

EXPERIMENTAL DETERMINATION OF THE INELASTIC NEUTRON
FORM FACTOR BY THE SCATTERING OF 12 GEV MUONS
ON HYDROGEN, CARBON AND COPPER*

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ABSTRACT

Results for the ratios of the muon inelastic scattering cross sections on carbon and copper to the cross sections on hydrogen, measured in the same experiment, are presented. The data are consistent with ratios of cross sections for real photoabsorption. In the kinematical region where coherent diffractive effects should be small or absent we find that the inelastic cross section varies as $A^{0.99 \pm .01}$ and that $\sigma_n / \sigma_p = .91 \pm .06$.

(Submitted to Physical Review Letters)

* Work supported by the U. S. Atomic Energy Commission.

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Cross sections for the inelastic scattering of 12 GeV/c positive muons on hydrogen, carbon and copper have been measured using a large aperture optical spark chamber spectrometer in the muon beam¹ at the Stanford Linear Accelerator Center. Preliminary results obtained from a small fraction of the data from this experiment have been reported previously,^{2,3,4} and we refer the reader to Refs. 2 and 4 for descriptions of the apparatus and experimental method. In this letter we are concerned with the ratio of the scattering from carbon and copper to that from hydrogen. Inelastic muon (or electron) scattering can be understood in terms of the one-photon exchange model as the absorption by the hadronic target of the cloud of virtual photons carried by the lepton.⁵ Indeed, the scattering cross section can be written:

$$\frac{d^2\sigma}{dq^2 dK} = \Gamma_T(\sigma_T + \epsilon\sigma_0) \equiv \Gamma_T \sigma_A(K, q^2) \quad (1)$$

where

$$\Gamma_T = \frac{\alpha}{2\pi |q^2|} \frac{K}{p^2} \left(1 - \frac{2\mu^2}{|q^2|} + \frac{2EE' - \frac{|q^2|}{2}}{\nu^2 + |q^2|} \right) \quad (2)$$

$$\epsilon = \left(\frac{2EE' - \frac{|q^2|}{2}}{\nu^2 + |q^2|} \right) / \left(1 - \frac{2\mu^2}{|q^2|} + \frac{2EE' - \frac{|q^2|}{2}}{\nu^2 + |q^2|} \right) \quad (3)$$

$E(E')$ is the energy of the incident (scattered) lepton, $\nu = E - E'$, $K = \nu - q^2/2M$, and is the energy of the real photon required to produce the same final state hadron. q^2 is the square of the four-momentum transfer and μ is the mass of the lepton. σ_T and σ_0 can be thought of as absorption cross sections for transverse and scalar virtual photons. In the limit as $q^2 \rightarrow 0$, $\sigma_T \rightarrow \sigma_\gamma$, the absorption cross section for real photons, and $\sigma_0 \rightarrow 0$. M is the mass of the target, assumed to be at rest in the laboratory. ϵ is the polarization of the virtual photon which for our data is close to unity.

There are two kinds of physical effect to be studied. The first we discuss is "shadowing," which causes $\sigma_A(K, q^2)$ to be less than A-dependent at low q^2 . This has been observed at $q^2 = 0$ in photon total cross section data.^{6,7} The effect was predicted⁸ on the basis of interference of the forward Compton amplitude for one-step direct photon scattering, and two-step forward Compton amplitudes involving real intermediate hadronic states (commonly regarded as almost entirely ρ^0) which would be strongly absorbed in nuclear matter. It should persist⁸ for virtual photons provided that there is an appreciable amplitude for the coherent diffractive production of an intermediate hadron of mass M_I , with a minimum 4-momentum transfer to the target, $t_{\min} = (|q^2| + M_I^2)^2 / 4\nu^2$, small enough for the nucleus to recoil without breakup. For a given element A, one can study shadowing by investigating R_A as a function of K and q^2 , where $R_A \equiv \sigma_A(K, q^2) / \sigma_p(K, q^2)$.

The second effect is the determination of the neutron inelastic form factor $\sigma_n(K, q^2)$. Diffraction models⁹ for inelastic scattering predict equality of σ_n and σ_p while some quark models¹⁰ predict different neutron and proton form factors. At sufficiently high q^2 the shadowing effects discussed above should disappear, and the virtual-photon cross section from a nucleus should be the incoherent sum of the nucleon cross sections:

$$\sigma_A(K, q^2) = Z\sigma_p(K, q^2) + (Z-A)\sigma_n(K, q^2) .$$

It is useful to define a quantity K_{eff} by the equation:

$$K_{\text{eff}} = \frac{\nu M_\rho^2}{|q^2| + M_\rho^2} \propto \frac{1}{\sqrt{t_{\min}}}$$

K_{eff} is the energy of the real photon which gives the same shadowing as a virtual photon of energy ν and mass q^2 , if a rho meson is the intermediate hadron. At

small values of K_{eff} , we can determine the ratio of neutron to proton inelastic form factors directly, while at high values of K_{eff} shadowing may have a significant effect.

Values of the virtual photon absorption cross section on hydrogen, carbon and copper were obtained from the data, as described in Ref. 1. Empty target subtractions were made event-by-event. For the hydrogen data, the subtraction averaged 4%. For the more massive and more localized complex nucleus targets, there were no events to be subtracted. Radiative corrections to the data were made using the methods of Tsai, and Mo and Tsai.¹¹ The major contribution to the radiative corrections in our region comes from the elastic and quasi-elastic scattering. These were calculated for carbon and copper using electron scattering data.¹² For copper, an equivalent form factor using a nuclear radius of 4.0 F was assumed. At high Z it is not entirely correct to treat the data in this fashion, since the higher order (multiphoton) terms should not be neglected. We estimate that this simple treatment leads to uncertainties of the order of 2-6% in the results at low q^2 and high K and less than 1% elsewhere.¹³

Radiative corrections for the continuum are small ($\sim 3\%$). They were estimated using a parameterization for the inelastic form factors. Uncertainties in these corrections are of the order of 15% in the correction for the hydrogen and complex nucleus data, or less than 1% in the cross section. Since the complex nucleus cross sections we observe are very close to A times the nucleon cross sections, the ratios are completely insensitive to the continuum corrections.

We present results for the ratios of the scattering cross sections from carbon and copper to the hydrogen cross sections computed bin-by-bin with no smoothing or interpolation.¹⁴ Table I lists the results and errors for each bin.

All the data were obtained using the same apparatus and analysis programs. Small changes were made for the various runs. Geometrical efficiency factors were calculated for each running condition using a Monte Carlo program which tracked muons through the apparatus. We are confident that the differences in geometry between the various conditions are so small that we can rule out any possible relative uncertainty. Similarly, the monitoring of the beam flux, the efficiency of all counters, electronics, spark chambers, and of the analysis programs should be identical in all cases. However, a 1.8% systematic normalization difference may exist in the heavy element data as a result of uncertainty in the relative measuring efficiencies.

Another feature which might give rise to a systematic difference was the addition of a veto counter with a hole through which the beam passed, just upstream of the complex nucleus targets. Pions produced at backward angles in the laboratory in inelastic interactions could possibly lead to an apparent reduction in the complex nucleus cross sections. We have studied the interactions of real photons at 1.44, 2.89 and 4.66 GeV in a hydrogen bubble chamber¹⁵ in order to estimate the size such an effect might have. It was assumed that the characteristics of the secondaries produced by real and virtual photons would be the same in the photon nucleon center-of-mass system at the same photon nucleon invariant mass. Each bubble chamber event was treated individually and the secondary particles were transformed from the center-of-mass into the laboratory system for several assumed values of q^2 . The results of this study show that pions in the appropriate angular range would be emitted in less than .5% of the interactions. We have made no correction to the data for this effect.

The data from Table I are shown in Fig. 1 plotted against the quantity K_{eff} , defined previously, so that they may be compared with data from experiments using real photons. It can be seen that our results are in good agreement with the real photon data⁶ of Meyer et al. and Caldwell et al., which are also shown in Fig. 1. The muon-carbon scattering data of Hoffman et al.⁷ show a somewhat higher overall normalization than this experiment but are not in serious disagreement with our data. However, their extrapolated result for $\sigma_{\gamma\text{C}}(q^2=0)$ depends on a steeper increase of the cross section at low q^2 than is indicated by our data.

Our data are mostly concentrated in the region of low K_{eff} , where shadowing effects should be small. However, no marked effect is seen at high K_{eff} .¹⁶ (For comparison, we show the simple ρ dominance model predictions of Ref. 8 also in Fig. 1.) If we assume that $\sigma_n = \sigma_p$ and that the dependence on K_{eff} is given correctly by the Brodsky-Pumplin model and use our data to determine the magnitude of the effect, we find that the amount of shadowing is $54 \pm 30\%$ of these predictions in the case of carbon, $21 \pm 24\%$ in the case of copper.

In Fig. 2 we show the data averaged over K , plotted as a function of q^2 , for $K > 3$, and for $K < 3$. The data are also shown averaged over q^2 , plotted versus K , for $|q^2| < .4$ and for $|q^2| > .4$. Again we see that the shadowing, if present, is weak and that the data support the simple assumption of a straightforward A -dependence independent of K and q^2 . If we exclude the data with $K_{\text{eff}} > 3$ GeV to eliminate any question of possible shadowing effects, we find that the mean values of the ratios to hydrogen are:

$$R_{\text{C}} = 11.0 \pm .4, \quad R_{\text{Cu}} = 63.1 \pm 2.2$$

The results establish an A -dependence for inelastic scattering of $A^{0.99 \pm 0.01}$. Since we see no evidence for shadowing in these data, the reaction must take place incoherently on the individual nucleons. We can, therefore, use them to

determine the ratio of neutron to proton virtual photoabsorption cross sections. From the carbon, copper and hydrogen data with $K_{\text{eff}} < 3$ GeV, we find

$$\sigma_n/\sigma_p = .91 \pm .06 .$$

In the determination of the A-dependence and the neutron/proton ratio we have included 2% systematic error in quadrature with the statistical errors.

Inclusion of the data with $K_{\text{eff}} < 3$ GeV does not materially change the result.¹⁶ We have estimated the effect of Fermi motion on this result, using a simple uniform phase space for the Fermi momenta. The σ_n/σ_p ratio is changed by at most .3%. Our result for virtual photons is in good agreement with the neutron/proton ratio obtained using real photons on H_2 and D_2 by Caldwell et al.¹⁷ of $.86 \pm .05$ at 4.1 GeV, and Meyer et al.,¹⁷ whose result is 0.93 ± 0.04 averaged from 1.5 to 3 GeV.

Many models for inelastic lepton scattering predict equality, or near equality, of neutron and proton cross sections. Among them are general diffractive models,⁹ the vector dominance model,¹⁸ and also the field theoretic model of Drell and Yan.¹⁹ Our result is in agreement with such models. The simplest quark model, with three quark model, with three quarks, predicts a ratio of .67,¹⁰ in clear disagreement with our result. It also fails to account for the shape of the inelastic cross section observed from hydrogen. More sophisticated quark models,¹⁰ which incorporate $Nq\bar{q}$ pairs in addition to the three quarks, predict a ratio of $\frac{2+N \cdot 4/3}{3+N \cdot 4/3}$ for the n-p ratio. Our data suggest a high value for N in such models.

To summarize:

1. We do not find strong evidence for (or against) coherent shadowing effects in our data.
2. In the kinematic region where scattering from nuclei should be incoherent ($K_{\text{eff}} < 3$), the carbon/hydrogen and copper/hydrogen

cross section ratios have little variation, indicating that σ_n and σ_p have roughly the same dependence on K and q^2 .

3. Our results:

$$\sigma_C/\sigma_H = 11.0 \pm 0.4$$

$$\sigma_{Cu}/\sigma_H = 63.1 \pm 2.2$$

$$\sigma_A = \sigma_p A^{0.99 \pm 0.01}$$

$$\sigma_n/\sigma_p = 0.91 \pm 0.06$$

have been determined using data having $K_{\text{eff}} < 3$ GeV. These data range over $0.6 \lesssim K \lesssim 5.0$ and $0.3 \lesssim q^2 \lesssim 2$ (GeV/c)² and have statistical weight which is centered near $K=2$ and $|q^2| = 0.6$ ($K_{\text{eff}} \sim 1$).

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14. Preliminary results from this experiment for the hydrogen data show good agreement of the absolute normalization with photoabsorption at low q^2 and with inelastic electron scattering at high q^2 .⁴ A detailed comparison of the absolute cross sections from hydrogen for muons and electrons will be made in a future publication.
15. Peter Seyboth (private communication). We thank the SLAC-Berkeley-Tufts Collaboration for making their raw data available.
16. The authors have previously reported (Ref. 4) 10 GeV carbon/hydrogen ratios with less statistics and greater systematic errors. Inclusion of these data, which show greater shadowing, changes $R_C(K_{\text{eff}} < 3 \text{ GeV})$ to 10.7 ± 0.30 . This results in slightly different A dependence and neutron/proton ratios: $\sigma_A = \sigma_p A^{0.98 \pm 0.01}$ and $\sigma_n / \sigma_p = 0.86 \pm 0.05$.
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TABLE I

Cross section ratios and statistical errors for various bins are listed.

The ratios are defined as the ratios of the differential cross sections integrated over the K and q^2 range of the bin.

$$R_A = \left(\int_{\text{bin}} \frac{d^2\sigma}{dq^2 dk} (\mu + A \rightarrow \mu + \text{all}) dq^2 dk \right) / \left(\int_{\text{bin}} \frac{d^2\sigma}{dq^2 dk} (\mu + p) \rightarrow \mu + \text{all}) dq^2 dk \right)$$

TABLE I

K	$ q^2 $	K_{eff}	R_C	R_{Cu}
.6 - 1.0	.3 - .4	0.617	11.0 ± 1.5	58.9 ± 8.4
	.4 - .6	0.574	13.4 ± 1.4	75.7 ± 8.3
	.6 - .8	0.533	9.0 ± 1.5	65.6 ± 11.2
	.8 - 1.2	0.491	10.2 ± 2.2	80.8 ± 17.1
	1.2 - 1.6	0.455	7.7 ± 3.3	52.3 ± 23.5
1.0 - 2.0	.3 - .4	1.054	11.0 ± 1.3	60.4 ± 7.4
	.4 - .6	0.951	9.7 ± 0.8	58.2 ± 5.2
	.6 - .8	0.852	9.3 ± 1.2	54.1 ± 7.9
	.8 - 1.2	0.749	13.4 ± 1.8	59.9 ± 10.2
	1.2 - 1.6	0.661	11.8 ± 3.1	72.8 ± 20.9
	1.6 - 2.0	0.602	7.2 ± 3.2	47.5 ± 22.6
2.0 - 3.0	.3 - .4	1.679	10.9 ± 1.7	65.1 ± 10.7
	.4 - .6	1.490	12.2 ± 1.4	75.1 ± 9.4
	.6 - .8	1.306	7.3 ± 1.3	58.1 ± 10.8
	.8 - 1.2	1.118	12.8 ± 2.3	43.2 ± 10.9
	1.2 - 1.6	0.955	12.5 ± 3.8	45.7 ± 18.8
	1.6 - 2.0	0.847	19.7 ± 7.9	91.0 ± 43.4
	2.0 - 3.0	0.725	4.7 ± 3.6	49.5 ± 31.7
3.0 - 5.0	.25- .4	2.681	11.0 ± 1.0	59.7 ± 5.9
	.4 - .6	2.298	10.1 ± 1.2	71.8 ± 8.6
	.6 - .8	1.988	11.1 ± 1.9	68.2 ± 12.7
	.8 - 1.2	1.671	14.1 ± 2.6	62.6 ± 13.8
	1.2 - 1.6	1.397	25.5 ± 9.6	135.9 ± 55.8
	1.6 - 2.0	1.214	6.4 ± 3.2	39.6 ± 21.7

K	$ q^2 $	K_{eff}	R_C	R_{Cu}
5.0 - 6.5	.1 - .2	4.638	9.4 ± 1.2	54.5 ± 7.5
	.2 - .4	3.904	12.4 ± 1.3	56.1 ± 7.6
	.4 - .6	3.241	12.6 ± 2.4	55.5 ± 13.6
	.8 - .8	2.784	6.4 ± 2.3	52.3 ± 18.7
	.8 - 1.2	2.316	10.2 ± 3.4	60.9 ± 22.5
	1.2 - 1.6	1.911	36.7 ± 49.1	194.5 ± 267.3
6.5 - 8.3	.1 - .2	5.951	11.4 ± 2.0	70.9 ± 13.3
	.2 - .4	4.993	10.3 ± 1.6	67.5 ± 11.4
	.4 - .6	4.129	11.7 ± 3.3	77.8 ± 23.9
	.6 - .8	3.534	9.8 ± 3.7	47.2 ± 22.5
.6 - 1.0	.3 - .4		11.0 ± 1.5	58.9 ± 8.4
1.0 - 2.0	.3 - .4		11.0 ± 1.3	60.4 ± 7.4
2.0 - 3.0	.3 - .4		10.9 ± 1.7	65.1 ± 10.7
3.0 - 5.0	.25- .4		11.0 ± 1.0	59.6 ± 5.9
5.0 - 6.5	.1 - .4		10.4 ± 1.0	55.0 ± 5.6
6.5 - 8.3	.1 - .4		11.0 ± 1.3	69.4 ± 9.0
.6 - 1.0	.4 - 1.6		11.4 ± 0.9	72.3 ± 6.1
1.0 - 2.0	.4 - 2.0		10.4 ± 0.6	58.3 ± 4.0
2.0 - 3.0	.4 - 3.0		11.1 ± 0.9	61.2 ± 5.8
3.0 - 5.0	.4 - 2.0		11.9 ± 1.0	70.7 ± 6.5
5.0 - 6.5	.4 - 1.6		11.5 ± 1.9	62.7 ± 11.6
6.5 - 8.3	.4 - .8		10.9 ± 2.5	64.2 ± 6.5

K	$ q^2 $	K_{eff}	R_C	R_{Cu}
.6 - 3.0	.3 - .4		11.0 ± 0.8	60.9 ± 5.0
	.4 - .6		11.3 ± 0.6	67.0 ± 4.0
	.6 - .8		8.6 ± 0.8	58.3 ± 5.6
	.8 - 1.2		12.6 ± 1.2	59.1 ± 7.0
	1.2 - 1.6		11.1 ± 2.0	59.7 ± 12.3
1.0 - 3.0	1.6 - 2.0		12.2 ± 3.5	65.1 ± 21.7
2.0 - 3.0	2.0 - 3.0		4.7 ± 3.6	49.5 ± 31.7
5.0 - 8.3	.1 - .2		10.1 ± 1.0	60.4 ± 6.7
3.0 - 8.3	.2 - .4		11.2 ± 0.7	60.2 ± 4.4
	.4 - .6		10.9 ± 1.0	68.9 ± 7.1
	.6 - .8		9.7 ± 1.4	60.0 ± 9.6
3.0 - 6.5	.8 - 1.2		12.9 ± 2.1	62.0 ± 11.8
	1.2 - 1.6		27.8 ± 11.4	148.1 ± 64.4
3.0 - 5.0	1.6 - 2.0		6.4 ± 3.2	39.6 ± 21.7
		0 - 1.0	10.8 ± 0.5	62.1 ± 3.1
		1.0 - 2.0	11.7 ± 0.7	64.7 ± 4.2
		2.0 - 3.0	10.4 ± 0.7	62.3 ± 4.7
		3.0 - 5.0	10.7 ± 0.7	58.2 ± 4.6
		5.0 - 6.5	11.4 ± 2.0	70.9 ± 13.3
		0 - 3.0	11.0 ± 0.4	63.1 ± 2.2
		3.0 - 6.5	10.8 ± 0.7	60.6 ± 4.4

FIGURE CAPTIONS

1. The ratio of cross sections for (muon-nucleus)/(muon-proton) inelastic scattering is plotted against the "equivalent photon energy" K_{eff} . The dotted line shows the shadowing predictions of Brodsky and Pumplin (Ref. 8) assuming ρ dominance. The parameters used for the calculation were $M_\rho = 760$ MeV, $\Gamma_\rho = 110$ MeV, $r_0 = 1.2$ F, $\sigma_{\rho N} = 25$ mb. Graphs (a) and (b) show the carbon/hydrogen cross section ratios and the copper hydrogen cross section ratios, respectively. Errors are statistical only and the points are as listed in Table I.
2. Graphs (a) - (d) show the data of Table I versus K for carbon and copper with $|q^2| < .4$ and $|q^2| > .4$ (GeV/c)². Graphs (e) - (h) show the same data plotted versus $|q^2|$ for $K < 3$ GeV and $K > 3$ GeV. Shadowing, if present, would cause a R_A to decrease where K is large and q^2 is small.

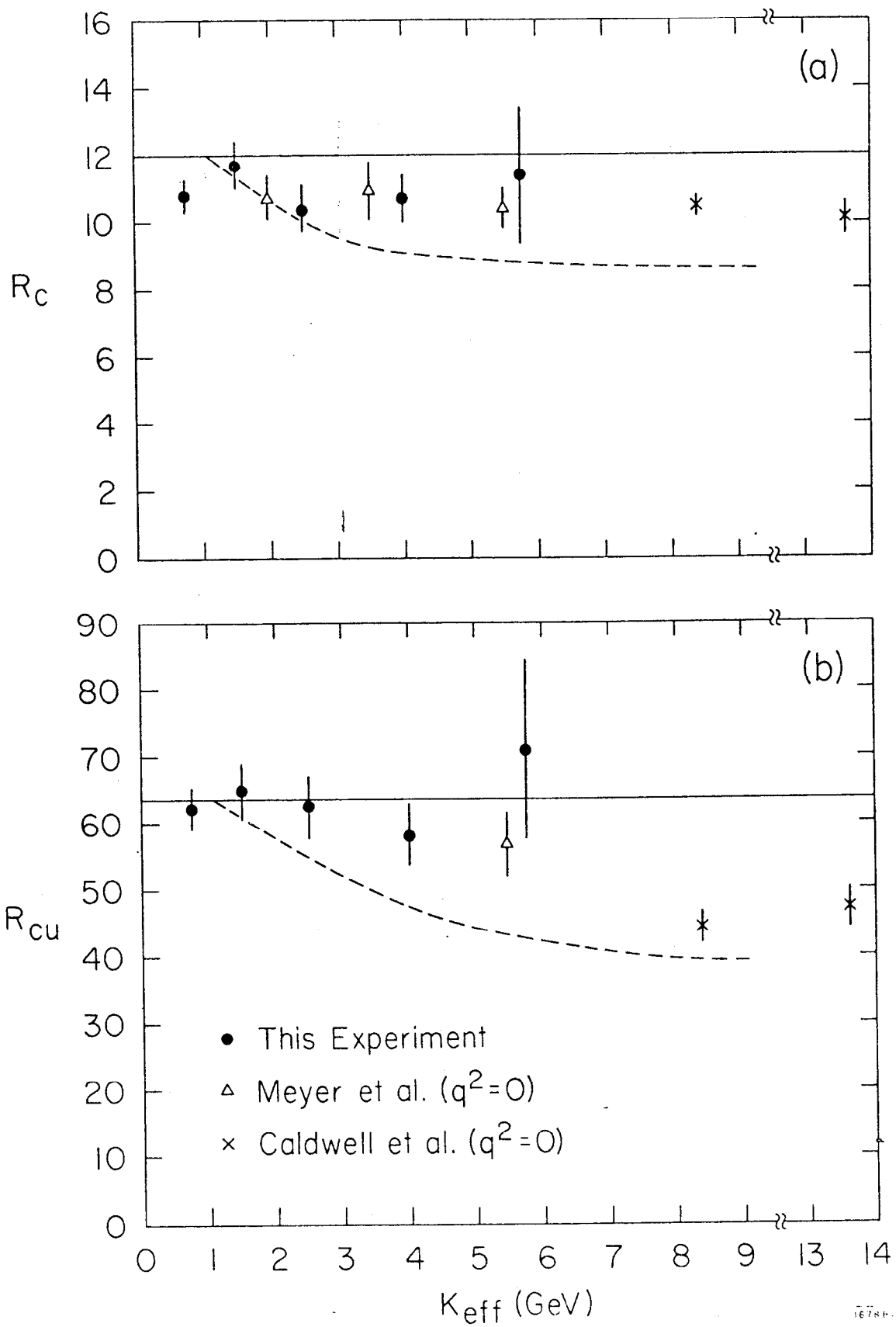


Fig. 1

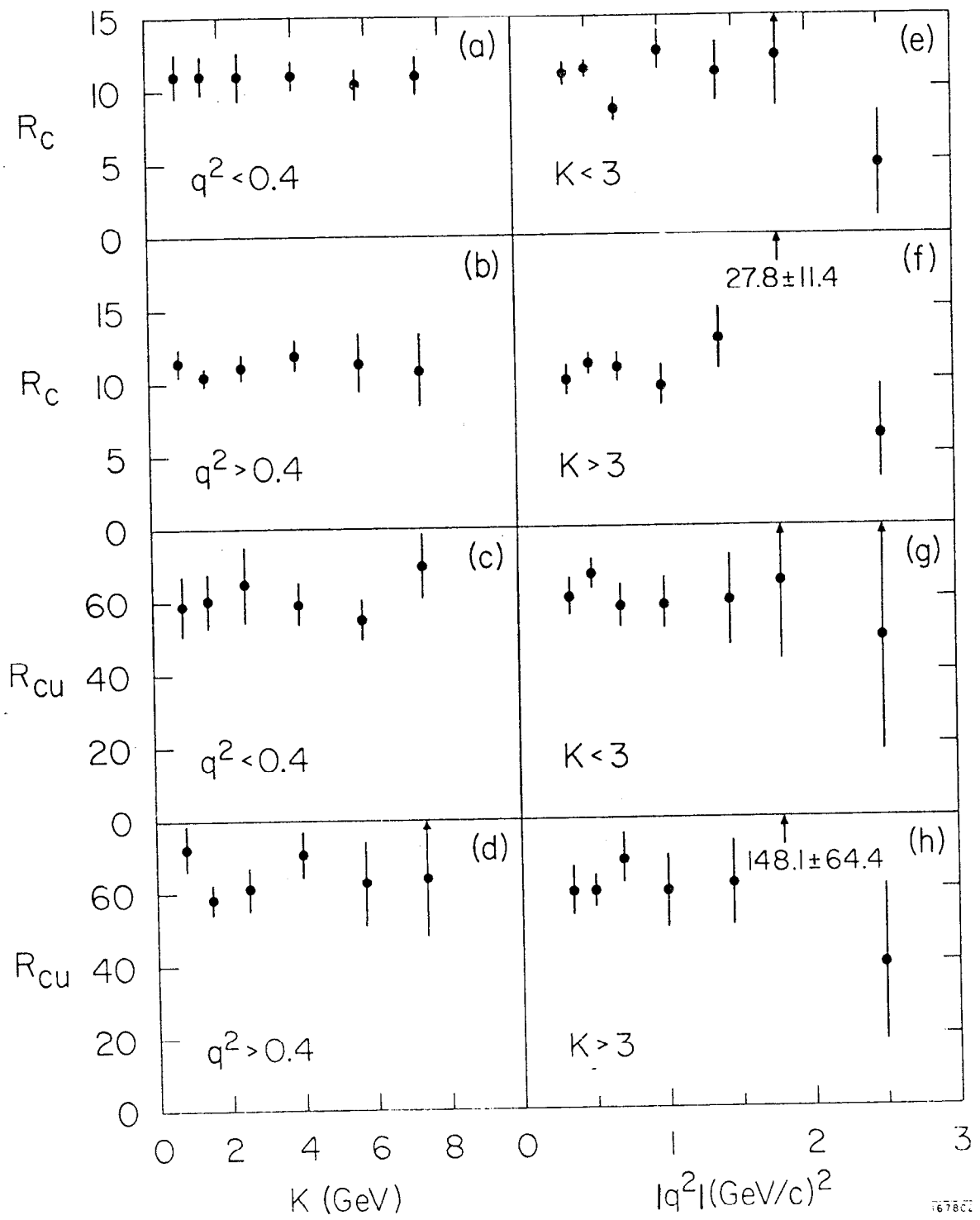


Fig. 2