SLAC-PUB-795
(TH) and (EXP)
August 1970

Quantum Electrodynamic Theory:
Its Relation to Precision Low Energy Experiments*

Stanley J. Brodsky+

Laboratory of Nuclear Studies, Cornell University

Ithaca, New York

and

Stanford Linear Accelerator Center, Stanford University

Stanford, California++

Invited paper presented to the International Conference on Precision Measurement and Fundamental Constants, August 3-7, 1970, National Bureau of Standards, Gaithersburg, Maryland.

*Supported in part by the U.S. Atomic Energy Commission and the National Science Foundation.

+Avco Visiting Associate Professor.

++Address after August 1, 1970.

(Portions of this paper were also presented at the Second International Conference on Atomic Physics, July 21-24, 1970, Oxford, England.)

1. INTRODUCTION

As far as we know, quantum electrodynamics provides a mathematically exact description of the electromagnetic properties of the electron and muon. To the extent that the electromagnetic properties of the nucleus are known and hadronic contributions to vacuum polarization are understood, the theory also provides the fundamental dynamical theory of the relativistic atom, including its external electromagnetic interactions.

At present quantum electrodynamics has reached the extraordinary state where not even one of its crucial tests indicates any serious discrepancy with its predictions. The key to the unraveling of previous conflicts of theory and experiment has been the independent determination of the fine structure constant α (via the ac Josephson junction measurements for e/h) and recent advances in algebraic and numeric computational techniques for the calculation of higher order radiative corrections--especially fourth order contributions to the Lamb shift and sixth order contributions to the anomalous magnetic moments of the electron and muon. In addition, recent measurements of the muon moment and the hyperfine splitting of muonium have now for the first time permitted a test of quantum electrodynamics free from complications of hadronic and weak interactions at a precision of 5 ppm.

In this talk I will review only the most recent and relevant developments in the precision tests of quantum electrodynamics--those which have had a role in the resolution of the most perplexing discrepancies between theory and experiment and those which seem to point the way to an understanding of the fundamental basis and possible limits of the theory.

Before beginning this review, however, we should note that the high energy—high momentum transfer experiments are now in essentially perfect agreement with the theoretical predictions. The most beautiful tests of this type are those from colliding beams: e^+e^-, e^-e^- elastic scattering and $e^+e^- \to 2\gamma$, $e^+e^- \to \mu^+\mu^-$ annihilation, all of which are free from hadronic complications. The Born approximation structure of the theory--i.e. the validity of the Dirac current for the lepton and Maxwell's equations--has now been verified to small distances approaching 10^{-15} cm. The experimentally obtained lower bounds [1] on possible high momentum transfer cut-offs for the various lepton and photon propagators are shown in figure 1.

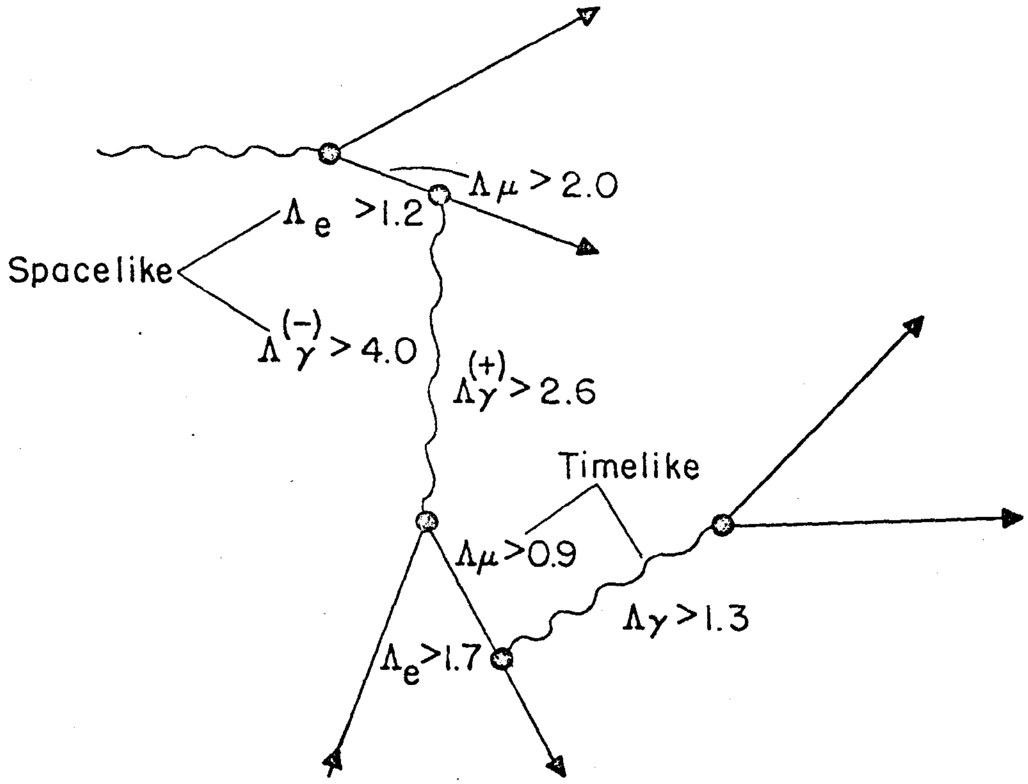


Figure 1. Composite picture of high energy QED measurements. The lower bounds for Λ (in GeV) correspond to 95% confidence limits on possible modifications of the photon and lepton space-like and time-like propagators. A review and references to individual experiments may be found in reference [1].

Although high momentum transfer tests are essential for detecting possible new interactions or deviations at short distances, they are in practice only sensitive to Born diagrams. Tests of the high order corrections, including those involving renormalization effects require the very high precision atomic hyperfine and fine structure measurements and precise determinations of the electron and muon anomalous magnetic moments. As we shall see, the Lamb shift and hyperfine measurements are sensitive to the dynamical effects of quantum electrodynamics through fourth order in perturbation theory, as well as relativistic recoil corrections which emerge from the covariant treatment of the hydrogen atom bound state. The measurements of the magnetic moment of the electron are on the threshold of checking quantum electrodynamics through sixth order in perturbation theory. Besides this, at the level of precision now possible in studying the muon's g-2 value, one is able to probe the effect in an isolated electrodynamic system of very interesting hadronic and weak interaction contributions buried in the vacuum polarization; this leads to limits on the e^+e^- annihilation cross section into the entire spectrum of hadrons. In addition, statements about the polarizability of the proton structure itself can be inferred from the fantastically precise measurements of the ground state hyperfine splitting of hydrogen.

The validity of QED of course pertains to the validity of our understanding of all atomic physics, the analysis of the fundamental constants, and possibly the understanding of elementary particles. But perhaps the underlying goal of the precision tests is aesthetic: the hydrogen atom is the fundamental two-body system and perhaps the most important tool of physics; fifty-seven years after the Bohr theory the challenge is still there to calculate its properties to the highest accuracy possible.

2. PRECISION TESTS OF QUANTUM ELECTRODYNAMICS

All of the precision tests of QED hinge on the value of the fine structure constant $\alpha = e^2/4\pi\hbar c$. Because of the precision measurements of e/h via the ac Josephson effect in superconductors and the massive reanalysis of the data relevant to the determination of the fundamental constants by Parker, Taylor, and Langenberg [2], α can now be determined to better than 2 ppm precision from experiments totally independent of QED input. The least square adjusted result--which has now almost been canonized--is [3]

$$\alpha^{-1} = 137.03608(26) \qquad (1.9\ \text{ppm}) \qquad (1)$$

Thus finally the input constants necessary for comparing theory and experiment are known sufficiently to permit critical and unambiguous tests of theory.

2.1 The Anomalous Magnetic Moments

The classic and basic test of quantum electrodynamics[1] is the anomalous magnetic moment of the electron $a_e = (g-2)/2$. Thus far it has been one of the most stunning triumphs of theoretical analysis.

Very recently Wesley and Rich [4] have reported a determination of a_e to a precision of 6 ppm:

$$a_{e^-}^{exp} = 0.001\ 159\ 644(7). \tag{2}$$

Although the new experiment is based on the same spin precession method used by Wilkinson and Crane [5], the reasons for the large difference with the final result [6] $a_{e^-}^{exp} = 0.001\ 159\ 549(30)$ for the older measurement is not understood. Promising new methods utilizing RF resonance techniques [7] or the change in flight time of ground state electrons in a magnetic field [8] have also been developed, although their precision is not yet comparable with the Wesley-Rich result.

[1] The experiments can be idealized as a measurement of the static electron in isolation from other dynamics. [The contribution from hadronic vacuum polarization is smaller than even the muon contribution: $(\alpha^2/45\pi^2)(m_e^2/m_\mu^2) \sim 10^{-10}$.]

Theoretically, the electron's interaction with the external electromagnetic field is completely specified by quantum electrodynamics. In general, contributions of order α^n to the anomalous moment are obtained in perturbation theory from Feynman diagrams for the free electron vertex containing n virtual photons, each diagram requiring up to 3n-2 non-trivial parametric integrations. The diagrams which have been computed thus far are shown in figure 2.

The present theoretical prediction for the electron anomalous moment is

$$a_e^{th} = \frac{\alpha}{2\pi} - 0.32848\frac{\alpha^2}{\pi^2} + [0.26(5) + 0.13(est)]\frac{\alpha^3}{\pi^3} . \quad (3)$$

The first two terms are the famous second and fourth order results obtained by Schwinger [9] and by Sommerfield [10] and Petermann [11]. The sixth order coefficient consists of (a) the contribution (0.055...) for fourth order vacuum polarization contributions to the sixth order moment calculated analytically by Mignaco and Remiddi [12], (b) a contribution -0.154±0.009 for second order vacuum polarization insertions in the fourth order vertex recently evaluated numerically by Kinoshita and myself [13], and (c) the contribution 0.36±0.04 from photon-photon scattering diagrams evaluated numerically by Aldins, Brodsky, Dufner and Kinoshita [14].

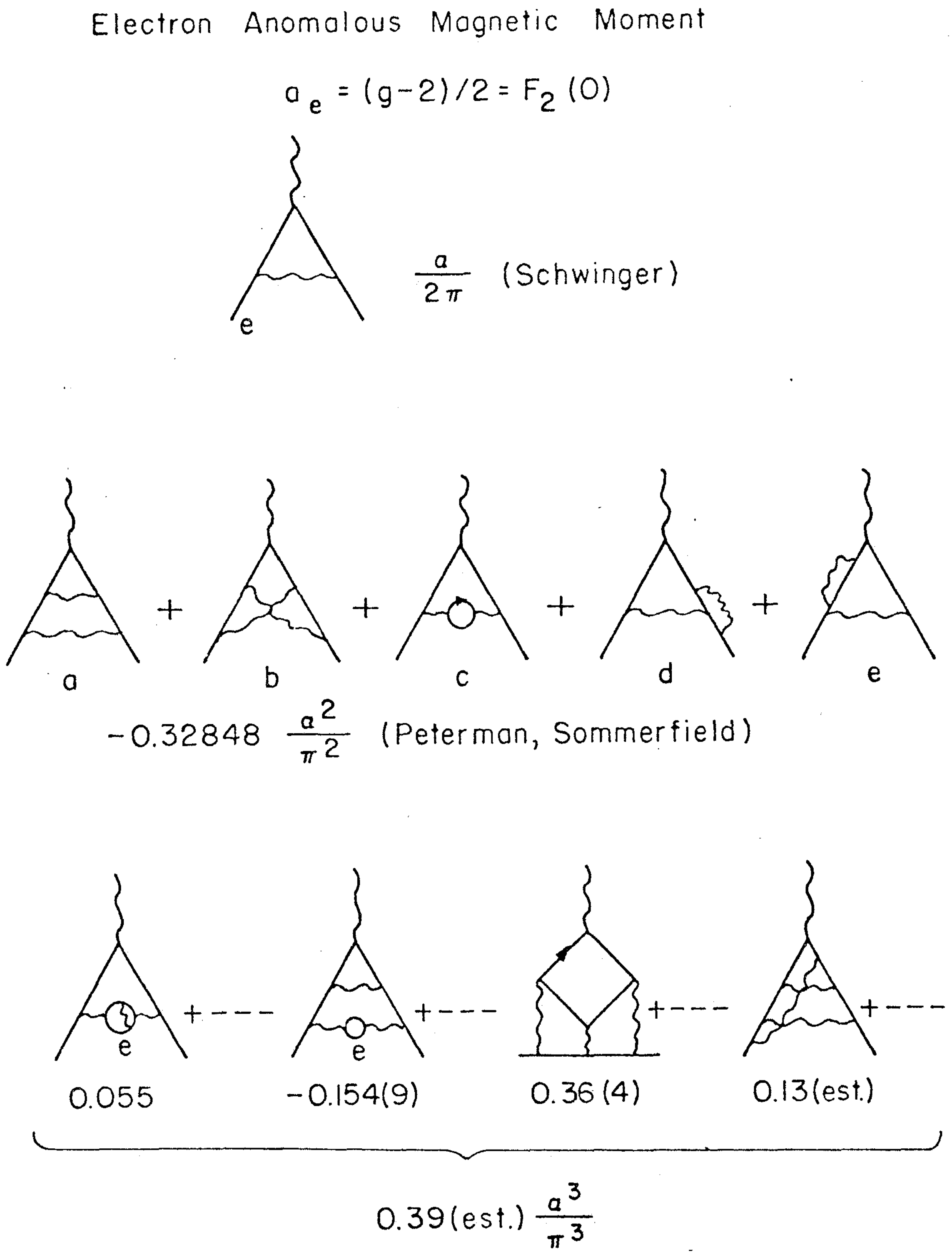


Figure 2. Types of Feynman diagrams which contribute to the anomalous magnetic moment of the electron. Only representative diagrams are shown for the sixth order contributions. Mass, charge, and wave function renormalization counter terms are understood.

The graphs which have not been explicitly evaluated are the sixth order vertices with no electron loop insertions [there are 28 distinct types of diagrams of this type]. The sixth order coefficient includes a dispersion theory estimate [15,16] $0.13 \frac{\alpha^3}{\pi^3}$ for these three photon radiative corrections.

Although it does not eliminate the necessity for a full calculation of the sixth order moment the estimate of Drell and Pagels [15] and Parsons [16] strongly suggests that the final result for the non-electron loop graphs will be positive and numerically small.

The new result obtained by Wesley and Rich (with $\alpha^{-1} = 137.03608(26)$ is

$$a_{e^-}^{exp}(\text{W.R.}) = \frac{\alpha}{2\pi} - 0.32848 \frac{\alpha^2}{\pi^2} + (0.54 \pm 0.58) \frac{\alpha^3}{\pi^2} \qquad (4)$$

which is very consistent with the present indicated sign and magnitude of the sixth order coefficient.

Further experiments and further development of the theoretical result[2] will be required before we can be confident that QED is confirmed through sixth order in perturbation theory. The complete evaluation of the remaining graphs, though difficult, seems technically feasible with presently available algebraic and numerical computation techniques.

[2]The exact calculation of sixth order radiative corrections to the lepton vertex is obviously a horrendous task. There are two central problems: (1) the reduction of matrix elements with three loop integrations to Feynman parametric form and (2) the multi-dimensional integration of the resulting integrand. In the photon-photon scattering contribution calculation of reference [14] all the trace algebra and substitutions required to accomplish step (1) were performed automatically using an algebraic computation program written by Hearn [17]. The resulting 7-dimensional integration was performed numerically using a program originally developed by G. Sheppey which on successive iterations improves the Riemann integration grid through a random variable sampling technique.

The muon anomalous moment is also an extremely valuable test of precision quantum electrodynamics. As a result of recent calculations, the difference of muon and electron anomalous moments has now been completely calculated through sixth order in quantum electrodynamics. The difference arises from the perturbation theory diagrams for the muon vertex with internal electron loops as shown in figure 3.

The complete QED result [18] is

$$\begin{aligned} a_\mu^{th} - a_e^{th} &= 1.09426 \frac{\alpha^2}{\pi^2} + [20.3 \pm 1.3] \frac{\alpha^3}{\pi^3} \\ &= 616(1) \times 10^{-8} \qquad \text{(QED)} \end{aligned} \tag{5}$$

including the sixth order contributions from new analytic-numerical evaluations of second order vacuum polarization insertions [13,19]; fourth order vacuum polarization [20,21]; and photon-photon scattering contributions [14] due to the electron current. This result differs by $[-0.9 \pm 0.3] \frac{\alpha^3}{\pi^3}$ from previous compilations [1,2] which did not include the complete non-logarithmic remainder of the second order vacuum polarization contributions. In addition, hadronic vacuum polarization calculated from the Orsay data for electron-positron annihilation in the ρ, ω, and ϕ regions gives the contribution [22]

$$\Delta a_\mu^{th}(\text{hadronic}) = 6.5(5) \times 10^{-8} . \tag{6}$$

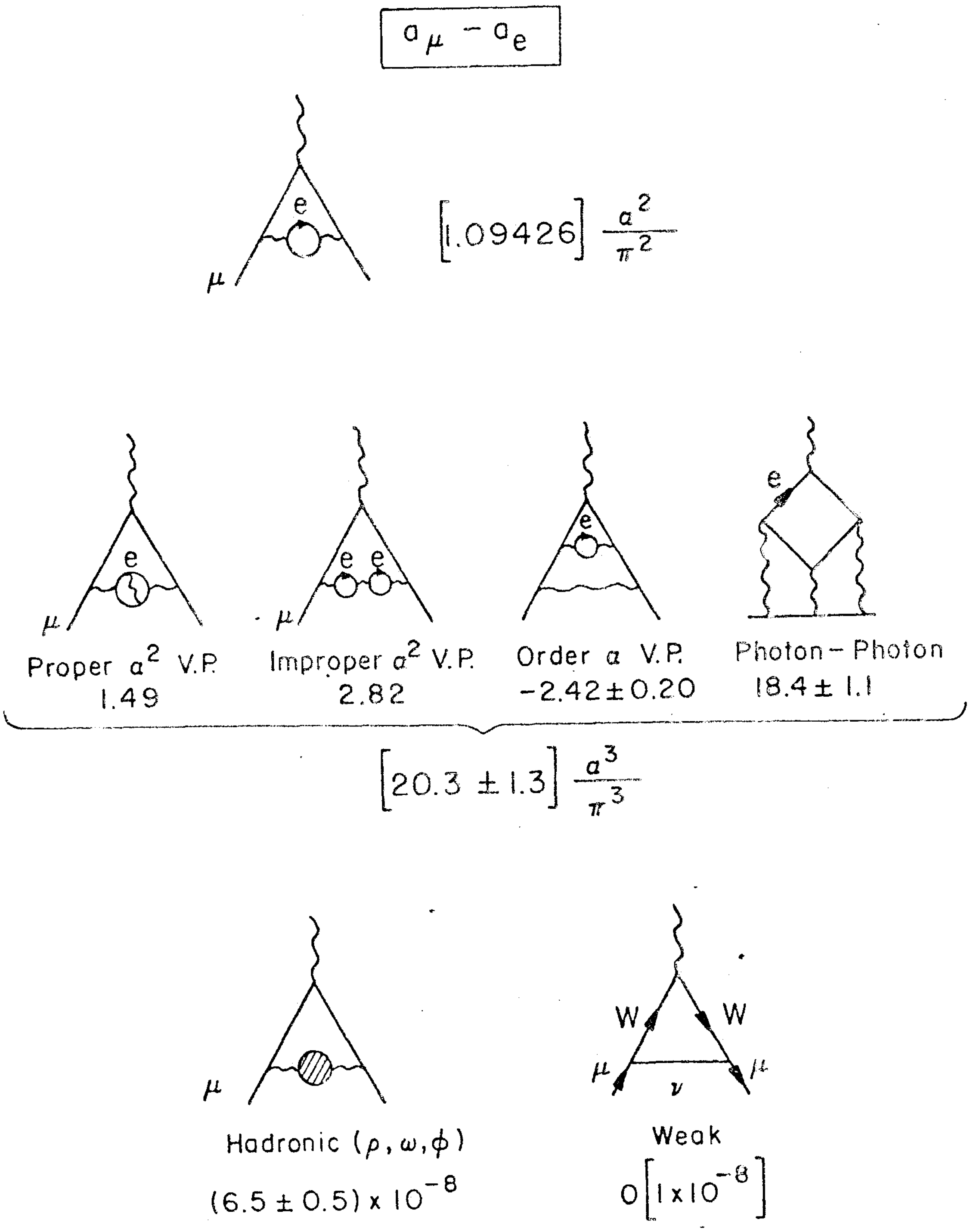


Figure 3. The distinct types of Feynman diagrams which contribute to the difference of muon and electron anomalous moments. The contributions of muon and hadronic vacuum insertions to the electron vertex are negligible.

The result of the CERN measurement [23] for the anomalous moment of the muon is

$$a_\mu^{exp} = 0.001\ 166\ 16(31)\ . \tag{7}$$

If we combine this with the Wesley-Rich result for a_e^{exp}, we can check the theoretical result for the difference of muon and electron moments directly:

$$a_\mu^{exp} - a_e^{exp} = 652(32)\ \times 10^{-8} \tag{8}$$

$$a_\mu^{th} - a_e^{th} = 623(2)\ \times 10^{-8}\ . \tag{9}$$

The theoretical uncertainty does not take into account further (positive) contributions from hadronic vacuum polarization beyond the ϕ resonance region or possible weak interaction contributions which may be of order 1×10^{-8}. Nevertheless the one standard deviation agreement between theory and experiment is a remarkable success for the application of QED to the muon. Even at the present precision this agreement gives an interesting bound on the electron-positron annihilation cross section integrated over the entire hadronic spectrum [14]. Photon propagator cutoffs or negative metric photon [24] mass less than 5 GeV are ruled out to 90% confidence.

2.2 The Lamb Shift and Fine Structure of Hydrogenic Atoms

The historic tests of quantum electrodynamics have been the energy levels of the hydrogen atom [see figure 4]. More recently the testing ground has been extended to other hydrogenic atoms and especially positronium and muonium for which the complications of hadron dynamics are remote. The dynamics of these atoms are completely specified by the interaction density of QED, $H_I = e\overline{\psi}\gamma_\mu\psi A^\mu$ plus the Maxwell and Dirac equations as expressed in perturbation theory in the form of the Feynman rules. The theoretical setting for the exact covariant treatment for the bound states of the hydrogenic atom is the Bethe-Salpeter equation [25]. If the atomic level experiments are idealized as photoabsorption measurements then as shown by Low [26] the line centers of the absorption spectra are determined by the eigenvalues of the full Bethe-Salpeter equation to at least order α^3.

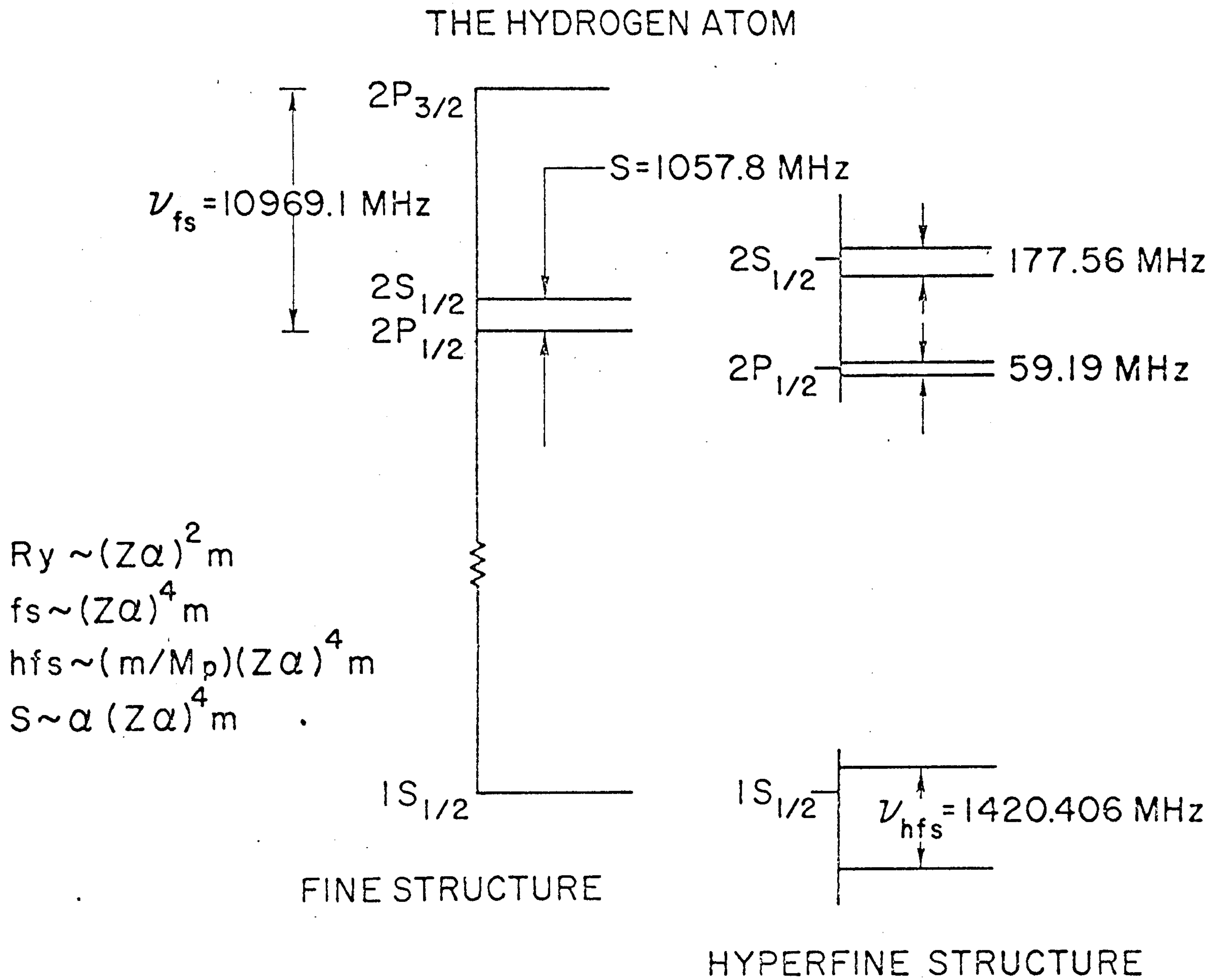


Figure 4. The n = 1 and n = 2 levels of the hydrogen atom.

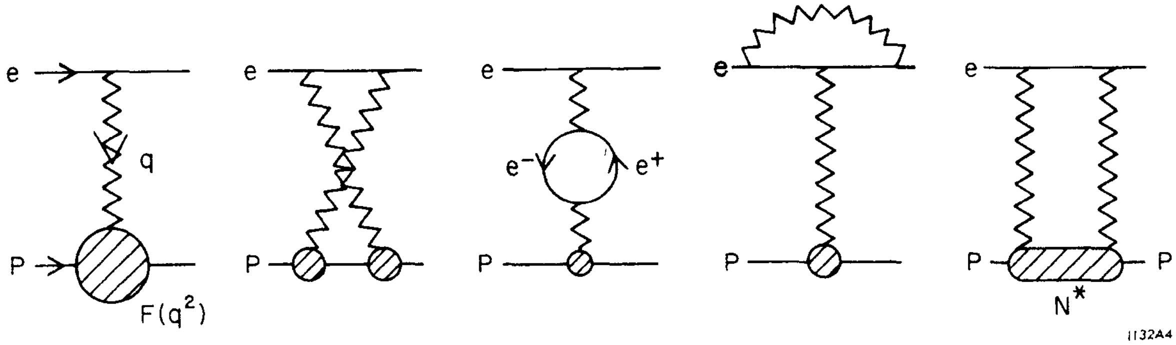
142581

The typical potential terms (irreducible kernels) which must be considered for the hydrogen atom are shown in figure 5. Of course the spectrum of the Bethe-Salpeter equation has not been solved exactly [28] and one must make recourse to the available small parameters. In particular, in the limit $m_e/m_p \rightarrow 0$, $R_p/a_o \rightarrow 0$, $\alpha \rightarrow 0$ (i.e. the neglect of recoil, the proton finite charge distribution, and radiative corrections) one recovers the Dirac equation for an electron in a Coulomb field, with the famous Dirac-Sommerfeld degeneracy of the $S_{1/2}$ and $P_{1/2}$ levels. This degeneracy, however, is rather delicate and is removed by any modification of the Coulomb interaction. In particular, the s-state binding is strengthened by vacuum polarization and weakened by the proton finite size, by non-reduced mass recoil corrections and, most important, by the QED modification of the electron's charge and magnetic interactions with the proton. Over the years there has been considerable technical progress extending Bethe's historic calculation [29] of the contribution to Lamb interval ($2S_{1/2}$-$2P_{1/2}$) [from the one photon self-energy correction to the electron] to include terms from order $\alpha(Z\alpha)^4 m \log(Z\alpha)$ up to order $\alpha(Z\alpha)^6 m$. [30] Also in the past two years Grotch and Yennie [31] have developed a very convenient effective potential method to handle the m_e/m_p corrections and have verified the previous calculations [32] of order $(Z\alpha)^5 m_e^2/m_p$ and $(Z\alpha)^5 \log(Z\alpha) m_e^2/m_p$.

Bethe-Salpeter Equation

$$(\not{p}_e - m_e)(\not{p}_p - m_p)\,\chi = G\chi$$

$$G = G_{1\gamma} + G_{\mathrm{CROSSED}} + G_{\mathrm{VAC.POL.}} + G_{\mathrm{SELF\ ENERGY}} + G_{\mathrm{NUC\text{-}POL}} + \cdots$$



$$G_{1\gamma} = G_{\mathrm{COULOMB}} + G_{\mathrm{TRANSVERSE}}$$

$$-\epsilon_\mu \frac{1}{q^2}\epsilon^\mu = \epsilon_0 \frac{1}{\vec{q}^{\,2}}\epsilon_0 + \sum_{\substack{\mathrm{TRAN} \\ i=1,2}} \epsilon_i \frac{1}{q^2}\epsilon_i$$

G_{COULOMB} $\longrightarrow$ Schrödinger equation, proton finite size correction

$+\ G_{\mathrm{TRANS}}$ $\longrightarrow$ reduced mass corrections, HFS splittings

$+\ G^{(\mathrm{all})}_{\mathrm{CROSSED}}$ $\longrightarrow$ Dirac equation, relativistic reduced mass correction

$+\ G_{\mathrm{VAC\text{-}POL}} + G_{\mathrm{SELF\ ENERGY}}$ $\longrightarrow$ Lamb shift, radiative corrections to HFS

$+\ G_{\mathrm{NUC\text{-}POL}}$ $\longrightarrow$ correction to HFS

Expansion Parameters: α, $Z\alpha$, m_e/M_p, R_p/a_0

Figure 5. Exact calculation of the hydrogen spectrum. The typical kernels required for calculation of the energy levels of the H-atom to the present precision are shown. The one photon exchange contribution can be separated into Coulomb and transverse parts in the CM frame. The effects of strong interactions are summarized by form factors in $G_{1\gamma}$ plus nuclear polarization contributions as indicated in $G_{\mathrm{NUC\text{-}POL}}$. The main effects of adding the higher order kernels are listed below the diagrams. The available small expansion parameters are also given. (From reference [27]).

At the ± 0.1 MHz precision level of the experiments, one is also sensitive to the two-photon (fourth order) QED correction. Recently Appelquist and myself [33], using algebraic and numeric computation techniques similar to those used for the higher order corrections to the anomalous magnetic moments [14], have obtained a new result for the fourth order contribution to the slope of the Dirac form factor of the electron--i.e. the order α^2 contribution to the effective mean square radius of the electron Dirac current distribution. The new result differs from the previous calculation [34] due to a change in overall sign and net numerical differences in the contribution of the non-logarithmic remainder of two of the five distinct Feynman diagram contributions. The largest numerical discrepancy was due to the results for the fourth order "corner" diagram [the fourth diagram in the second line of figure 2] to the free electron vertex. Our numerical result for this contribution has very recently been confirmed by de Rafael, Lautrup, and Peterman [35] at CERN and also by Barbieri, Mignaco, and Remiddi [36] who have obtained a completely analytic result for the crucial corner diagram.

The new results increase the $nS_{1/2}$-$nP_{1/2}$ separation in hydrogenic atoms by 0.35(7) MHz x $[Z^4(2/n)^3]$ (3σ conf.) over previous compilations [30,2]. A tabulation of the theoretical contributions for the Lamb interval in hydrogen is given in table 1.

A comparison with the most precise Lamb shift measurements is presented in figure 6 and table 2. I have rather arbitrarily taken ±1/2 of the limit of error (L.E.) to indicate the uncertainty in the theory. The recent measurements of the large interval $nP_{3/2}$-$nS_{1/2}$ are also included in the comparison with theory, utilizing the theoretical formula for the $nP_{3/2}$-$nP_{1/2}$ fine structure separation and $\alpha^{-1} = 137.03608(26)$.

The comparison of theory with experiment now shows quite satisfactory agreement because of the recent modification of the theoretical result for dF_1/dq^2 in fourth order. The remaining inconsistencies appear more as contradictions between the various experiments than with theory.

2.2 The Hyperfine Structure of Hydrogenic Atoms

Very exciting recent experimental developments have now made the hyperfine splitting in muonium (μ^+e^-) the most precise test of quantum electrodynamics.

VARIOUS CONTRIBUTIONS TO THE LAMB SHIFT IN H (n = 2)

DESCRIPTION	ORDER	MAGNITUDE (MHz)
2^{nd}ORDER — SELF-ENERGY	$\alpha(Z\alpha)^4 m\{\log Z\alpha,\ 1\}$	1079.32 ± 0.02
2^{nd}ORDER — VAC. POL.	$\alpha(Z\alpha)^4 m$	- 27.13
2^{nd}ORDER — REMAINDER	$\alpha(Z\alpha)^5 m$	7.14
	$\alpha(Z\alpha)^6 m\{\log^2 Z\alpha, \log Z\alpha,\ 1\}$	- 0.38 ± 0.04
4^{th}ORDER — SELF-ENERGY	$\alpha^2(Z\alpha)^4 m \begin{cases} F_1'(0) \\ F_2(0) \end{cases}$	0.45 ± 0.07 - 0.10
	$\alpha^2(Z\alpha)^5 m$	± 0.02
4^{th}ORDER — VAC. POL.	$\alpha^2(Z\alpha)^4 m$	- 0.24
REDUCED MASS CORRECTIONS	$\alpha(Z\alpha)^4 \frac{m}{M} m\{\log Z\alpha,\ 1\}$	- 1.64
RECOIL	$(Z\alpha)^5 \frac{m}{M} m\{\log Z\alpha,\ 1\}$	0.36 ± 0.01
PROTON SIZE	$(Z\alpha)^4 (mR_N)^2 m$	0.13

$$\mathscr{L} = \Delta E(2S_{\frac{1}{2}} - 2P_{\frac{1}{2}}) = 1057.91 \pm 0.16 \quad \text{(L. E.)}$$

$$\alpha^{-1} = 137.03608(26) \qquad \Delta E(2P_{\frac{3}{2}} - 2S_{\frac{1}{2}}) = 9911.12 \pm 0.22 \quad \text{(L. E.)}$$

$$\Delta E(2P_{\frac{3}{2}} - 2P_{\frac{1}{2}}) = 10969.03 \pm 0.12 \quad \text{(L. E.)}$$

Table 1. (From Brodsky and Appelquist, reference [33].) The previous result for the F_1' contribution to the H (n = 2) Lamb interval was 0.102 MHz. In view of the new analytic work by Barbieri, Mignaco, and Remiddi [36], this contribution can be further revised to 0.44±0.04 MHz.

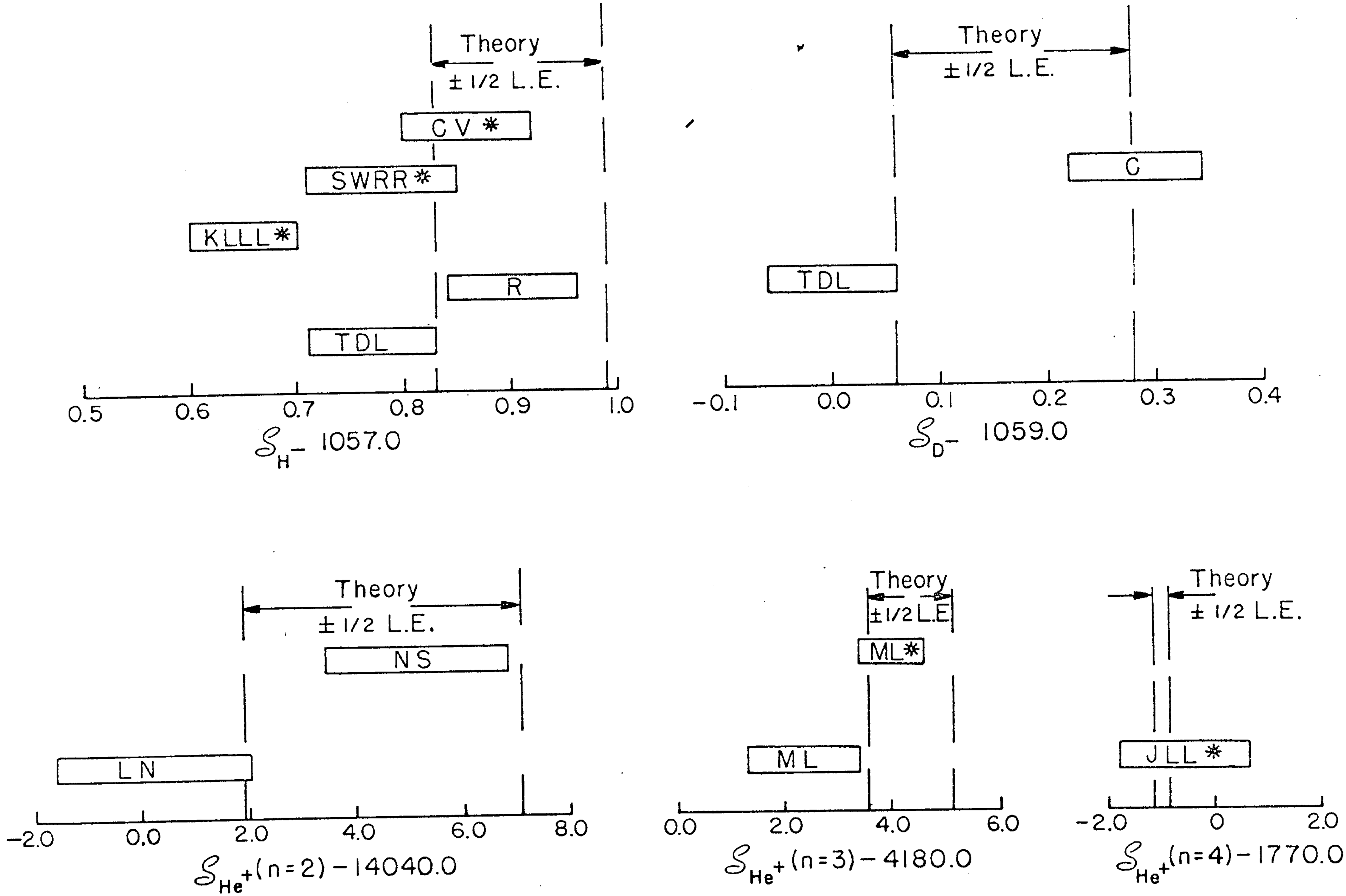


Figure 6. Comparison of Lamb shift theory and experiment (see table 2).

THE LAMB SHIFT IN HYDROGENIC ATOMS (in MHz)

	$\mathcal{S}_{exp}$ ($\pm 1\sigma$)	Theory ($\pm$L.E.)	Exp-Th ($\pm 1\sigma$)
H (n = 2)		1057.91±0.16	
Triebwasser, Dayhoff, Lamb (a)	1057.77±0.06		-0.14±0.08
Robiscoe (b)	1057.90±0.06		-0.01±0.08
Kaufman, Lea, Leventhal, Lamb (c) $[(\Delta E-\mathcal{S})_{exp} = 9911.38 \pm 0.03]$	(1057.65±0.05)		-0.26±0.07
Shyn, Williams, Robiscoe, Rebane (d) $[(\Delta E-\mathcal{S})_{exp} = 9911.25 \pm 0.06]$	(1057.78±0.07)		-0.13±0.09
Cosens and Vorburger (e) $[(\Delta E-\mathcal{S})_{exp} = 9911.17 \pm 0.04]$			-0.05±0.08
D (n = 2)		1059.17±0.22	
Triebwasser, Dayhoff, Lamb (f)	1059.00±0.06		-0.17±0.09
Cosens (g)	1059.28±0.06		+0.11±0.09
He^+ (n = 2)		14044.5±5.2	
Lipworth, Novick (h)	14040.2±1.8		-4.3 ±2.5
Narasimham, Strombotne (i)	14045.1±1.7		0.6 ±2.4
He^+ (n = 3)		4184.4±1.5	
Mader, Leventhal (j)	4182.4±1.0		-2.0 ±1.1
Mader, Leventhal (j) $[\Delta E-\mathcal{S} = 47843.8 \pm 0.5]$	(4184.0±0.6)		-0.4 ±0.8
He^+ (n = 4)		1769.0±0.6	
Hatfield, Hughes (k)	1776.0±7.5		-3.0 ±7.5
Jacobs, Lea, Lamb (ℓ)	1768.0±5.0		-1.0 ±5.0
Jacobs, Lea, Lamb (ℓ) $[\Delta E-\mathcal{S} = 20179.7 \pm 1.2]$	(1769.4±1.2)		0.4 ±1.3
Li^{++} (n = 2)		62771.0±50.0	
Fan, Garcia-Munoz, Sellin (m)	63031.0±327.0		260.0 ±333.0

Table 2. Comparison of Lamb shift experiments and theory. (From reference [33]).

Telegdi's group at the University of Chicago [37] has now reported the first direct measurement of the muon moment using the double-resonance method. Two ground state Zeeman transitions of muonium $\nu_1(F=1,\ m=1 \leftrightarrow F=1,\ m=0)$ and $\nu_2(F=1,\ m=-1 \leftrightarrow F=0,\ m=0)$ are measured at a field B_o where the frequencies are to first order field independent: $\partial\nu_1/\partial B = \partial\nu_2/\partial B = 0$. The ratio of sum to difference of the frequencies determines the muon moment μ_μ in units of the electron's magnetic moment in the atom. The results correspond to a determination of the muon moment to proton moment ratio:

$$\mu_\mu/\mu_p = 3.183337(14) \quad . \tag{10}$$

A comparison of the muonium hfs measurement with theory is thus finally free of questions of possible chemical shifts in previous determinations of the ratio for muons stopped in water. Using their own best result for $\nu_{\mu e}^{hfs}$ [38], the Chicago group obtains the value

$$\alpha^{-1} = 137.03568(33) \quad [2.5\ \text{ppm}] \tag{11}$$

which is in good agreement with the non-QED determination of Taylor et al. [2]: $\alpha^{-1} = 137.03602(26)$ [1.9 ppm].

The paradoxical fact, however, is that the chemical shift discussed by Ruderman [39] (whereby muons in the intermolecular space experience ~15 ppm less diamagnetic shielding than the H_2O-bound protons) and which was of great concern in interpreting the μ_μ'/μ_p' measurements in water is actually not applicable.

In a recent work, a group from the University of Washington and LRL [40] has measured the μ'_μ/μ'_p ratio in various chemical environments to the extraordinary precision of 2.5 ppm. As in earlier measurements [41] the proton resonance ω_p frequency and the muon (decay asymmetry) precession frequency ω_μ of stopped polarized positive muons are measured in the same magnetic field. The direct results are [42]

$$\mu'_\mu/\mu'_p = \omega_\mu/\omega_p = \begin{matrix} 3.183350(8) & H_2O \\ 3.183355(8) & \text{NaOH solution.} \end{matrix} \qquad (12)$$

If the Ruderman model for magnetic shielding were applicable, then the muon frequency in the NaOH solution would be expected to be ~15 ppm lower than in H_2O--since the positive muon would rapidly combine with the NaOH and suffer about the same shielding as a proton. In fact it is 1.6 ppm higher. The correct picture, as discussed by the Univ. of Washington/LRL group [40], seems to be that the slowed muons are first neutralized by capturing an electron. The "hot" muonium atom is then combined into neutral molecules with the substitution of the muon for a proton. The shielding of the muon in this situation is then very similar (within ~2 ppm) as the proton. In this new picture, the muon is rarely found in the intermolecular weak shielding region.

The final corrected value for the muon/proton moment ratio from the Univ. of Washington measurements is

$$\mu_\mu/\mu_p = 3.183347(9) \quad (2.8 \text{ ppm}) \tag{13}$$

in excellent agreement with the Chicago group's result.

If we use this value and $\alpha^{-1} = 137.03608(26)$, then the predicted value for the muonium hyperfine splitting is [1,31, 43]

$$\nu_{\mu e}^{th} = 4463.289(19) \text{ MHz} \quad [4.3 \text{ ppm}] . \tag{14}$$

This is very close to the weighted average of the two most recent results

$$\nu_{\mu e}^{exp} = 4463.317(21) \text{ MHz} \quad (\text{Chicago}) \tag{15}$$

$$\nu_{\mu e}^{exp} = 4463.249(31) \text{ MHz} \quad (\text{Yale}) . \tag{16}$$

Thus μ/e universality and quantum electrodynamics, free from hadronic effects, is now being tested at close to the 5 ppm level. The nuclear recoil corrections [44]

$$\delta_\mu = -\frac{3\alpha}{\pi}\frac{m_e}{m_\mu}\log\frac{m_\mu}{m_e} = -179.7 \text{ ppm} \tag{17}$$

which emerge from the Bethe-Salpeter covariant formalism are being checked to ~3% accuracy. The radiative corrections have been evaluated up to the 1 ppm level [43]. Finally we note that the new measurements [40] determine the muon to electron mass ratio to 3 ppm:

$$m_\mu/m_e = 206.7683(6) .$$

The theory of the hyperfine splitting of the ground state of atomic hydrogen is also in a state of quite satisfactory agreement [1,31,43]:

$$\frac{\nu_H^{exp} - \nu_H^{th}}{\nu_H^{exp}} = 2.5 \pm 4.0 \text{ ppm} - \delta_N^{pol} . \tag{18}$$

The results are consistent with a small proton polarization correction [45].

The theory of the positronium ground state splitting has now been extended to terms of order $\alpha^6 m \log \alpha^{-1}$ by a remarkable calculation of Fulton, Owen, and Repko [46]:

$$\begin{aligned} \nu_{e^+e^-}^{th} &= \alpha^2 Ry_\infty \left[\frac{7}{6} - \frac{\alpha}{\pi}\left(\frac{16}{9} + \log 2\right) + 3/4\, \alpha^2 \log \alpha^{-1} + O(\alpha^2)\right] \\ &= 2.03415 \times 10^5 \text{ MHz} + O(\alpha^6 m) \end{aligned} \tag{19}$$

using $\alpha^{-1} = 137.03608(26)$. The most recent experimental value [47] is

$$\nu_{e^+e^-}^{exp} = 2.03403(12) \times 10^5 \text{ MHz} \quad [50 \text{ ppm}] . \tag{20}$$

A calculation of the terms of order α^6 will be necessary for comparison with new experiments in progress at Yale which should attain a precision of 10 ppm [48].

3. IMPLICATIONS FOR FUNDAMENTAL PHYSICS

Quantum electrodynamics has never been more successful in its confrontation with experiment than it is now. This becomes especially apparent when one compares the values of α which can be derived to 5 ppm or better from the QED tests with its canonical non-QED value [see figure 7 and table 3]. In addition to the values derived from the muonium and hydrogen hfs (assuming $\delta_p = 0 \pm 5$ ppm), α can be determined to high precision from the various level crossing measurements of the hydrogen fine structure separation [49]. Further work, both experimental and theoretical for the muonium hfs [50], positronium hfs, the fine structure of atomic helium [51], and the anomalous moment of the electron can conceivably produce values for the fine structure to a precision of 1 ppm. It is also possible that further high energy spin-analyzed electron-proton elastic and inelastic scattering data will eventually lead to an unambiguous determination of the nuclear polarization contribution to the hydrogen hyperfine splitting [52]. Measurements of the hyperfine splitting or Lamb interval in muonic-hydrogen (μ^--p) would be of incredible interest for solving these hadronic problems [53].

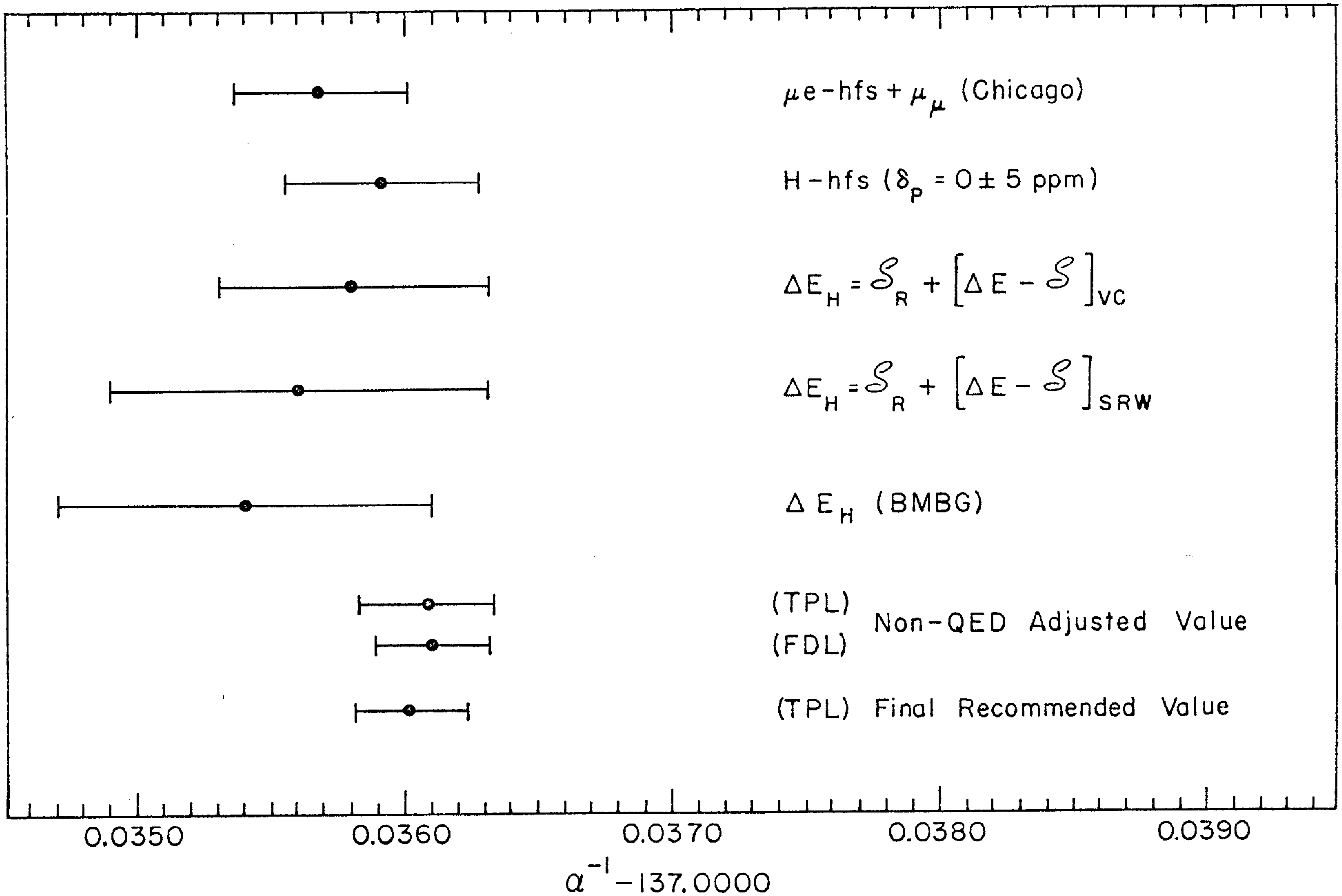


Figure 7. Recent determinations of the fine structure constant. The value for α obtained using the high precision "atomic bottle" measurement [54] of the $2P_{3/2}$-$2P_{1/2}$ interval depends critically on the value taken for the Lamb interval and is not included here. (See reference [2]).

Determination of α^{-1} to 5 ppm or better

TPL	Final Recommended Value (1969)	137.03602(21)	1.5 ppm
TPL	WQED Least Square Value (1969)	137.03608(26)	1.9 ppm
FDL	WQED Least Square Value (1970)	137.03610(22)	1.6 ppm
H-hfs ($\delta_p = 0 \pm 5$ ppm, 1969)		137.03591(35)	2.6 ppm
μ_μ + μ-e hfs (Chicago, 1970)		137.03568(33)	2.5 ppm
ΔE_H (BMBG, 1970)		137.0354(7)	5 ppm
H-fs	$[\mathcal{S}_R + (\Delta E - \mathcal{S})_{SWR}$, 1970]	137.0356(7)	5 ppm
H-fs	$[\mathcal{S}_R + (\Delta E - \mathcal{S})_{VC}$, 1969]	137.0358(5)	4 ppm

Table 3. Recent determinations of the fine structure constant. The value for α obtained using the high precision "atomic bottle" measurement [54] of the $2P_{3/2}$-$2P_{1/2}$ interval depends critically on the value taken for the Lamb interval and is not included here. (See reference [2]).

A further essential element of our understanding of the hydrogen atom concerns its interaction with external fields. The general theory of the electromagnetic interaction of relativistic weakly-bound composite systems is given within the Bethe-Salpeter formalism [55,56]. Of greatest current interest is the general reliability of the Zeeman theory [56] (which is now required to 1 ppm accuracy) and the specific question of finite nuclear mass, radiative, and binding corrections to the electron's g_J value.

The basic formulae for these problems were given by Lamb in his famous papers [57]. More recently Grotch [58] and Hegstrom [59] have computed higher order corrections to the electron's g_J value. The dependence on nuclear mass is reflected in the ratio of g_J values for atomic hydrogen and deuterium. The prediction is

$$\begin{aligned} g(H)/g(D) &= [1+\tfrac{1}{2}(Z\alpha)^2\frac{m_e}{m_p}]/[1+\tfrac{1}{2}(Z\alpha)^2\frac{m_e}{m_D}] \\ &= 1 + 7.3 \times 10^{-9} \quad [58] \end{aligned} \tag{21}$$

compared to the measured values:

$$\frac{g(H)}{g(D)} = \begin{cases} 1+(9.4\pm 1.4)\times 10^{-9} & \text{(Larsen, Valberg, Ramsey [60])} \\ 1+(7.2\pm 3.0)\times 10^{-9} & \text{(Robinson and Hughes [61])} \end{cases} \tag{22}$$

The radiative-binding corrections of order $\alpha(Z\alpha)^2$ to the g_J value can be tested by measuring the ratio in H to He^+.[58]

Quantum electrodynamics is the main link between the physics of the atom and elementary particle physics. In atomic physics it is the fundamental theory and the basis of all calculations. In particle physics and field theory it serves as the model and guide to the weak and strong interactions [62]. Despite its successes and despite the essential simplicity of its equations, it is clear that we are still uncomfortable with the theory as it is. For one thing, it is difficult to accept the infinite renormalization procedure as an essential part of a physical theory.[3] Ingenious extensions of the theory, especially the introduction by Lee and Wick of negative metric photons and leptons [24], and the possible non-polynomial modifications due to gravitational effects, as proposed by Salam and Strathdee [64], can lead to finite physical theories.

[3]The infinities may, of course, only be symptomatic of an incorrect or asymptotic expansion. A convergent expansion in α may take a Padé form, for example [63]. On the other hand, in some mathematical field theory models studied by Jaffe and Glimm [64] the renormalization constants are infinite with or without a perturbation expansion. The best experimental clue we have to the possibilities of an asymptotic expansion is the test at sixth order of the electron anomalous moment.

As we discussed above, the comparison of the muon anomalous moment with conventional theory already rules out (to 90% conf.) negative metric photons with mass less than 5 GeV. The gravitational modifications of the theory seem to have no direct tests, but are interesting because the electromagnetic self-mass of the electron can, in principle, be calculated from the gravitational constant in such an approach.

From a second point of view, it is frustrating to have a theory which--as far as we know--provides an exact mathematical description of the physical world and yet tells us nothing about so many fundamental questions, especially the origin of charge quantization [66], the numerical value of α, and the problems understanding the existence of the muon and the symmetry of its interactions with those of the electron.

Despite these fundamental problems, the successes of quantum electrodynamics are phenomenal. Perhaps the most dramatic evidence of its validity is the electron's g-factor. Considering that there is no a priori reason for the bare lepton to have a Dirac moment ($g = 2$), theory and experiment for $g = 2(1 + a_e)$ can be said to agree to nine significant figures.

The calculation of the entire sixth order contribution to the anomalous moment will almost certainly be completed within a few years--probably to within a numerical precision of $\pm 0.1\ \alpha^3/\pi^3$. The necessity for measurement of a_e to a comparable precision will then be critical. The electron anomalous moment is perhaps the most precious and unique precision test of quantum electrodynamics; it is the only way we have to check the theory--and the correctness of the Taylor expansion in powers in α--through sixth order in perturbation theory.

In order to carry out this program, the value of α will be required to a precision near ± 0.5 ppm. Thus the first order of business is to push the fundamental constants and precision tests as hard as possible, especially measurements of the muonium hfs, the hydrogen fs, and even the helium fs and positronium hfs.

An additional dividend of a muonium hfs measurement at this precision will be the determination of the proton polarization in the H hfs to ~1 ppm; this could well be of fundamental importance to hadron physics [45]. Additionally, it should be emphasized that a high precision determination of the difference of muon and electron anomalous moments will yield invaluable information on the total contribution of the hadronic current to the vacuum polarization as well as a limit on the magnitude of the weak interaction correction to the electromagnetic current of the muon.

Although the comparison of theory with the Lamb shift measurements now shows satisfactory agreement, it is clear that further work is needed to improve the precision of the experiments and theory. Measurements in medium and high Z hydrogenic atoms could be of great value in checking out the various components and Z-dependence of the total theoretical result.

References for table 2

a. S. Triebwasser, E. S. Dayhoff, and W. E. Lamb, Jr., Phys. Rev. 89, 98 (1953).

b. R. T. Robiscoe and T. W. Shyn, Phys. Rev. Letters 24, 559 (1970); R. Robiscoe, Phys. Rev. 168, 4 (1968).

c. S. L. Kaufman, W. E. Lamb, Jr., K. R. Lea, and M. Leventhal, Phys. Rev. Letters 22, 507 (1969).

d. T. W. Shyn, W. L. Williams, R. T. Robiscoe, and T. Rebane, Phys. Rev. Letters 22, 1273 (1969).

e. T. V. Vorburger and B. L. Cosens, Phys. Rev. Letters 23, 1273 (1969).

f. S. Triebwasser, E. S. Dayhoff, and W. E. Lamb, Jr., Phys. Rev. 89, 98 (1953).

g. B. L. Cosens, Phys. Rev. 173, 49 (1968); see also T. V. Vorburger and B. L. Cosens (submitted to International Conference on Precision Measurement and Fundamental Constants, National Bureau of Standards, Gaithersburg, Maryland, 1970).

h. E. Lipworth and R. Novick, Phys. Rev. 108, 1434 (1957).

i. M. A. Narasimham and R. L. Strombotne (submitted to International Conference on Precision Measurement and Fundamental Constants, National Bureau of Standards, Gaithersburg, Maryland, 1970).

j. D. Mader and M. Leventhal, International Conf. on Atomic Physics, New York University (1968).

k. L. L. Hatfield and R. N. Hughes, Phys. Rev. 156, 102 (1967); see also reference [2].

ℓ. R. R. Jacobs, K. R. Lea, and W. E. Lamb, Jr., Bull. Amer. Phys. Soc. 14, 525 (1969).

m. C. Y. Fan, M. Garcia-Munoz, and I. A. Sellin, Phys. Rev. 161, 6 (1967).

Note: The one standard deviation error limits assigned by reference [2] are used here for those experiments for which only "limits of error" are given.

REFERENCES

[1] S. J. Brodsky and S. D. Drell, Ann. Rev. Nucl. Science (1970, to be published). Further details on the present status of quantum electrodynamics are given in this and the following reference.

[2] B. N. Taylor, W. H. Parker, and D. N. Langenberg, Rev. Mod. Phys. 41, 375 (1969).

[3] An even more precise value of e/h (to 0.46 ppm) has recently been obtained by T. F. Finnegan, A. Denenstein, and D. N. Langenberg, Phys. Rev. Letters 24, 738 (1970). This leads to $\alpha^{-1} = 137.03610(22)$ [1.6 ppm].

[4] J. C. Wesley and A. Rich, Phys. Rev. Letters 24, 1320 (1970).

[5] D. T. Wilkinson and H. R. Crane, Phys. Rev. 130, 852 (1963).

[6] The recalculated result given here is obtained from analyses by A. Rich, Phys. Rev. Letters 20, 967, 1221 (1968); G. R. Henry and J. F. Silver, Phys. Rev. 180, 1262 (1969); and F.J.M. Farley, Cargese Lectures in Physics, edited by M. Levy (Gordon and Breach, New York, 1968), Vol. 2.

[7] G. Gräff, F. G. Major, R.W.H. Roeder, and G. Werth, Phys. Rev. Letters 21, 340 (1968); A. Dehmelt (unpublished).

[8] B. Kincaid, W. M. Fairbank, and L. V. Knight (unpublished).

[9] J. Schwinger, Phys. Rev. 73, 416 (1948), 76, 790 (1949).

[10] C. M. Sommerfield, Phys. Rev. 107, 328 (1957); Ann. Phys. (N.Y.) 5, 26 (1958).

[11] A. Petermann, Helv. Phys. Acta 30, 407 (1957).

[12] J. A. Mignaco and E. Remiddi, Nuovo Cimento 60A, 519 (1969).

[13] S. J. Brodsky and T. Kinoshita, submitted to the XVth Intl. Conf. on High Energy Physics, Kiev, 1970.

[14] J. Aldins, T. Kinoshita, S. J. Brodsky, and A. Dufner, Phys. Rev. Letters 23, 441 (1969), and Phys. Rev. (to be published).

[15] S. D. Drell and H. R. Pagels, Phys. Rev. 140, B397 (1965).

[16] R. G. Parsons, Phys. Rev. 168, 1562 (1968).

[17] A. C. Hearn, Stanford Univ. Report ITP-247 (1969).

[18] The fourth order contribution to the muon-electron moment difference is the result obtained by: H. Suura and E. H. Wichmann, Phys. Rev. 105, 1930 (1957); A. Petermann, Phys. Rev. 105, 1931 (1957); Fortschr. Physik 6, 505 (1958); H. H. Elend, Phys. Letters 20, 682, 21, 720 (1966); G. W. Erickson and H. Liu (unpublished).

[19] The contribution of second order vacuum polarization insertions to the sixth order muon anomalous moment has also been calculated by E. de Rafael, B. E. Lautrup, and A. Peterman (private communication). The results agree with those of reference [13].

[20] B. E. Lautrup and E. de Rafael, Phys. Rev. 174, 1835 (1968).

[21] B. E. Lautrup and E. de Rafael, CERN preprint TH.1042 (1969).

[22] M. Gourdin and E. de Rafael, Nuclear Phys. 10B, 667 (1969).

[23] J. Bailey, W. Bartl, G. von Bochman, R.C.A. Brown, F.J.M. Farley, H. Jöstlein, E. Picasso and R. W. Williams, Phys. Letters 28B, 287 (1968).

[24] T. D. Lee and G. C. Wick, Nuclear Phys. 39, 209 [1969); T. D. Lee, talk given at Topical Conf. on Weak Interactions, CERN (1969). J. Bailey and E. Picasso (to be published in Nuclear Phys.) have summarized the limits which can be obtained on various speculative theories from the agreement of theory and experiment for a_μ.

[25] E. E. Salpeter and H. A. Bethe, Phys. Rev. 84, 1232 (1951); E. E. Salpeter, Phys. Rev. 87, 328 (1952); M. Gell-Mann and F. Low, Phys. Rev. 84, 350 (1951); S. Mandelstam, Proc. Roy. Soc. (London) A238, 248 (1952).

[26] F. Low, Phys. Rev. 88, 53 (1952).

[27] S. J. Brodsky and J. R. Primack, Ann. Phys. (N.Y.) 52, 315 (1969), Phys. Rev. 174, 2071 (1968).

[28] Relativistic two-body bound state spectra in quantum electrodynamics have recently been derived by E. Brezin, C. Itzikson and J. Zinn-Justin (to be published, Phys. Rev.). The proposed formulae, which are based on an analytic continuation of the eikonal forward scattering amplitude only take into account the long-range Coulomb potential and do not properly incorporate relativistic recoil corrections. The resulting spectra are incorrect beyond order $(Z\alpha)^4$ (S. J. Brodsky and D. R. Yennie, to be published).

[29] H. A. Bethe, Phys. Rev. 72, 339 (1947).

[30] G. W. Erickson and D. R. Yennie, Ann. Phys. (N.Y.) 35, 271, 447 (1965). These papers give a complete systematic evaluation of the order α radiative corrections to the Lamb shift.

[31] H. Grotch and D. R. Yennie, Rev. Mod. Phys. 41, 350 (1969).

[32] E. E. Salpeter, Phys. Rev. 87, 328 (1952).

[33] T. Appelquist and S. J. Brodsky, Phys. Rev. Letters 24, 562 (1970), and Phys. Rev., to be published.

[34] M. F. Soto, Jr., Phys. Rev. Letters 17, 1153 (1966). See also J. Weneser, R. Bersohn, and N. M. Kroll, Phys. Rev. 91, 1257 (1953).

[35] E. de Rafael, B. Lautrup and A. Peterman, CERN preprint Th.1140, 1970 (to be published).

[36] R. Barbieri, J. A. Mignaco, and E. Remiddi, Univ. of Pisa preprint, 1970. A complete analytic calculation of the fourth order vertex is now in progress (E. Remiddi, private communication).

[37] R. de Voe, P. M. McIntyre, A. Magnon, D. Y. Stowell, R. A. Swanson, and V. L. Telegdi (submitted to the International Conference on Precision Measurement and Fundamental Constants, National Bureau of Standards).

[38] R. D. Ehrlich, H. Hofer, A. Magnon, D. Stowell, R. A. Swanson, and V. L. Telegdi, Univ. of Chicago Preprint EFINS-69-71 (1969).

[39] M. A. Ruderman, Phys. Rev. Letters 17, 794 (1966).

[40] J. F. Hague, J. E. Rothberg, A. Schenk, D. L. Williams, R. W. Williams, and K. M. Crowe (submitted to the International Conference on Precision Measurement and Fundamental Constants, National Bureau of Standards).

[41] D. P. Hutchinson, J. Menes, A. Shapiro and A. M. Patlack, Phys. Rev. 131, 1351 (1963).

[42] Another recent measurement of the ratio in water is

$$\mu'_\mu/\mu'_p = 3.183362(30)$$

[D. P. Hutchinson, F. L. Larsen, N. C. Shoen, D. I. Sober, and A. S. Kanofsky, Phys. Rev. Letters 24, 1254 (1970)].

[43] W. Newcomb and E. E. Salpeter, Phys. Rev. 97, 1146 (1955); R. Arnowitt, Phys. Rev. 92, 1002 (1953).

[45] Theoretical work on the magnitude of the proton polarization correction is reviewed in references [1 and 31]. The contribution of nuclear polarization to the hfs of H [see figure 5] is an extremely important subject since its calculation involves, in principle, most of the uncertainties of proton dynamics. For the hfs we are interested in spin-dependent dynamical quantities (whose spin-independent but isotopic-dependent parts are required for the calculations of the n-p mass difference, a program of limited success thus far). These current matrix elements also enter the study of inelastic e-p scattering which is currently of great interest and under intense study.

[46] T. Fulton, D. A. Owen, and W. W. Repko, Phys. Rev. Letters 24, 1035 (1970), and to be published.

[47] E. D. Theriot, Jr., R. H. Beers, V. W. Hughes, and K.O.H. Ziock, Phys. Rev. (to be published).

[48] F. R. Carlson, V. W. Hughes, Jr., and E. D. Theriot, Jr. (abstract submitted to the International Conference on Precision Measurement and Fundamental Constants, National Bureau of Standards).

[49] H. Metcalf, J. R. Brandenberger, and J. C. Baird, Phys. Rev. Letters 21, 165 (1968), and references b, c, and d of table 2.

[50] A new experiment is being designed at Yale to determine both $\nu_{\mu e}^{hfs}$ and μ_μ to a precision between 0.1 and 0.5 ppm [P. Crane, V. W. Hughes, G. zuPutlitz, and P. A. Thomson (abstract submitted to the International Conference on Precision Measurement and Fundamental Constants, National Bureau of Standards)].

[51] The large J = 0, J = 1 interval in the 2^3P state of helium has been measured to 2 ppm. When calculations of the $O(\alpha^4 Ry)$ contribution are completed, it should be possible to determine α to 1 ppm. See V. W. Hughes, C. E. Johnson, A. Kponoy, S. A. Lewis, and F.M.J. Pichanick (abstract submitted to the International Conference on Precision Measurement and Fundamental Constants, National Bureau of Standards).

[52] V. L. Cherniak, R. N. Faustov, G. M. Zinojev, and B. V. Struminski, Dubna preprints (1969).

[53] A. Di Giacomo, Nuclear Phys. B11, 411 (1969).

[54] Reference e of table 2.

[55] S. Mandelstam, Proc. Roy. Soc. (London) A238, 248 (1955).

[56] S. J. Brodsky and J. R. Primack, Ann. Phys. (N.Y.) 52, 315 (1969), Phys. Rev. 174, 2071 (1968); S. J. Brodsky and R. G. Parsons, Phys. Rev. 163, 134, 176, 423 (1967).

[57] W. E. Lamb, Jr., Phys. Rev. 85, 259 (1952); W. E. Lamb, Jr. and R. C. Retherford, Phys. Rev. 79, 549 (1950), 81, 222 (1951), 86, 1014 (1952).

[58] H. Grotch, Phys. Rev. Letters 24, 39 (1970), and to be published.

[59] R. A. Hegstrom, Phys. Rev. 184, 17 (1969) and to be published.

[60] D. J. Larsen, P. A. Valberg and N. F. Ramsey, Phys. Rev. Letters 23, 1369 (1969).

[61] W. M. Hughes and H. G. Robinson, Phys. Rev. Letters 23, 1209 (1969).

[62] For a striking example see H. Cheng and T. T. Wu, Phys. Rev. Letters 24, 1456 (1970).

[63] R. Chisholm (private communication).

[64] A. Jaffe and J. Glimm, Commun. Math. Phys. 11, 9 (1968).

[65] A. Salam and J. Strathdee (to be published).

[66] See especially J. Schwinger, Phys. Rev. 173, 1536 (1968).