SLAC-PUB-794
October 1970
(TH) & (EXP)

COULOMB PRODUCTION METHOD FOR STUDYING π - π INTERACTIONS*

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ABSTRACT

We consider the production of pion pairs in the Coulomb field by photons and pions, a method which can be used to study the π - π interaction. Simple general formulas are given for the angular correlations of the π pairs, the behavior of the cross section with angle and energy, and the effects of the ρ^0 coherent production background.

(Submitted to Phys. Rev.)

*Work supported by the U. S. Atomic Energy Commission.

The question of the nature value of the $\pi - \pi$ interaction, particularly in the s-wave, continues to be of the greatest interest in elementary particle physics. It is relevant to such diverse questions as the final state interaction in K^0 decays, chiral symmetry, and nuclear force calculations. A considerable effort has been expended in studying πp reactions[1] in situations where π exchange is dominant, allowing for a separation of the process $\pi + \pi \rightarrow \pi + \pi$. This method, particularly for s-wave $\pi\pi$ interactions is troubled by the facts that the ρ resonance in the p-wave is the very dominant feature of the $\pi\pi$ system, and the existence of possible background processes other than π exchange, some of which can behave like π exchange, makes things somewhat "dirty."[2] Pais and Treiman[3] have proposed a method, using $K \rightarrow \pi\pi\ell\nu$ decay, which is "clean," involving only relatively well founded theoretical assumptions and which essentially can yield information on the $\pi\pi$ phase shifts in the region $m^2_{\pi\pi}$ to 12 m^2_{π} for s and p waves, isospin T = 0 and 1, respectively. Creutz and Einhorn[4] have also discussed the colliding beam experiment $e^+ + e^- \rightarrow \pi + \pi + \gamma$ which can be used to study C = +1 states (even partial waves and T = 0, 2) of the $\pi\pi$ system.

Here we would like to discuss another method involving Coulomb production of π pairs with incident photons or pions via photon exchange ("Primakoff Effect") (see Fig. 1). This method, already used for determining π^0 and η lifetimes,[5] if experimentally feasible,[6] would have several good features.

It is "clean" from the theoretical point of view. The production takes place at large impact parameters, far from the nucleus, which is reflected in the production angular distribution of the 2π system by a sharp peak very near the forward direction. If this peak is clearly seen, the events in it can only come from the photon exchange process and we know the process took place outside the nuclear matter. Furthermore the Watson theorem, i.e., unitarity together with time reversal invariance, tells us that the phase of $\gamma + \gamma \rightarrow \pi + \pi$ or $\pi + \gamma \rightarrow \pi + \pi$ has the phase of $\pi - \pi$ scattering. These phases may be examined by looking at interference effects between partial waves of the $\pi - \pi$ distributions in their center-of-mass, just as for the π exchange mechanism.

Since the most interesting problem is the production of the $\pi - \pi$ s-wave, which cannot be reached with an incident π (see below) we first discuss photon induced reactions. Since two photons give a state of even C, C invariance says that the 2π system can only be in states of even angular momentum, $s = 0, 2, 4, \ldots$ In particular the mechanism does not give the ρ^{o} state. The isospin states reached are then $T = 0$ and $T = 2$ and in particular the method could be used to look for a $T = 0$, $s = 0$, "σ" resonance. The method is applicable when the minimum momentum transfer to produce the $\pi\pi$ system, $\Delta_0 \cong m_{\pi\pi}^2/2k$, is on the order of or less than (R nucleus)$^{-1}$, when the peak rises very sharply due to Δ_0 approaching the Coulomb pole at $t = 0$. Higher $\pi - \pi$ masses such as in the f^{o} region are thus accessible since for an incident photon energy k, of 15 GeV for example, we have $m_{f^o}^2/2k =$ 50 MeV. It must be kept in mind, however, that the peak narrows with diminishing Δ_0 so that very good momentary transfer resolution is necessary to take advantage of the rise of the Coulomb peak.

Whether there is any practical point to these considerations depends entirely on whether there will be a distinguishable Primakoff effect peak which rises above the background due to nuclear production. For $m_{\pi\pi}$ in the ρ region, it is necessary to overcome the strong coherent ρ production at small angles; we discuss this below.

To get an idea of the possible rates of s-wave production we assume that σ exists; the unknown part of Fig. 1 is then the matrix element corresponding to the decay $\sigma \rightarrow \gamma + \gamma$. On general grounds the form of this matrix element is

$$M = f \, \epsilon^{\mu}(k) \left[k \cdot q \; g_{\mu\nu} - q_{\mu} k_{\nu} \right] \epsilon^{\nu}(q) \qquad (1)$$

where q, k and are the momenta and polarizations of the respective photons. This corresponds to a decay width $\Gamma(\sigma \rightarrow \gamma\gamma) = \frac{1}{64\pi} (f)^2 \; m_{\sigma}^3$ and gives a production differential cross section for σ, in terms of q^2, the momentum transfer squared, $q^2 \cong -\underline{\Delta}^2 \cong - \left[\Delta_0^2 + (k\theta)^2 \right]$,

$$\frac{d\sigma}{dq^2} \cong \frac{8\pi \, \Gamma(\sigma \rightarrow 2\gamma)}{m_{\sigma}^3} \, Z^2 \alpha \, \frac{(k\theta)^2}{\left[\Delta_0^2 + (k\theta)^2 \right]^2} \, F^2(\Delta^2) \qquad (2)$$

where we have used high energy kinematic approximations. The cross section goes to zero at $\theta = 0$ because there is no helicity 1 state for a σ. It turns out, as we shall see, that this behavior holds regardless of the spin of the final state. The form factor F gives the effects of the nuclear shielding and charge distribution. Its effects are not significant.[7] This cross section, of course, is integrated over the mass spread of the σ. These calculations show that the Primakoff effect rate for the σ meson bears the same relation to $\sigma \rightarrow \gamma + \gamma$ as in the π^0 or η^0 case. Hence it is perhaps feasible as far as rates are concerned to look for the elusive σ in this way. Similar ideas have been mentioned by Rosen.[8] Let us now go on to consider the production of the general $\pi - \pi$ system.

KINEMATICS

Certain kinematic simplifications in the high energy, highly relativistic limit are very useful. First of all, the energy of the incident particle ω is taken to be much greater than the mass of the $\pi - \pi$ system and we always use small angle approximations. In this case the energy in the lab system of the 2π's is almost exactly that of the incident particle $\omega \equiv \omega(\gamma) \approx \omega(2\pi)$. Near the forward direction the three momentum transfer in the lab has a longitudinal part Δ_0 and a transverse part Δ_T. These have values $\Delta_0 \cong \frac{m_{\pi\pi}^2}{2\omega}$ and $\Delta_T \cong k\theta \approx \omega\theta$, θ being the angle of the 2π system's line-of-flight with respect to the incident beam direction. The invariant four-momentum transfer is then $-\underline{\Delta}^2 = -\left[\Delta_0^2 + \Delta_T^2\right]$. The characteristic peak for this process, as given by Eq. (2) occurs around $\Delta_T \approx \Delta_0$ so we are always concerned with small values of q^2.

Since the analysis of the $\pi\pi$ system will be done in its rest frame, the $\pi\pi$ c.m., it is convenient to know certain quantities there. We refer to these quantities with an asterisk (*). The energy of the incoming beam, ω^* may be estimated as follows: we see from Fig. 1 that the upper vertex may be thought of as a "decay" $(\pi\pi) \longrightarrow \gamma + \gamma$, where both photons have approximately zero mass; this gives immediately $\omega^* \approx \frac{m_{\pi\pi}}{2}$. Another useful quantity is the angle X between the beam and the target nucleus as seen in the $\pi - \pi$ c.m. In Fig. 2 we sketch the configuration of vectors in that system. For comparison we also show the lab. configuration. That the angle X is small can be seen by choosing an axis in the lab along the $\pi - \pi$ system's line-of-flight. The beam's momentum transverse to this axis is $\simeq \omega\theta \approx \Delta_T$. Now performing a Lorentz transformation along this axis to the $\pi - \pi$ c.m. this transverse component is unchanged. On the other hand, we know that the total length of the beam momentum vector is $\omega^* \simeq \frac{m_{\pi\pi}}{2}$. Then the angle with this axis, which is the same as the vector labeled "target nucleus," is

$$X = \frac{\Delta_T}{\omega^*} \approx \frac{2\Delta_T}{m_{\pi\pi}} . \tag{3}$$

Since Δ_T in the peak region will be on the order of $\Delta_0 \simeq \frac{m_{\pi\pi}^2}{2\omega}$ typical values of this angle will be $X \sim \frac{m_{\pi\pi}}{\omega}$. Finally, increasing energy for the beam is reflected in the $\pi - \pi$ c.m. by increasing values of the nucleus' momentum $\approx M\omega/m_{\pi\pi}$. This means that the value of the nuclear current vector (sum of initial and final nuclear momenta) transverse to the beam direction in this frame is

$$N_T^* = 2M \frac{\omega}{m_{\pi\pi}} \quad X = 4 \frac{M\omega}{m_{\pi\pi}^2} \Delta_T \tag{4}$$

We will need this result in the next section. In the region of the peak then N_T^* will always be on the order $N_T^* \sim 2M$.

GENERAL SHAPE OF THE DIFFERENTIAL CROSS SECTION

The general matrix element corresponding to Fig. 1 is essentially $\sim \frac{1}{q^2} J^\mu N_\mu$ where J_μ is the electromagnetic current operator connecting the beam particle to the $\pi - \pi$ state, and $N_\mu = (N_1 + N_2)_\mu$ is the current of the nucleus. It has been shown in general work [9] on this subject that in a frame where $\underline{N}$ is large, such as the one we will use (viz. the 2π rest frame) that only virtual photons which are transverse to $\underline{N}$ contribute significantly, while longitudinal virtual photons can be neglected. This classical result means that matrix element is proportional to the transverse momentum transfer and gives the characteristic peak at $\Delta_T = \Delta_0$ and zero at $\theta = 0^o$.

Since the natural "Z axis" for the problem at hand is the beam particle direction in the 2π rest frame (Fig. 2b), let us show directly that this kind of result is also true with respect to this axis; this is possible because of the smallness of X.

The essential point is current conservation $q^\mu J_\mu = 0$, and $q^\mu N_\mu = 0$. This allows us to write

$$N^\mu J_\mu = - \frac{q^2}{q_0^2} N_Z J_Z - N_T J_T \tag{5}$$

where "Z" and "T" (for transverse) are referred to the beam (or exchange) direction in Fig. 2b. Now we show that $\frac{q^2}{q_0^{*2}} N_Z^*$ is negligible compared to N_T^*. Since $N_T^* \approx N_Z^* \chi$ the point is to show that $q^2/q_0^{*2} \ll \chi$. From the estimates in the previous section this means $2\left[\Delta_0^2 + \Delta_T^2\right]/m_{\pi\pi} \ll \Delta_T$. This inequality is clearly good, due to $\Delta_0/m_{\pi\pi} \ll 1$, except in the insignificantly small region in Δ_T, $0 < \Delta_T < \Delta_0^2/m_{\pi\pi}$. Since $\Delta_0 \approx m_{\pi\pi}^2/2\omega$ these approximations improve with energy. Finally we note that since J_Z^* and J_T^* are in the $\pi - \pi$ rest frame and refer only to the beam particle and the $\pi - \pi$ system they are essentially independent of the energy ω, and therefore a large value of J_Z^* cannot in some way compensate for the smallness of $\left(q^2/q_0^{*2}\right) N_Z^*$. Thus we can write in the $\pi\pi$ rest frame

$$N^{\mu} J_{\mu} \approx - N_T^* J_T^* \quad . \tag{6}$$

Since N_T^* is proportional to Δ_T by Eq. (4), the cross section will always have the characteristic form

$$d\sigma \sim \frac{(N_T^*)^2}{q^4} \sim \frac{\Delta_T^2}{\left[\Delta_0^2 + \Delta_T^2\right]^2} \tag{7}$$

This is a general result and applies to any inelastic photon exchange process with $J_T^* \neq 0$.

COULOMB PRODUCTION BY PHOTONS

With the general result (6) in hand, we now have only to construct the matrix element J connecting the beam particle by photon exchange to the final 2π state of definite spin s.

For an incoming photon k and an exchange photon q this amplitude will have two indices μ, ν for coupling initial and exchange photon respectively. Current conservation dictates

$$k^{\mu} A_{\mu\nu}^{(s)} = 0 = A_{\mu\nu}^{(s)} q^{\nu} \tag{8}$$

The state of spin s may be represented[10] by a tensor $\Theta\ldots\mu\ldots\nu$ of rank s which reduces to a traceless, symmetric tensor with no "time" components in the $\pi-\pi$ c.m. That is, Θ obeys

$$P^{\mu}\,\Theta\ldots\mu\ldots\nu = 0 \tag{9}$$

$$g^{\mu\nu}\Theta\ldots\mu\ldots\nu = 0 \tag{10}$$

$$\Theta\ldots\mu\ldots\nu = \Theta\ldots\nu\ldots\mu\ldots \tag{11}$$

where P is the total momentum for the $\pi\pi$ system, $P = p_1 + p_2$. The actual construction of Θ from the vectors internal to the $\pi-\pi$ system does not concern us for the moment.

For a virtual photon (3 helicities) and a real photon (2 helicities) combining to form the final state we must have, with parity conservation, $1/2 \times (2\times 3) = 3$ independent amplitudes. Thus we must construct 3 independent amplitudes, with two indices, containing Θ once, and obeying Eq. (8). We have at our disposal in addition to Θ the vectors k and q, (P being eliminated by $k + q = P$) and the tensor $g_{\mu\nu}$. We can form lower rank tensors from Θ by contracting with k. There is no need to contract Θ with q since by Eq. (9) and $k + q = P$ we have

$$k^{\mu}\Theta\ldots\mu\ldots = -q^{\mu}\Theta\ldots\mu\ldots \tag{12}$$

For a contraction of Θ with say two k's leaving indices μ,λ free we adopt the notation

$$(kk\Theta)_{\mu\lambda} \equiv k^{\alpha}k^{\beta}\,\Theta_{\alpha\beta\mu\lambda} \tag{13}$$

Note by Eq. (11) that the order in saturating the indices does not matter. The reader may now verify that the following amplitudes satisfy all the necessary conditions for $A^{(s)}_{\mu\nu}$, where $\Theta^{(s)}$ is of rank s.

$$A_1^{(s)}\left[k\cdot q\, g_{\mu\nu} - q_\mu k_\nu\right](kk\ldots k\,\Theta^{(s)}) \tag{14}$$

$$A_2^{(s)}\left[k\cdot q(kk\ldots k\Theta^{(s)})_{\mu\nu} - q_\mu(kk\ldots k\,\Theta^{(s)})_\nu + (kk\ldots k\Theta^{(s)})_\mu k_\nu - \frac{q_\mu k_\nu}{k\cdot q}(kk\ldots k\,\Theta^{(s)})\right] \tag{15}$$

$$A_3^{(s)}\left[k\cdot q(kk\ldots k\Theta^{(s)})_\mu - (kk\ldots k\,\Theta^{(s)})q_\mu\right]\left[k\cdot qq_\nu - q^2 k_\nu\right] \tag{16}$$

That the three amplitudes are independent is established by looking at the number of free Θ indices present in the final result; (15) starts with two, (16) with one and (14) has none. For the special case of s = 0 only (14) is needed and we recover Eq. (1). The coefficients $A_i^{(s)}$ are functions of the $m_{\pi\pi}$ and q^2, although q^2 will always be so small that we can ignore its variation in the A's.

Now note that (16) corresponds to the exchange of a longitudinal photon in the $\pi\pi$ c.m., thus it is negligible by Eq. (6) since it gives no contribution to J^*_T. Similarly note that only the first term in each tensor contributes to the transverse current in the π - π c.m.

Thus for an incoming photon of polarization i the Coulomb production vertex has, in the π - π c.m., the form

$$A_1^{(s)}\,\underline{\epsilon}^i\cdot \underline{N}^*_T(kk\ldots k\,\Theta^{(s)}) + A_2^{(s)}\epsilon^i_j\left(N^*_T\right)_k(kk\ldots k\,\Theta^{(s)})_{jk} \tag{17}$$

Now the tensors $\Theta^{(s)}$ are easily constructed for low partial waves s. Since we are to evaluate Eq. (17) in the π - π rest frame, we only need the rest frame reduction of Θ.

The only vector internal to the $\pi - \pi$ system from which to construct Θ is the relative $\pi - \pi$ momentum $\underline{p}^* = \underline{p}_1 - \underline{p}_2$. Thus Θ has the form

$$\Theta^{(0)}_{(1)} = 1$$

$$\Theta^{(1)}_{i} = p^*_i$$

$$\Theta^{(2)}_{ij} = \frac{1}{2}\left[3\, p^*_i p^*_j - \underline{p}^{*2}\, \delta_{ij}\right]$$

$$\Theta^{(3)}_{ijk} = \frac{5}{2}\left[p^*_i\, p^*_j\, p^*_k - \frac{p^{*2}}{5}\left(p^*_i\, \delta_{jk} + p^*_j\, \delta_{ik} + p_k\, \delta_{ij}\right)\right] \quad (18)$$

The fully saturated Θ is just the Legrendre polynomial, $(kk \ldots k\, \Theta^{(s)}) = \underline{p}^{*s} \underline{k}^{*s}\, P_s(\hat{p}^* \cdot \hat{k}^*)$. For the Coulomb photoproduction problem we need only s even, and M becomes

$$M = (\underline{\epsilon}^i \cdot \underline{N}^*_T)\left[A^{(0)} + A^{(2)}_1\, \underline{p}^{*2}\, \underline{k}^{*2} \quad P_2(\hat{p}^* \cdot \hat{k}^*) - \frac{1}{2}\, A^{(2)}_2\, \underline{p}^{*2}\right]$$

$$+ \frac{3}{2}\, (\underline{\epsilon}^i \cdot \underline{p}^*)\, (\underline{N}^*_T \cdot \underline{p}^*)\, A^{(2)}_2 + (\text{spin 4 and higher}) \quad (19)$$

At relatively low values of the $\pi - \pi$ mass where the $A^{(2)}$ (d wave amplitudes) and also p wave interference from the ρ are unimportant, the interesting thing to study will presumably be simply the magnitude of $A^{(0)}$ to see if there is a bump corresponding to a "σ" resonance. Note there should be no production with polarized photons if $\underline{\epsilon}^i \perp \underline{N}^*_T$. At high energy, in the $\pi - \pi$ mass region beyond the ρ, approaching the f^o, Eq. (19) may be applicable without interference from the nuclear production amplitude. In this case note that the various amplitudes may be partially separated by studying the correlations in $|M|^2$ between the photon polarization ϵ^i, the momentum transfer direction $\underline{N}^*_T$ and the $\pi - \pi$ relative momentum $\underline{p}^*$. If we take $\underline{\epsilon}^i \perp N^*_T$ then only $\left|A^{(2)}_2\right|^2$ enters in the cross section and its magnitude may be found.

Interference terms between different amplitudes will generally be of the type Re $A^{(0)}\, A^{(2)*}$, involving the cosine of phase differences. If we consider using circularly polarized photons it is amusing that the sine enters from relations like $d\sigma_R - d\sigma_L \sim \mathrm{Im}\, A^{(0)} A^{(2)*}$. The unpolarized cross section corresponding to Eq. (19) is given by

$$\sum_i |M|^2 = \underline{N}_T^{*2} \left| A^{(0)} + A_1^{(2)} \underline{p}^{*2} \; \underline{k}^{*2} \; P_2(\hat{p}^* \cdot \hat{k}^*) - \frac{1}{2} A_2^{(2)} \underline{p}^{*2} \right|^2$$
$$+ \frac{3}{2} (\underline{p}^* \cdot \underline{N}_T^*)^2 \left[2 \,\mathrm{Re} \left\{ \left(A^{(0)} + A_1^{(2)} \underline{p}^{*2} \; \underline{k}^{*2} \; P_2(\hat{p}^* \cdot \hat{k}^*) - \frac{1}{2} A_2^{(2)} \right) A_2^{(2)*} \right\} \right.$$
$$\left. + \frac{3}{2} \underline{p}_T^{*2} \left| A_2^{(2)} \right|^2 \right] \tag{21}$$

If we bring out the nuclear charge Z and form factor $F(q^2)$ explicitly, the cross section corresponding to M is

$$\frac{d\sigma}{dq^2} = \frac{1}{\omega^2} \left[Z F(q^2) \right]^2 \frac{1}{q^4} \; |M|^2 \; d\Omega_{p^*} dm_{\pi\pi} \tag{22}$$

which has the form as claimed in Eq. (7).

While we expect (judging by the known $\pi - \pi$ resonances) the T = 0 forces to be more important than T = 2 forces, each amplitude $A_1^{(s)}$ and $A_2^{(s)}$ contains contributions from both isospins. Although the amplitude to a state of definite isospin must have the phase of $\pi - \pi$ scattering, they can have different magnitudes for $A_1^{(s)}$ and $A_2^{(s)}$ and so when combined to give a definite charge state $A_1^{(s)}$ and $A_2^{(s)}$ need not have the same phase. If one or another of the isospins is dominant A_1 and A_2 have the same phase. The general analysis, however, involves study of the experimentally difficult $\pi^o\pi^o$ state. The isospin relations for the amplitude are

$$A_i^{(s)}(\pi^+\pi^-) = \sqrt{\frac{1}{3}}\, A_i^{(s)}\,(T = 0) + \sqrt{\frac{1}{6}}\, A_i^{(s)}\,(T = 2)$$
$$A_i^{(s)}(\pi^o\pi^o) = -\sqrt{\frac{1}{3}}\, A_i^{(s)}\,(T = 0) + \sqrt{\frac{2}{3}}\, A_i^{(s)}\,(T = 2)$$

If only T = 0 is present the production of $\pi^+\pi^-$ is equal to $\pi^0\pi^0$ for a given orientation of $\underline{p}^*$; the total number of charged pairs is twice that for $\pi^0\pi^0$. Interference between the different isospins may be eliminated by using the combination of cross sections $2\, d\sigma(\pi^+\pi^-, \underline{p}^*) + d\sigma(\pi^0\pi^0, \underline{p}^*)$ since

$$2|A(\pi^+\pi^-)|^2 + |A(\pi^0\pi^0)|^2 = |A(T = 0)|^2 + |A(T = 2)|^2$$

Note that the kinematic analysis of this section applies equally well to $K\overline{K}$ pairs (with C even). It is no longer true, however, that the phase of production of a state of definite isospin has the phase of $K\overline{K}$ elastic scattering, since the channels $n\pi$ are open to K pairs.

In the region near the ρ^0 meson the interference with the ρ^0 diffractively produced on the nucleus will be important and in fact may be useful through interference with the Coulomb produced s-wave $\pi - \pi$ state. We may take the amplitude in the $\pi - \pi$ c.m. for this ρ^0 production to be $\sim \underline{\epsilon}^i \cdot \underline{p}^*$ since the polarization of the ρ^0 is that of the incident photon. This means that to Eq. (19) we must add, normalizing to Eq. (22)

$$M_\rho = \frac{q^2}{Z\,F(q)}\, e^{i\alpha}\, C\, \underline{\epsilon}^i \cdot \underline{p}^* = D\underline{\epsilon}^i \cdot \underline{p}^* \tag{23}$$

where α is the phase of ρ production on the nucleus, presumably near $\pi/2$, and C is the production magnitude times the Breit-Wigner form for the ρ.

On lead at 15 GeV the nuclear coherent ρ^0 production[11] at t = 0 is $\sim 586\,\text{mb}/(\text{GeV})^2$. This quantity is approximately energy independent. If we assume that σ exists and the respective π^0 and σ coupling constants are the same so that $(\sigma \to \gamma\gamma)/m_\sigma^3 = \Gamma(\pi^0 \to \gamma\gamma)/m_\pi^3$ then Eq. (2) gives on lead for a σ mass of 750 MeV

$$\frac{d\sigma}{dq^2} = 1.3 \times 10^{-2}\, k^2\, \frac{(k\theta/\Delta_0)^2}{\left[1 + (k\theta/\Delta_0)^2\right]^2}\, \frac{\text{mb}}{\text{GeV}^2}$$

with the lab. momentum k in GeV. Thus it would seem that even with a $k = 20$ GeV photon the Coulomb production of a σ which lies under the ρ will be one or two orders of magnitude below the nuclear coherent production. Note, however, that Eq. (23) indicates that the ρ contribution may be strongly suppressed by taking $\underline{p}^* \perp \underline{\epsilon}$, either with polarized or unpolarized photons. Furthermore for lower masses the ρ contribution drops substantially while the Coulomb production rises as $\Delta_0^{-2} \sim (m_{\pi\pi})^{-4}$.

The presence of a small s-wave term may be magnified through its interference with the p-wave; it is noteworthy that the interference exists even with unpolarized photons.

Restricting ourselves to s-wave Coulomb production and the nuclear ρ production we have in the $\pi - \pi$ c.m.

$$M = \underline{\epsilon}^{i} \cdot (\underline{N}_T^* A^{(0)} + \underline{p}^* D) \tag{24}$$

Now $\underline{\epsilon} \cdot \underline{N}_T^*$ or $\underline{\epsilon} \cdot \underline{p}^*$ eliminates s-wave or p-wave respectively. With unpolarized photons

$$\sum_i |M|^2 = \underline{N}_T^{*2} \, |A^{(0)}|^2 + \underline{p}_T^{*2} \, |D|^2 + 2 \, \mathrm{Re}(A^{(0)} D^*) \, \underline{p}^* \cdot \underline{N}_T^* \tag{25}$$

This is then to be inserted in Eq. (22) with D given by Eq. (23).

The interesting s-p wave interference may be examined by studying the asymmetry in the $\underline{p}^* \cdot \underline{N}_T^*$ distribution, i.e., by comparing rates with say the π^+ on the "inside" or "outside" of the momentum transfer direction. An interference effect of this type, peaked near Δ_0, would be evidence for the Coulomb production of the s-wave state, and may be the most promising method for a σ near the ρ.

COULOMB PRODUCTION BY PIONS AND KAONS

A complementary process to the one we have been discussing is Coulomb production by a pion (or kaon) beam, so that the upper vertex in Fig. 1 now corresponds to $\pi + \gamma \rightarrow \pi + \pi$. This has been discussed by several authors in the past[9, 12] we include it for completeness and to emphasize its possible use for studying $\pi - \pi$ interactions. Here again the process takes place predominantly outside the nuclear matter, so the situation is "clean" and the amplitudes have the phase of $\pi - \pi$ scattering. Particularly interesting may be the possibility of eventually doing $\pi + e \rightarrow e + \pi + \pi$ "electroproduction" on the pion.[9] In this case there is no nuclear production and very good resolution is not necessary. A 300 GeV π beam as at Batavia could reach 500 MeV in the π-e c.m. so that pions can be produced. Unfortunately production of s even $\pi - \pi$ states are forbidden so that the first state to be formed will be the well-known p-wave, and the f-waves probably do not enter until substantially heavier masses. That only s odd can be produced may be seen from the following simple argument: the exchanged photon may be thought of as a combination of ρ and ω mesons, but by G parity only the $T = 0, \omega$ component enters. But then the isospin of the final pair is one and only s odd is allowed. Note that this reasoning does not apply to incident kaons, so that a d-wave K-π pair, for example, can be Coulomb produced. The s-wave, however, is always forbidden by parity conservation.

In any case for each spin s there is just one amplitude for the current J_μ connecting the incident particle to the final pair of spin s

$$J_\mu = \sum_s A^{(s)} \epsilon_{\mu\nu\lambda\rho} (k + q)^\nu k^\lambda (kk\ldots \Theta^{(s)})_\rho \tag{26}$$

where k is now the momentum of the incident π. In the $\pi - \pi$ c.m. this becomes

$$\sum A^{(s)} m_{\pi\pi} (\underline{k}^* \times \underline{N}^*)_j (kk\Theta^{(s)})_j \tag{27}$$

Thus for incident pions we have

$$M = m_{\pi\pi}(\underline{k}^* \times \underline{N}^*) \cdot \underline{p}^* \left\{ A^{(1)} + \frac{5}{2}\left[(\underline{k}^* \cdot \underline{p}^*)^2 - \frac{\underline{p}^{*2}\underline{k}^{*2}}{5} \right] A^{(3)} + \ldots \right\} \tag{27}$$

In this case the amplitudes have directly the phase of $T = 1$ π -π scattering in the appropriate partial wave. Pion Coulomb production is thus complementary to that by photons in that odd partial waves are accessible. Estimates have been made for the rate for ρ Coulomb production,[12] giving, for example, $8\mu b$ on Al at 10 GeV.

For incident kaons even partial waves are to be included so that

$$M = m_{K\pi}(\underline{k}^* \times \underline{N}^*) \cdot \underline{p}^* \left\{ A^{(1)} + \frac{3}{2} \underline{p}^* \cdot \underline{k}^* A^{(2)} + \ldots \right\} \tag{28}$$

and the A^i for $K^o\pi$ or $K\pi^o$ contains contributions from $T = 1/2$ and $T = 3/2$ states. Note that Eq. (28) coming from Eq. (20), leads to the general form, Eq. (7), even though the earlier assumption that the mass of the beam particle is negligible may not be true when kaons produce low mass states.

ACKNOWLEDGEMENTS

Leo Stodolsky would like to acknowledge the hospitality of the Laboratoire de Physique Theorique at Orsay where part of this work was done, and to thank his colleagues there for useful discussions on the subject. We would also like to thank R. Blankenbecler, J. Bjorken and A. Suri for helpful comments on several matters, and J. Rosen for discussions on the subject of Coulomb pro-production.

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FIGURE CAPTIONS

1. Photon exchange graph.
2. Kinematic configurations in lab and $\pi - \pi$ c. m. systems.

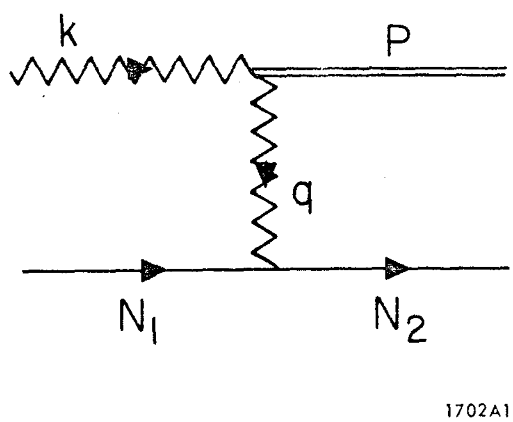

Fig. 1

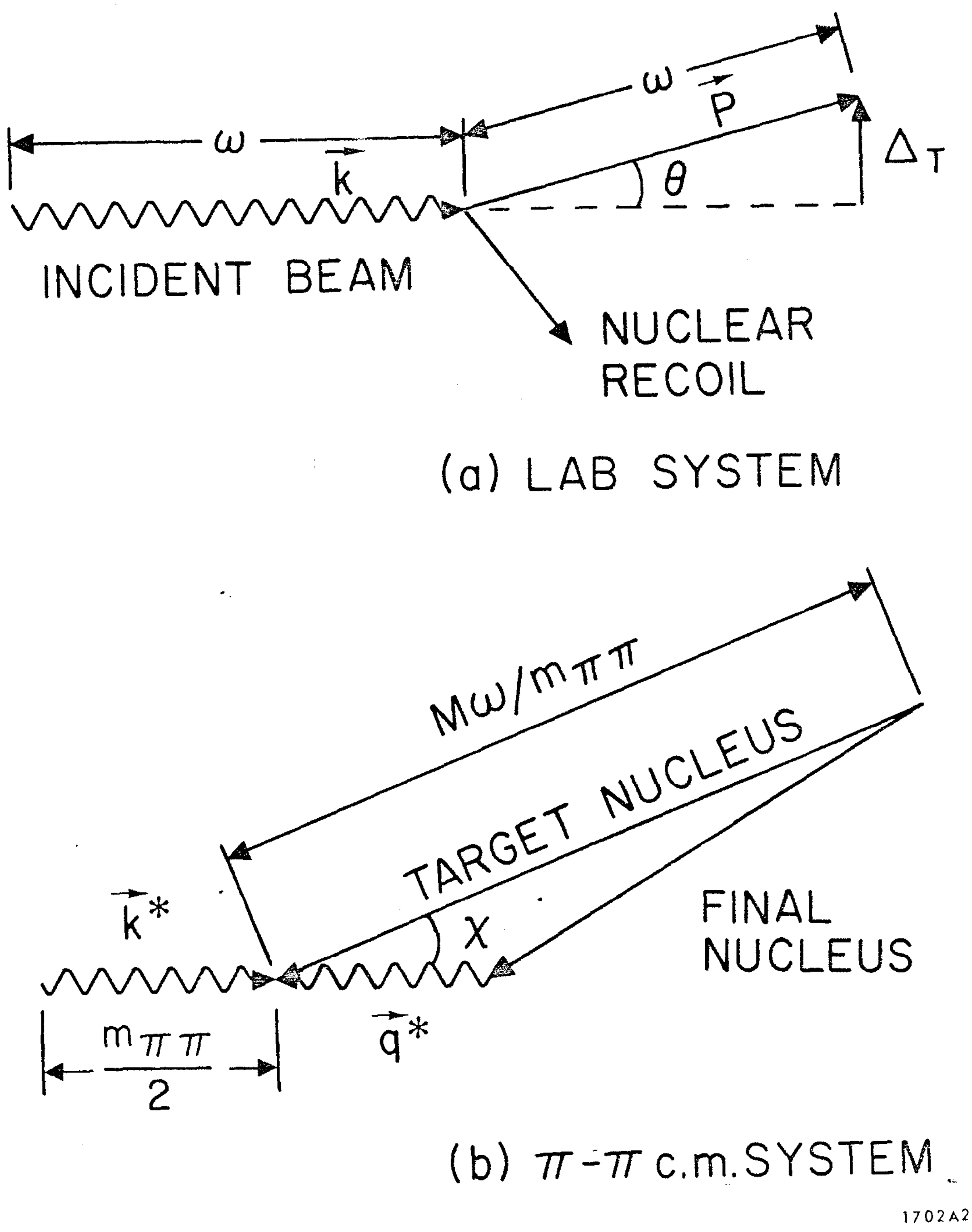
ω
k
ω
P
θ
Δ T
INCIDENT BEAM
NUCLEAR
RECOIL
(a) LAB SYSTEM
Mω/m ππ
TARGET NUCLEUS
k*
χ
FINAL
NUCLEUS
m ππ
2
q*
(b) π-π c.m. SYSTEM
1702A2

Fig. 2