ADDENDA AND CORRIGENDA TO SLAC-PUB-817

THE QUARK-PARTON MODEL AND THE NEW ELECTRON DEUTERON SCATTERING DATA

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page 5: Add Ref. 11 after the words "... significant role."

page 5: Last line: insert "many" before "predictions."

Replace pages 7 - 10 in your copy of this paper with the new pages 7 - 11 attached.

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ERRATUM

The theorem $|\Delta| \leq 1/3$ in this paper is incorrect. This was pointed out by J. S. Bell who provided a counter example. Coherent superpositions can be constructed out of the states considered with the particle labels interchanged; the theorem fails because of cross-terms in this case.

It is true, however, that if $\Delta \neq 1/3$ the models considered previously are excluded.

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ABSTRACT

An inequality is derived which is valid in all quark parton models. The new (preliminary) electron deuteron scattering data satisfies the inequality. The fact that the inequality is apparently not saturated indicates that rather complicated quark configurations (not previously considered) must be present in this model. Implications are discussed.

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I. AN INEQUALITY

Recent highly inelastic electron scattering data[1,2] can be interpreted in terms of a simple model[3,4,5] in which the nucleon is supposed to behave as a free gas of bare constituents (or 'partons') from which the electron scatters incoherently. In order to obtain the results of current algebra it is natural to assume that the partons which participate in the weak and electromagnetic interactions are quarks. Five relations are known to follow from these assumptions[6] (see Eqs. (2-6) below); a further inequality follows from the requirement that the gas of quarks which form the nucleon belong to an isodoublet:

$$\left| \int \frac{d\omega}{\omega} \left(F_2^{\gamma p}(\omega) - F_2^{\gamma n}(\omega) \right) \right| \leq \frac{1}{3} \tag{1}$$

where

$$F_2(\omega) = \lim_{\nu \to \infty} \nu W_2$$

$$\omega = \frac{2\nu}{-q^2} \quad \text{fixed}$$

in the standard notation (see, e.g., Ref. 5)(we take $M_p = 1$).

To prove this result, recall that in all parton models[5]:

$$\int \frac{d\omega}{\omega} F_2(\omega) = \sum_N P_N \sum_{i=1}^{N} Q_i^2$$

where

P_N — probability of there being N partons

Q_i — charge of the ith parton in the N parton configuration.

(Note that this expression may be infinite.) Consider now

$$\Delta_N = \sum_{i=1}^{N} Q_i^2 \Big|_{\text{proton}} - \sum_{i=1}^{N} Q_i^2 \Big|_{\text{neutron}}.$$

λ quarks, being isosinglets, do not contribute to Δ_N and may be ignored. The proton must be built from n + 3 proton and neutron quarks (described by isospinors q^a) and n proton and neutron antiquarks (q_b). In reducing the product state:

$$q^{a_1} q^{a_2} \ldots q^{a_{n+3}} q_{b_1} \ldots q_{b_n}$$

the indices may be contracted with the invariant SU(2) tensors ϵ^{ab}, ϵ_{cd} or δ^e_f. Two sorts of isodoublet configuration can be formed:

1) A state with one upper index uncontracted; in the proton this index must correspond to a p type quark:

$$\begin{aligned} \text{proton} &\sim p \otimes \text{SU(2) scalar} \\ &\sim p(pn - np)^{m+1} (\overline{pn} - \overline{np})^m (p\bar{p} + n\bar{n})^{n-m}. \end{aligned}$$

In this case: $\Delta_N = 1/3$.

2) A state with one lower index uncontracted:

$$\begin{aligned} \text{proton} &\sim \bar{n} \otimes \text{SU(2) scalar} \\ &\sim \bar{n}(pn - np)^{m+2} (\overline{pn} - \overline{np})^m (p\bar{p} + n\bar{n})^{n-m-1}. \end{aligned}$$

In this case: $\Delta_N = -1/3$.

If the N parton state has a fraction $1 - \epsilon_N$ of configuration 1 and ϵ_N of configuration 2 ($1 \geq \epsilon_N \geq 0$) then:

$$-\frac{1}{3} \leq \Delta_N = \frac{1}{3} - \frac{2\epsilon_N}{3} \leq \frac{1}{3}$$

which gives Eq. (1).

In the model of Drell, Levy, and Yan (DLY)[7] the partons are pions and (integrally charged) nucleons and antinucleons. As far as their SU(2) properties are concerned the pions may be regarded as nucleon-antinucleon bound states, as in the Fermi-Yang model. When $\sum_i Q_i^2$ is constructed the contribution of the π^0's depends crucially on whether they are 'fundamental' or bound states; their contribution to Δ_N is zero, however, since there must be an equal number of π^0's in the proton and neutron. The DLY model and the Fermi-Yang model are therefore equivalent as far as Δ_N is concerned; the argument above can be applied and gives 1 on the right-hand side of Eq. 1 for both models.

II. COMPARISON WITH THE DATA

The new (preliminary) results give :[1]

$$\Delta = \int_1^\infty \frac{d\omega}{\omega}\left(F_2^{\gamma p} - F_2^{\gamma n}\right) = 0.13 + \frac{0.03}{\alpha}$$

where $F_2^{\gamma p} - F_2^{\gamma n} = 0.03\left(\frac{12}{\omega}\right)^{\alpha}$ has been used in the unmeasured region $12 < \omega < \infty$. Taking $\alpha = 1/2$ (as suggested by Regge models) and errors of 40% (as estimated for the contribution of the measured region in Ref. 1);

$$\Delta = 0.19 \pm 0.08$$

and the inequality is satisfied. Defining

$$\epsilon = \sum_N P_N \epsilon_N$$

then

$$\epsilon = 0.22 \pm 0.12 \quad \text{(quark models)}$$

$$= 0.40 \pm 0.04 \quad \text{(DLY model).}$$

The fact that apparently $\epsilon \neq 0$ in the quark model excludes the simple three quark model,[8] the model with three quarks plus an SU(3) symmetric $q-\bar{q}$ sea and other simple models which have been considered.[6,9] Although we do not know what occurs for $\omega > 12$ so that $\epsilon = 0$ is not definitely excluded we shall assume $\epsilon \neq 0$ below.

Before continuing, it is appropriate to remark that the new data[1] can be fitted by

$$R = 0.18 \pm 0.05 \quad \text{or} \quad R = \frac{-q^2}{\nu^2} \quad \text{or} \quad R = 0.031\ q^2/M_p^2$$

(and probably several other forms) where $R = \frac{\sigma_L}{\sigma_T}$. It is often stated that models with spin 1/2 constituents require $R = 0$ (which is unlikely but not impossible experimentally[1]); this is strictly only true, however, in the limit $\nu \to \infty$ with ω fixed.[10] For finite q^2/ν^2 the parton model is not applicable and R depends on the partons' mass etc. It is amusing to note, however, that the conventional method of calculation[5] (which, if taken literally outside its domain of validity, implies that the photon interacts only with light partons of mass $-q^2/2\nu$) gives $R = -q^2/\nu^2$ in agreement with the data.

The DLY model gives

$$z \, \frac{\nu^2 W_2}{-q^2} = W_1$$

with

$$z = \frac{\sum_N P_N \sum_i{}' Q_i^2 \, f_i^N(1/\omega)}{\sum_N P_N \sum_i Q_i^2 \, f_i^N(1/\omega)} \, ,$$

$f_i^N(x)$ — probability of parton i in the N parton configuration carrying a fraction x of the proton's longitudinal momentum,

$\sum'$ — runs over the nucleons and antinucleons

$\sum$ — runs over the nucleons, anitnucleons and pions.

$z = 1$ is expected at $\omega = \infty$ (where the nucleon behaves as a single bare nucleon in this model) but $z < 1$ must obtain whenever the pions play a significant role. The data (Table III of Ref. 1) indicates that for $1 < \omega < 12$, $z \simeq 1$ (albeit with large errors). We shall therefore only consider the quark parton model henceforth.

III. IMPLICATIONS

The complicated configuration mixing implied by the fact that $\epsilon \neq 0$ and the fact that the value of $\langle Q^2 \rangle$ requires the presence of neutral particles[5] makes it hard to make predictions beyond those which must be true in all quark parton models:

$$2F_1 = \omega F_2 \qquad \text{Callan Gross relation}^{10} \tag{2}$$

(for γ, ν, or $\overline{\nu}$ scattering)

$$\int \frac{d\omega}{\omega}\left(F_2^{\nu n} - F_2^{\nu p}\right) = 2 \qquad \text{Adler sum rule}^{12} \tag{3}$$

$$-\int \frac{d\omega}{\omega^2}\left(F_3^{\nu p} + F_3^{\nu n}\right) = 6^{(13)} \tag{4}$$

$$12\left(F_1^{\gamma p} - F_1^{\gamma n}\right) = F_3^{\nu p} - F_3^{\nu n} \tag{5}$$

$$F_2^{\gamma p} + F_2^{\gamma n} \geq \frac{5}{18}\left(F_2^{\nu p} + F_2^{\nu n}\right) \tag{6}$$

Equations (2-4) have been derived from more general considerations in the references indicated (Eq. (2) actually only for $F^{\gamma p}$, $F^{\gamma n}$ and $F^{\nu p} + F^{\nu n (13)}$) while the more general gluon model gives[6]:

$$12\int \frac{d\omega}{\omega^3}\left(F_1^{\gamma p} - F_1^{\gamma n}\right) = \int \frac{d\omega}{\omega^3}\left(F_3^{\nu p} - F_3^{\nu n}\right) \tag{7}$$

$$\int \frac{d\omega}{\omega^2}\left(F_2^{\gamma p} + F_2^{\gamma n}\right) \geq \frac{5}{18}\int \frac{d\omega}{\omega^2}\left(F_2^{\nu p} + F_2^{\nu n}\right) \tag{8}$$

Equations (7) and (8) are the most simply tested consequences of Eqs. (5) and (6) since these are the integrals which occur in the expressions for the total neutrino cross sections (a weaker form of Eq. (8), which is valid more generally, is given in Ref. 14). The neutrino structure functions are defined here by:

$$\lim_{\substack{\nu \to \infty \\ \omega \text{ fixed}}} \quad W_1 = \cos^2\theta_c \, F_1 + \sin^2\theta_c f_1$$

$$\lim_{\substack{\nu \to \infty \\ \omega \text{ fixed}}} \quad \nu W_2 = \cos^2\theta_c \, F_2 + \sin^2\theta_c f_2$$

$$\lim_{\substack{\nu \to \infty \\ \omega \text{ fixed}}} \quad \nu W_3 = \cos^2\theta_c \, F_3 + \sin^2\theta_c f_3$$

where θ_c is the Cabibbo angle which we take to be zero henceforth (a large number of sum rules for the f_i are catalogued in footnote 72 of Ref. 9).

The further reasonable assumption that the functions $f_i^N(x)$ are not too different for quarks and antiquarks allows us to make a few additional (weak) predictions. In this case $\sigma^{\bar{\nu}n} < \sigma^{\nu p}$ and $\sigma^{\bar{\nu}p} < \sigma^{\nu n}$. Configuration 1 discussed above gives $\sigma^{\gamma p} > \sigma^{\gamma n}$, $\left|F_3^{\nu n}\right| > \left|F_3^{\nu p}\right|$ and $\sigma^{\nu n} > \sigma^{\nu p}$ while the inequalities are reversed for configuration 2. Since $\sigma^{\gamma p} > \sigma^{\mu n}$ in the measured region we expect $\sigma^{\nu n} > \sigma^{\nu p}$ and $-\left(F_3^{\nu p} + F_3^{\nu n}\right) > -\left(F_3^{\nu n} - F_3^{\nu p}\right) = 12\left(F_1^{\gamma p} - F_1^{\gamma n}\right)$. Hence the data (extrapolated to $\omega = \infty$ as above) gives[15]

$$\sigma^{\nu p} + \sigma^{\nu n} - \sigma^{\bar{\nu}p} - \sigma^{\bar{\nu}n} = \frac{2G^2ME}{3\pi} \int \frac{d\omega}{\omega^3}\left(F_3^{\nu p} + F_3^{\nu n}\right) > (0.17 \pm 0.10)\,\frac{G^2ME}{\pi}$$

Together with the CERN data[16] this implies that

$$\frac{\sigma^{\bar{\nu}p} + \sigma^{\bar{\nu}n}}{\sigma^{\nu p} + \sigma^{\nu n}} < 0.83\begin{array}{l}+0.10\\-0.13\end{array}$$

Since we do not expect this inequality to be saturated the difference between ν and $\bar{\nu}$ cross sections is therefore probably sufficiently large to be measurable in the forthcoming CERN experiment.

Besides requiring the nucleon to be a member of an isodoublet we can demand that it belongs to an SU(3) octet. In this case there are three types of configurations:

1) proton $\sim$ ppn $\otimes$ S

2) proton $\sim (p\lambda) \otimes \{pn\lambda\} \otimes S$

3) proton $\sim (\bar{n}\bar{\lambda}\bar{\lambda}) \otimes \{pn\lambda\} \otimes \{pn\lambda\} \otimes S$

where S is an SU(3) scalar state:

$$S \sim \{pn\lambda\}^a \; \{\bar{p}\bar{n}\lambda\}^a \quad (p\bar{p} + n\bar{n} + \lambda\bar{\lambda})^b$$

and $\{\ \}$ implies antisymmetrization.

This does not lead to useful new results beyond the statement that the presence of configuration 3 (required by $\epsilon \neq 0$) and the indication that $a \neq 0$ or $b \neq 0$ (required by the relative smoothness of νW_2 at large ω) implies that the inequality of Eq. (6) will not be saturated (which would occur if there were no λ or $\bar{\lambda}$ quarks); this agrees with the data available.

Finally we remark that if indeed $\epsilon \neq 0$ the parton model may not be very useful except as an heuristic device. In any case neutrino experiments which can test Eq. (2 - 8) will be of great interest.

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11. I am indebted to Prof. S. D. Drell for pointing this out to me.

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16. I. Budagov et al., Phys. Letters 30B, 364 (1969). Note that Eq. (4) is wrong and does not follow from Eq. (3). We find σ/nucleon = $(0.51 \pm 0.13)\,\frac{G^2 ME}{\pi}$. The error is only statistical and systematic errors may be considerable. Since the propane target does not contain an exactly equal number of neutrons and protons we have implicitly assumed in the text that $\sigma^{\nu n}$ and $\sigma^{\nu p}$ are not too different in the scaling region.