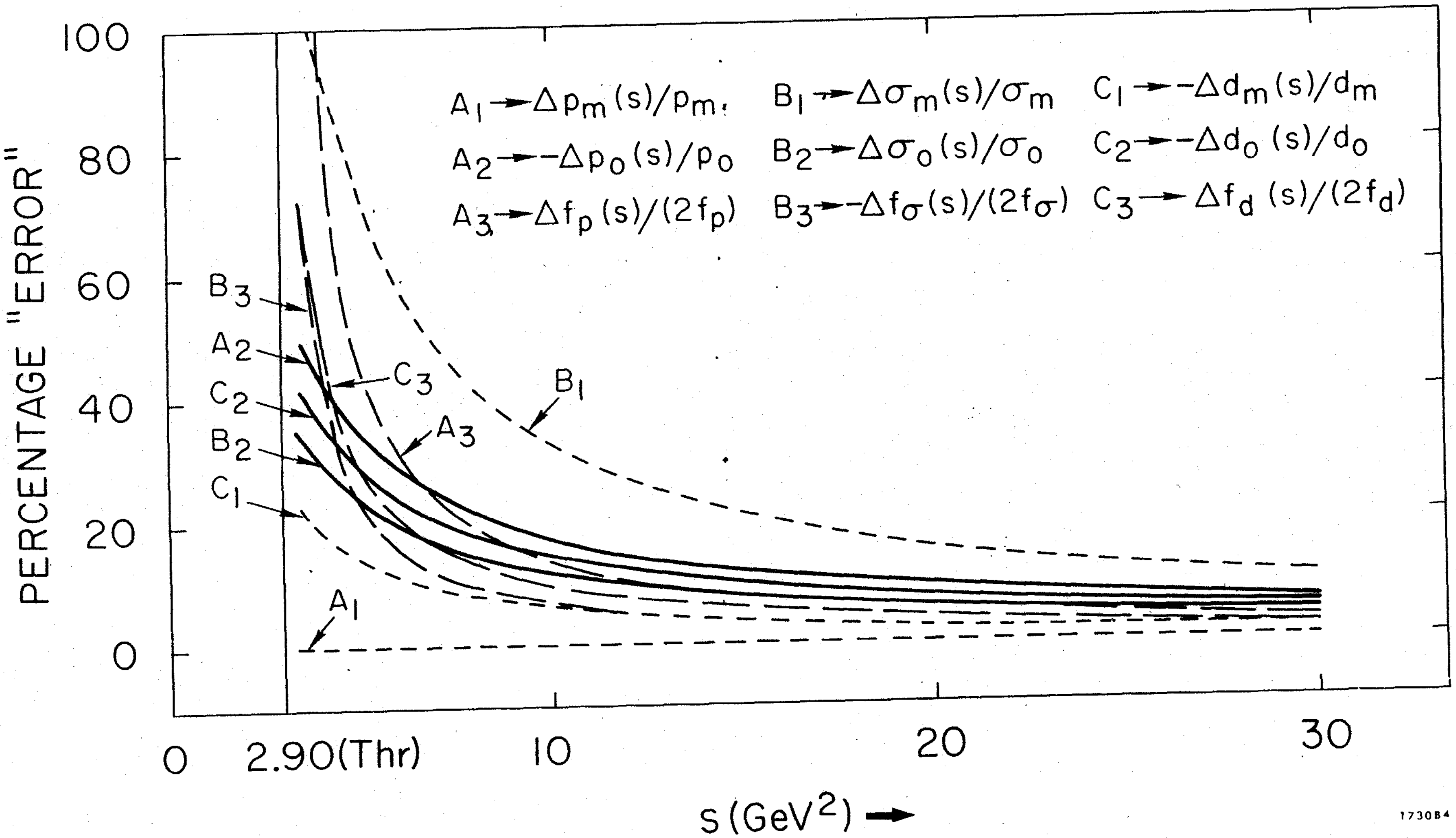
N
1730B5

Fig. 2



PERCENTAGE "ERROR"
100
80
60
40
20
0
A1 → Δpm(s)/pm,
B1 → Δσm(s)/σm
C1 → −Δdm(s)/dm
A2 → −Δpo(s)/po
B2 → Δσo(s)/σo
C2 → −Δdo (s)/do
A3 → Δfp (s)/(2fp)
B3 → −Δfσ(s)/(2fσ)
C3 → Δfd (s)/(2fd)
B3
A2
C2
B2
C1
A1
C3
A3
B1
0
2.90(Thr)
10
20
30
s(GeV2) →
1730B4

Fig. 3