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A LAGRANGIAN CALCULATION OF "SOFT" MESON PRODUCTION AT 12.3 GeV/c*

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ABSTRACT

A simple Lagrangian calculation of a differential cross section for the process $pp \longrightarrow pp\pi^0$ at 12.3 GeV/c is found to agree quantitatively with experiment. Qualitative agreement if found in the case of $pp \longrightarrow pp\omega$.

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The elucidation of processes involving the production of many mesons is an inescapable requirement of any theory of relativistic quantum physics. Field theory can provide the conceptual framework for such an understanding, and has recently served as the basis of some rather interesting speculations[1,2,3,4] concerning the high energy behavior of production processes. However, the questions remain: which field theory, and how to use it.

The present work is an attempt to extend to the high energy domain the phenomenological philosophy represented by the low-energy theorems of electrodynamics[5] and the effective Lagrangians based on chiral invariance.[6] These developments have demonstrated how one can "clothe" an arbitrary hadronic process with soft pions or photons without opening the black box. Since the four-momentum of a real π meson is not zero, it can, at high energies be nonsoft with respect to several of the external lines. We would like to test the following rule to calculate with effective Lagrangians at high energies: vertices at which mesons are produced are inserted on only those lines where the legs can be close to their mass shells. More precisely, the meson of four-momentum k is to be attached only to external lines whose momenta p satisfy $k \cdot p \lesssim O(1\ \mathrm{GeV}^2)$.[7] The effective Lagrangian is presumably totally damped for large relative momenta.[8] Because of the low mass of the π-meson, this does not place much of an absolute cutoff on meson momenta. For example, a pion with momentum 280 MeV/c in the c.m. traveling parallel to a nucleon with 10 GeV/c in the c.m. (corresponding to a collision at 200 MeV/c in the lab) will have a $k \cdot p = 0.35\ \mathrm{GeV}^2$ with respect to this nucleon. If it were perpendicular, then $k \cdot p = 2.8\ \mathrm{GeV}^2$, and it could presumably be cut off.

Our immediate aim is to try to assess the relevance of the above reasoning to the real world. In this I have been fortunate to have had brought to my attention[9] an experiment done in 1967 by H. L. Anderson et al.[10] In it was measured a differential cross section in pp collisions at 12.3 GeV/c for the production of single ω's and π^0's which are slow both in the laboratory and, to a certain extent, with respect to the recoil (slow) proton. Hence there exists experimental data on which to anchor any theory. We refer the reader to Ref. 10 for the experimental details. For our present purposes it is sufficient to state that what is measured for the case of single meson production is $d^2\sigma/d\Omega_c d\Omega_d = \int_\Delta dp_d (d^3\sigma/d\Omega_c d\Omega_d dp_d)$ where $d\Omega_c$ and $d\Omega_d$ are the solid angle acceptances for the two outgoing protons and Δ is the momentum acceptance for the slow proton d (see Fig. 1). With θ_c and θ_d fixed in the lab at 5° and 35°, respectively (the events are coplanar), and with the acceptance on p_d in the range $1.2 \text{ GeV/c} \leq p_d \leq 2.8 \text{ GeV/c}$, the cross sections measured were

$$\frac{d^2\sigma}{d\Omega_c d\Omega_d} \, (pp \longrightarrow pp\pi^0) \simeq 150 \, \mu b/\text{ster}^2 \tag{1}$$

$$\frac{d^2}{d\Omega_c d\Omega_d} \, (pp \longrightarrow pp\omega) \simeq 110 \, \mu b/\text{ster}^2 \tag{2}$$

Since the kinematics reveals that the mesons are slow in the laboratory, and proton d is also quite slow in the laboratory, we use our ansatz to keep only the graphs in Fig. 1. For ω mesons we use a vertex $g_\omega \not{\epsilon}$, where ϵ is the polarization vector of the ω;[11] for the π^0, we use the chiral vertex $(G/2M)\,\gamma_5 \not{k}$,[12] where k is the π^0 momentum. The elastic pp Feynman amplitude can be written as[13]

$$F_{\lambda_c \lambda_d ; \lambda_a \lambda_b} = \sum_{i=1}^{5} F_i \left(\bar{u}_{\lambda_c}(p_c) \Gamma_i u_{\lambda_a}(p_a) \right) \left(\bar{u}_{\lambda_a}(p_d) \Gamma^i u_{\lambda_b}(p_b) \right) \tag{3}$$

where $\Gamma_i = 1, \gamma_5, \gamma_\mu, \gamma_5\gamma_\mu, (1/\sqrt{2})\sigma_{\mu\nu}$, with corresponding invariant amplitudes F_S, F_P, F_V, F_A, F_T. Hence, with the above vertices, the Feynman amplitudes for the graphs in Fig. 1 may be written as

$$\mathscr{M} = \sum_{i=1}^{5} F_i \left(\bar{u}_{\lambda_c}(p_c) \Gamma_i u_{\lambda_a}(p_a) \right) \left[\frac{\bar{u}_{\lambda_d}(p_d) \mathscr{L} (\not{p}_d + \not{k} + M) \Gamma^i u_{\lambda_b}(p_b)}{2k \cdot p_d + \mu^2} - \frac{\bar{u}_{\lambda_d}(p_d) \Gamma^i (\not{p}_b + \not{k} + M) \mathscr{L} u_{\lambda_b}(p_b)}{2k \cdot p_b - \mu^2} \right] \tag{4}$$

where $\mathscr{L} = g_\omega \not\epsilon$ or $(G/2M)\gamma_5 \not k$, μ is the meson mass, and M is the proton mass. Using the expansion $\not p + M = 2M \sum_\lambda u_\lambda(p)\, \bar{u}_\lambda(p)$, we obtain $\mathscr{M} = \mathscr{M}_0 + \mathscr{M}_1$, where

$$\mathscr{M}_0 = \sum_{i=1}^{5} F_i\left(\bar{u}_{\lambda_c}(p_c)\Gamma_i u_{\lambda_a}(p_a)\right) \sum_{\lambda=\pm 1/2}\left(\bar{u}_{\lambda_d}(p_d)\mathscr{L} u_\lambda(p_d)\right)\left(\bar{u}_\lambda(p_d)\Gamma^i u_{\lambda_b}(p_b)\right)\left(\frac{M}{k\cdot p_d}\right)$$

$$- \left(\bar{u}_{\lambda_d}(p_d)\Gamma^i u_\lambda(p_b)\right)\left(\bar{u}_\lambda(p_b)\mathscr{L} u_{\lambda_b}(p_b)\right)\left(\frac{M}{k\cdot p_b}\right) \qquad (5)$$

$\mathscr{M}_1$ contains terms of $(O(\mu^2/2k\cdot p))$ and the contribution of the $\not k$ term in the propagator.

A consideration of $\mathscr{M}_1$ is delayed to the Summary. In contributes negligibly to π^0 emission, not so negligibly to ω emission.

We rewrite $\mathscr{M}_0$ in the factorized form

$$\mathscr{M}_0 = \sum_\lambda \bar{F}_{\lambda_c\lambda;\lambda_a\lambda_b}(p_a, p_b, p_c, p_d)\left(\bar{u}_{\lambda_d}(p_d)\, \mathscr{L} u_\lambda(p_d)\right)\left(\frac{M}{k\cdot p_d}\right)$$

$$- \bar{F}_{\lambda_c\lambda_d;\lambda_a\lambda}(p_a, p_b, p_c, p_d)\left(\bar{u}_\lambda(p_b)\, \mathscr{L} u_{\lambda_b}(p_b)\right)\left(\frac{M}{k\cdot p_d}\right) \qquad (6)$$

The reason for the bar over the F's is that they are not exactly the elastic amplitudes. This is due to the fact that $p_a + p_b \neq p_c + p_d$, and hence $s_{ab} \neq s_{cd}$, $t_{ac} \neq t_{bd}$. The proton legs, however, are on shell in this approximation. We now launch into a series of approximations made necessary by a) the fact that $k \neq 0$ and b) our ignorance of the full pp amplitude.

1) We approximate $\bar{F}$ by the elastic amplitude F at $\bar{t} = 1/2(t_{ac} + t_{bd})$, $\bar{s} = s_{ab}$. We do this because it gives the reasonable answers for both ω and π^0. It is consistent with our philosophy of trying to find relevant rules for such calculations but, although plausible, has very little basis in theory. Clearly, this approximation is truly believable only if t_{ac} is not too different from t_{bd}, and if the elastic amplitudes do not vary too rapidly with t. For the experiments under consideration in the region of low meson momenta,

$t_{ac} \simeq -1\ (\mathrm{GeV}/c)^2$, $-2.8\ (\mathrm{GeV}/c)^2 \leq t_{bd} \leq -1\ (\mathrm{GeV}/c)^2$. At this energy and in this region of $\bar{t}$, $1.0 \leq |\bar{t}| \leq 1.9$, the cross section may be interpolated from the data[14] in the parametric form $(d\sigma/dt)_{el} \simeq 110\ e^{1.63t}\ \mu b/\mathrm{GeV}^2$, so that the variation in amplitude is far slower than in the diffraction peak, but is still considerable. We have no idea why $\bar{s} = s_{ab}$ works consistently better than $\bar{s} = 1/2(s_{ab} + s_{cd})$. In the kinematic region under consideration, the latter choice would correspond to a lab energy of ≈ 9.5 GeV/c.

2) Consistent with the discussion in the introductory paragraphs, and with the omission of graphs with insertions on protons a and c, we do not, in calculating $d^2\sigma/d\Omega_c d\Omega_d$, integrate over values of p_d giving large values of $k \cdot p_d$ or $k \cdot p_b$. Our (arbitrary) rule is to cut off all contributions where the kinetic energy of the meson in the proton rest frames is greater than 1 GeV. This is equivalent to $k \cdot p_d$, $k \cdot p_b \leq 1.0$ for the π^0, ≤ 1.8 for the ω, and roughly cuts off contributions from $p_d \geq 2.3$ or 2.4 GeV/c. In terms of this covariant cutoff, the graphs with insertions on protons a and c are omitted because $k{\cdot}p_a > k{\cdot}p_c > 2.0\ (3.0)\ \mathrm{GeV}^2$ for $\pi^0(\omega)$ production.

3) We assume that the two independent nonflip elastic amplitudes ($\lambda_a = \lambda_c$, $\lambda_b = \lambda_d$) are approximately equal, and dominate all the flip ones in the kinematic region of interest. This has been conjectured by Gilman et al.,[15] and finds some experimental support in the small magnitude of the pp polarization data at 12 GeV/c and $|t| \simeq 1\ (\mathrm{GeV}/c)^2$.[16]

With these assumptions, we arrive at our working formula

$$\mathcal{M}_0 \simeq F_{\frac{1}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2}}(\bar{s},\bar{t})\ R(\lambda_b,\lambda_d,p_b,p_d,k) \tag{7}$$

and

$$\left|\mathcal{M}_0\right|^2 = |F|^2 |R|^2 \approx \pi\left(\frac{s}{M}\right)^2 \left(\frac{d\sigma}{dt}\right)_{el} |R|^2$$

where

$$R = \left(\frac{M}{k \cdot p_d}\right)\left(\bar{u}_{\lambda_d}(p_d)\, \mathscr{L}\, u_{\lambda_b}(p_d)\right) - \left(\frac{M}{k \cdot p_b}\right)\left(\bar{u}_{\lambda_d}(p_b)\, \mathscr{L}\, u_{\lambda_b}(p_b)\right) \tag{8}$$

Finally,

$$\frac{d^3\sigma}{d\Omega_c d\Omega_d dp_d} = \frac{1}{(2\pi)^5} \frac{M^4}{Mp_a} \frac{p_d^2}{E_d} \int dp_c \frac{p_c^2}{E_c} \delta\left((p_a + p_b - p_c - p_d)^2 - \mu^2\right) \frac{1}{4} \sum_{\text{spins}} \left|\mathcal{M}_0\right|^2 \tag{9}$$

Single π^0 Emission

With $\mathcal{L} = (G/2M)\gamma_5 \not{k}$, $G^2/4\pi = 14.6$, we can perform all the necessary operations to obtain $d^3\sigma/d\Omega_c d\Omega_d dp_d$ (Eq. 9). Integrating over the range of p_d consistent with Assumption (2) we obtain

$$\left(\frac{d^2\sigma}{d\Omega_c d\Omega_d}\right)_{\pi^0} \approx 140\,\mu b/\text{ster}^2 \tag{10}$$

in very good agreement with the experimental value (1).

Single ω Emission

Since $\bar{u}_{\lambda'}(p)\,\gamma^\mu\,u_\lambda(p) = (p^\mu/M)\,\delta_{\lambda\lambda'}$, the matrix element R for $\mathcal{L} = g_\omega \not{\epsilon}$ is the well known bremsstrahlung form[5]

$$R = g_\omega\left(\frac{\epsilon \cdot p_d}{k \cdot p_d} - \frac{\epsilon \cdot p_b}{k \cdot p_b}\right)\delta_{\lambda_b \lambda_d} \tag{11}$$

We take $g_\omega^2/4\pi \simeq 5.0$.[17] Proceeding as with the π^0 case, we obtain

$$\left(\frac{d^2\sigma}{d\Omega_c d\Omega_d}\right)_\omega \approx 250\,\mu b/\text{ster}^2 \tag{12}$$

The value (11) consists roughly of 130 μb/ster^2 for transverse ω's, 120 μb/ster^2 for longitudinal. If for some reason the Lagrangian is valid for transverse ω's alone (making for a closer analogy to QED), the agreement with experiment is much better.

We now proceed to comment on our results.

REMARKS:

1) The aim has been to find a reasonable procedure to do a "zero" parameter soft meson calculation. We feel that we have succeeded, especially with the π^0; the

rules are essentially to keep only those insertions which have small meson momentum relative to the emitting line, and to average the elastic "black box" in a "factorizable" way, i.e., evaluate at $t = \bar{t} = \frac{1}{2}\left(t_{ac} + t_{bd}\right)$.

2) In this formulation the π^0's coming from resonances are negligible, since resonance production in the black box is about 5% of the elastic scattering.

3) $\mathcal{M}_1$ can be estimated; it is about a 10% correction in the π^0 calculation, less than 40% in the case of the ω.

4) It would be of interest to know if the ω's are predominantly transverse. In any case, both the experimental and theoretical indications are that multi-ω production is an important process at high energies.

5) In the case of single π emission, further tests of the model are possible:
(a) In an identical experiment, but with a neutron (somehow) detected as particle d, isospin invariance and the model predict $(d^2\sigma/d\Omega_c d\Omega_d)_{\pi^+} = 2 \times (d^2\sigma/d\Omega_c d\Omega_d)_{\pi^0}$. (This also involves neglect of n-p charge exchange scattering.)
(b) A numerical evaluation of $|R|^2$ (Eq. 8) for the case of the π^0 reveals that there is a sharp dip in $d^3\sigma/d\Omega_c d\Omega_d dp_d$ at $p_d \simeq 1.6$ GeV/c. If the experiment were set up again, this could be observed, since the lack of background in the case of the π^0 makes this triply differential cross section measurable.

6) An example of double counting within the context of the effective Lagrangian philosophy would be to add a contribution from the Deck graph (1c). This is outside the scheme since it opens the "black box." It may in fact be the dominant *dynamical* contribution, *equivalent* to the *phenomenological* one embodied in the effective Lagrangian approach.

7) The calculation of multipion emission with chiral Lagrangians is very difficult[18] due to multipion vertices. We find the results of the present study encouraging to further efforts in that direction but using the rules for insertion

stated in Assumption (2). We hope to present some work in this direction in the not too distant future.

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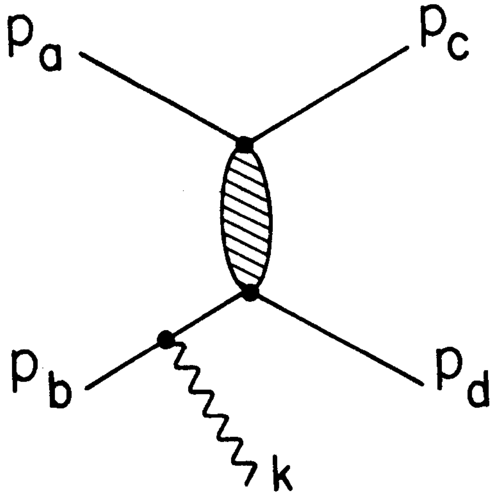
17. This value is arrived at as follows: From SU_3 with standard $\omega - \phi$ mixing, g_ω and g_ϕ may be written as linear combinations of g_1 and g_8, the singlet and octet couplings to the proton. A further condition that the ϕ decouple from the proton (based on the quark model) gives $g_\omega = 3g_{\rho^0}$. Universality in turn gives $g_{\rho^0} = \frac{1}{2} g_{\rho\pi\pi}/4\pi \simeq 2.2$. The experiment, incidentally, lends credence to all these assumptions, in that ρ^0 and ϕ production seem to be strongly suppressed. A further verification of this reasoning would be a suppression of ρ^0 and ϕ relative to ω in backward photoproduction.

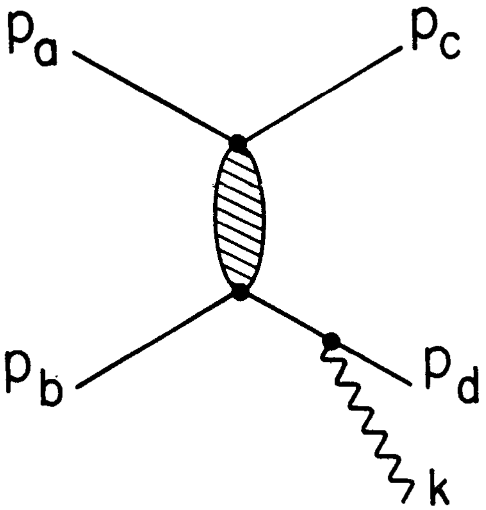
18. S. Weinberg, MIT preprint, "Summing Soft Pions," to be published in Phys. Rev.

FIGURE CAPTION

1. (a) and (b) Graphs contributing to single meson production. In the experiment protons a (fast) and d (slow) are detected at fixed angles. (c) A Deck graph. (See Remark No. 6 in text.)



(a)



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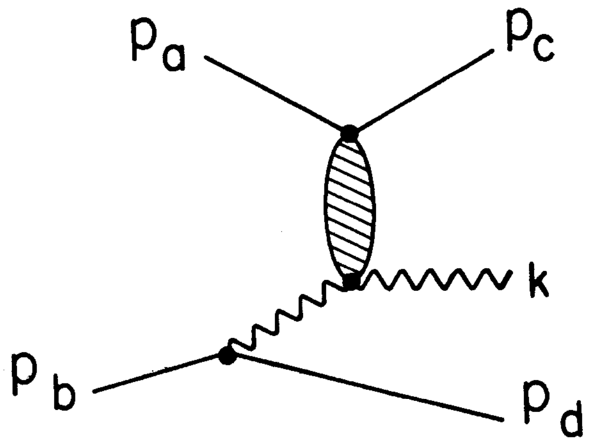

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