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PHENOMENOLOGICAL PREDICTIONS FOR DEEP INELASTIC ELECTRON SCATTERING**

D. M. Ritson*
Physics Department
University College
London, England

ABSTRACT

With preliminary information from colliding beams relative to the coupling of photons with "high mass" hadron states, we have extended the 1960 Drell model for photons interacting with nucleons, to estimate off the mass-shell photon cross sections. Based on kinematic considerations as to the domain in which the propagator of the virtual intermediate particle is large, one can predict cross sections and configurations for the final produced events.

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*On Sabbatical leave from the Physics Department and the Stanford Linear Accelerator Center, Stanford, California.

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The theory for inelastic scattering off nucleons can be formulated in terms of $\sigma_t(K,Q^2)$ and $\sigma_o(K,Q^2)$, the total cross sections for transverse and longitudinal or scalar "off the mass-shell" photons interacting with nucleons[1]. K is the energy of an "on the mass-shell" photon that produces the same C. of M. energy of the final hadron system, and Q^2 is the negative of the four momentum transfer squared.

The most natural class of theories to account for these cross sections has been the Vector Dominance theories based on the assumption that the ρ^o, ω and ϕ mesons saturate the photon coupling[2].

The rough prediction of Vector Dominance models is:

$$\sigma_t(K,Q^2) \approx \left(\frac{m_\rho^2}{m_\rho^2 + Q^2}\right)^2 \cdot \sigma_t(K,0) \qquad (1)$$

whereas the experimental observation is[3]

$$Q_t(K,Q^2) \approx \left(\frac{m_\rho^2}{m_\rho^2 + Q^2}\right) \cdot \sigma_t(K,0) \qquad (2)$$

In view of the fact that Vector Dominance is approximately true on the mass-shell, it would be surprising if this theory did not remain approximately valid out to Q^2s of the order of m_ρ^2. At high Q^2s however the contribution to the cross sections from the ρ, ω and ϕ mesons should have become small and it is necessary to look elsewhere to account for the experimental results.

In the belief that the photon coupling was almost completely saturated by the vector mesons, most theories have postulated a new form of fundamental interactions involving partons or constituent-like substructures of the nucleon[4,5,6]. These theories are decoupled from other strong interaction theories and involve a number of implicit as well as explicit assumptions. For example, it is implicitly assumed that the "effective mass" of a constituent does not change after a collision, and for those theories which involve fractionally charged, or spin one half, constituents it is assumed that these constituents are prevented by some mechanism from appearing in the lab. However, with the successful operation of the Adone e^+e^- colliding beam storage ring, we now know that the photon does indeed couple strongly to "high mass" hadron states other than the ρ, ω and ϕ mesons(7). The preliminary observations suggest that multipion final states are produced with cross sections of the order of those expected for "point" Dirac particles.

The purpose of this note is to explore whether by using this fact we can explain inelastic electron scattering in conventional terms. We shall write the total cross section in the form:

$$\sigma_t(K,Q^2) \approx \sigma_t(K,0) \cdot \left[.75\left(\frac{m_\rho^2}{m_\rho^2 + Q^2}\right)^2 + .25 f(K,Q^2)\right] \qquad (3)$$

The first term on the R.H.S. of equation (3) is assumed to be the Vector Dominance contribution and the second term is that due to the higher mass states coupled to the photon. The factors .75 and .25 are estimates based on the present tests as to the extent to which the Vector Dominance holds. The $f(K,Q^2)$ is evaluated below in the spirit of the 1960 Drell model(8) for evaluating total photon nucleon cross section.

Fig. 1 shows the Drell model, diagrammatically. In this model "on the mass-shell" photons couple to a pion pair, one member of which is real and the other virtual but close to the mass-shell. This virtual pion then interacts with the nucleon. This theory assumed the pion pair interacted with the photon via a point like cross section. (Vertex coupling of e), and predicted

$$\sigma_{total}\left(\gamma P, Q^2=0\right) \sim e^2 \cdot \sigma_{total}\left(\pi P\right) \qquad (4)$$

Fig. 2 shows the analogous process for an off the mass-shell photon. The photon is assumed to interact with a group of bosons with an effective coupling constant of e. One of the bosons is virtual and procedes to the second vertex where it interacts with the nucleon. The photon has energy and four momentum (ν, Q^2), the group of produced bosons energy $(1-r)\nu$, and the intermediate particle $(r\nu, t)$ and mass μ. As has been pointed out by West[9] the domain for which this diagram Fig. 2 gives an appreciable contribution should be determined by the kinematic region for which the propagator of the virtual particle is large. At large values of Q^2 and ν the minimum value of the four momentum squared t_{min} is given by[9]:

$$|t_{min}| \approx \frac{x}{(1-x)} \left[(M_n^2 - M^2) + x M^2\right] \qquad (5)$$

where the parameter x is the conventional $x = Q^2/2M\nu$, M_n is the residual mass of the products at the nucleon vertex, and M is the nucleon mass.

In terms of r, the energy partition factor

$$M_n^2 = 2r\nu M + M^2 + t_{min} \qquad (6)$$

and by substitution into equation (5)

$$t_{min} = -x\left[2r\nu M + xM^2\right] \qquad (7)$$

If $|t_{min}| < \mu^2$ the intermediate particle is close to the mass-shell. If $|t_{min}| > \mu^2$ the propagator of the intermediate particle will be small and the contribution to the cross section should be small. We first consider values of $x<\mu/M$. The range r values which lead to $|t_{min}| \leq \mu^2$ and which thus contribute appreciably to the cross section is then given by:

$$r \leq r_{max} = \left(\frac{\mu^2}{2x\nu M} - \frac{xM}{2\nu}\right) \tag{8}$$

with $$r_{max} \approx \frac{\mu^2}{2x\nu M} = \frac{\mu^2}{Q^2}$$

To estimate the cross section σ'_t due to states other than the ρ, ω, and ϕ we assume a one dimensional phase space distribution for r of the form $(\langle n\rangle - 1)(1 - r)^{\langle n\rangle-2}\,dr$ where $\langle n\rangle$ is the average multiplicity. Then if the contribution to the cross section comes predominantly from $|t_{min}| \leq \mu^2$.

$$\frac{\sigma'_t(K,Q^2)}{\sigma'_t(K,0)} \approx \frac{\int_0^{t_{max}} (1-r)^{\langle n\rangle-2}dr}{\int_0^1 (1-r)^{\langle n\rangle-2}dr} \approx (\langle n\rangle-1)r_{max} \tag{9}$$

and substituting from equation (8) into equation (9)

$$\sigma'_t(K,Q^2) \approx \sigma'_t(K,0)\frac{(\langle n\rangle-1)\mu^2}{Q^2} \tag{10}$$

If $\langle n\rangle \sim 5$ and $\mu^2 \sim m_\rho^2$, substitution of the result of equation (10) into equation (3) gives for large Q^2

$$\sigma_t(K,Q^2) \approx \sigma_t(K,0)\,\frac{m_\rho^2}{Q^2} \tag{11}$$

which is indeed what is observed experimentally[3]. The essential point to note is that the restriction that the dominant contribution to the cross section comes from four momenta transfers squared less than μ^2 has modified the on the mass shell cross section by the factor m_ρ^2/Q^2.

As the r values contributing to the cross section are small, the average event will consist of a collimated group of high energy bosons carrying most of the energy and a low energy boson-nucleon collision with energy

E_{eff} of the order of $r_{max}\nu/2$ or,

$$E_{eff} \approx \frac{\mu^2 \nu}{2Q^2} = \frac{M}{x}\left(\frac{\mu^2}{4M^2}\right) \tag{12}$$

Using a value of μ^2 of the order of 0.5 GeV^2, $E_{eff} < 2$ GeV for values of $x > 0.06$. Therefore for $x > 0.06$ there is no reason to expect identical cross sections for neutrons and protons. For $x < 0.06$ neutron and proton cross sections should be approximately identical and the overall behaviour should be closed to that predicted by "diffraction" models(11).

For the kinematic region in which $x > \mu/M$, $|t_{min}|$ is always greater than μ^2 but is minimized when $M_n = M$

or

$$t_{min} = -\frac{x^2}{(1-x)} M^2 \tag{13}$$

As x increases beyond μ/M the cross section will be sharply cut into by the propagator of the intermediate virtual particle. This is indeed observed experimentally, σ_t dropping drastically for $x > .3$. If it is assumed that the major contribution of the cross section comes from the neighborhood where $|t|$ is mimimized, the typical event configuration will consist of a collimated jet of bosons and a recoiling nucleon carrying kinematic energy along the direction of the momentum of the virtual photon of the order of $x^2M/2(1-x)$. There is no reason to expect the neutron and proton cross sections to be identical.

In analogy with the theory for vector meson photo-production at low energies where the minimum momentum transfer is large we would expect the form for $\sigma'_t\,(K,Q^2)$ to be(12)

$$\sigma'_t(K,Q^2) \sim \sigma'_t(K,0)e^{Bt_{min}} \tag{14}$$

In order to fit the experimentally observed cross sectional drop off for inelastic electron scattering with increasing x, B should be $\sim 3(\mathrm{GeV}/c)^{-2}$. An interpolation formula which connects equations (3), (11) and (14) so that they are correct in their respective regions of validity is:

$$\sigma_t(K,Q^2) \approx \sigma_t(K,0) \left[.75\left(\frac{m_\rho^2}{m_\rho^2 + Q^2}\right)^2 + \left(\frac{m_\rho^2}{4m_\rho^2 + Q^2}\right) e^{3t_{min}} \right] \qquad (15)$$

where $t_{min} = -\frac{x^2}{(1-x)} M^2$

The first term corresponds to the contribution from Vector Dominance through the ρ, ω and ϕ mesons, the second term corresponds to the contribution from the higher mass states with an assumed final state multiplicity of five and the exponent multiplying the second term allows for the large minimum momentum transfer region. We believe that this equation should closely represent the actual Physics situation, and that event configurations should be as predicted by us above[13].

It is clear if the foregoing is correct that experiments in different kinematic regions of x and Q^2 will check very different physics and no single experiment will unequivocally select a "correct" theory.

One further result follows from these considerations. On the basis of this model inelastic electron scattering should connect to e^+e^- annilation into protons or antiprotons plus anything by crossing, in the range where $|t_{min}| > 4M^2$ or for $x > .8$. Accordingly the cross sections would be connected in the region for which νW_2 as determined from inelastic electron scattering[3] is less than .01 and we would therefore expect asymptotic cross sections for e^+e^- colliding beam machines to produce nucleons of the order of 10^{-2} of the point Dirac cross section and therefore of the order of 10^{-2} the actually experimentally observed e^+e^- colliding beam cross sections for multipion production.

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10. Implicit in these estimates is the assumption that there are no Regge "damping" factors $s^{\alpha(t)}$ for the intermediate particles. Considerations advanced by R. Feynman suggest that such factors should appear for any specific final channel ("inclusive state") but that summed over all final states ("exclusive state") they should not appear c.f. Feynman. Phys. Rev. Letters 23, 1415 (1969)

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13. Our predicted event configurations are very different from those given by the interesting field theoretic model of Ref. (6), which expects most of the four momentum transfer to be given to the recoiling nucleon. We note that in contradistinction to our model this model[6] would predict small cross sections for inelastic electron scattering from a pion target $e + \pi \rightarrow e + \pi$ + anything, and that by crossing this model of Ref. (6) would then predict much smaller cross sections for e^+e^- annihilation into hadrons than are observed experimentally[7].

Figure Captions

Figure 1 Diagrammatic representation of the interaction of a photon ("on the mass-shell") with a nucleon to give "anything" as in Ref. (8). The cross sectional contributions are assumed to come predominantly from $t < m_\pi^2$, where t is the exchanged four momentum squared.

Figure 2 Diagrammatic representation of an off the mass-shell photon interacting with a nucleon to give "anything". The particles at the top vertex take off energy $(1-r)\nu$, the C. of M. energy of the products of the lower vertex are M_n. The effective coupling of the top vertex is assumed to be of the order of e. The cross sectional contributions are assumed to come predominantly from $|t| < \mu^2$, where μ is the effective mass of the exchanged particle. This results in values of r less than μ^2/Q^2 giving the dominant contribution to the cross section.

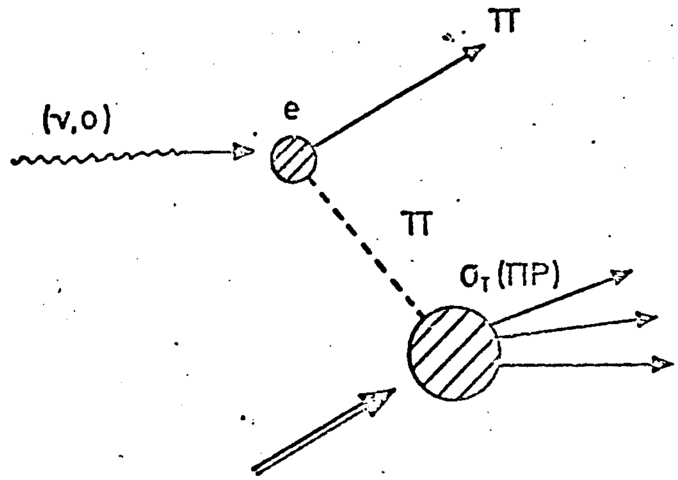

Figure 1

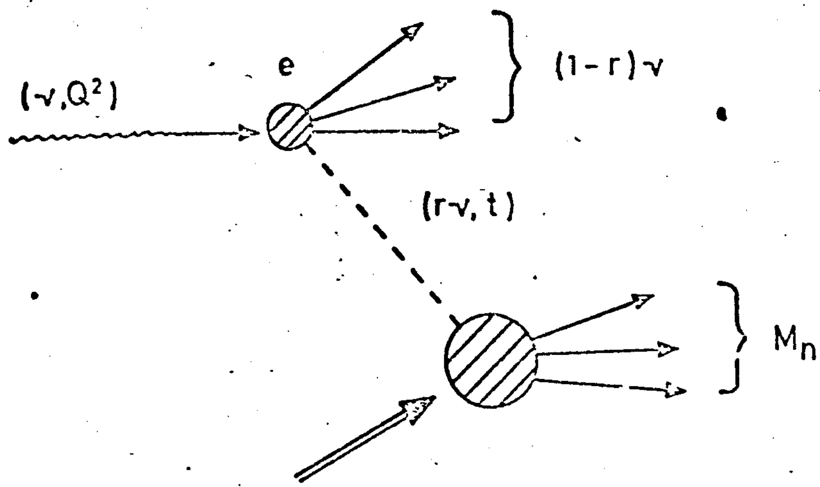

Figure 2