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RESONANCE MIXING, "DIPOLES", AND "ZOMBIES"

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If the lively discussion of the ρ-ω system and the A_2 splitting at this conference is any guide, the problems of resonance mixtures will continue to be an active one for some time. Therefore I would like to review here a simple formalism for treating the problem and in particular to emphasize certain points:

1. The parameter which gives the nonorthogonality of the underlying states caused by their mixing is in principle directly measurable in a "missing-mass" type experiment, being given by the interference term between the Breit-Wigner shapes. Thus the need for a large interference term in the A_2 rules out any kind of "weak mixing" explanation.
2. Perhaps even more striking than the "dipole" is the apparent need in the A_2 problem for a "dead" state, which couples neither to production nor decay. This seems forced by the similiarity of the A_2 effect in all decay modes. Perhaps such states should be looked for in isolation. Simple formulae are given for the cross section shapes for such phenomena.

THE MASS MATRIX FORMALISM

The problem of resonances and their mixing is simply and explicitly attacked by thinking of two kinds of states. First there are the discrete

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eigenstates $|\alpha\rangle$, $|\beta\rangle$, ... of a Hamiltonian H^o, where H^o contains all interactions except that causing the decay of the states. The second kind of states, which we call $|m\rangle$, $|n\rangle$, ... are the eigenstates of a "mass matrix" M. M arises in the course of considering the effect of an interaction, V, which makes the levels $|\alpha\rangle$, $|\beta\rangle$, ... unstable. M is a matrix in the finite dimensional space $|\alpha\rangle$, $|\beta\rangle$, ... and the $|n\rangle$, $|m\rangle$, ... are simple linear combinations of them. The importance of these eigenstates of M is that they decay as an exponential in time or correspond to a simple Breit-Wigner term in the energy variable. The eigenvalues of M are complex, $M|n\rangle = E_n|n\rangle = (E_n^o - i/2\,\Gamma_n)|n\rangle$, representing the energy and width of $|n\rangle$. Since H_0 is Hermitian, its eigenstates are orthogonal

$$\langle\alpha|\beta\rangle = 0$$

but (1)

$$\langle n|m\rangle \neq 0$$

in general, since M is not Hermitian, as we'll see.

We note in advance that if the resonances in question are well separated then the non-Hermitian ("width") part of M is small compared to the Hermitian (real energy) part and the problem reduces to the familiar one of diagonalizing an effective Hamiltonian. Thus nothing special is to be expected unless the resonances are overlapping.

The problem of a set of levels $|\alpha\rangle$, $|\beta\rangle$, ... which become unstable through the "turning on" of a V can be solved explicitly and without the use of perturbation theory[1] (subject to the proviso that the effects of turning on V are not so violent so as to completely disrupt the spectrum set up by H^o or at least that V can be separated into parts to give a new H_0 so that this does not happen). The resulting formula for M, expressed in the basis of states $|\alpha\rangle$, $|\beta\rangle$, ... is

$$M_{\alpha\beta} = M^o_{\alpha\beta} + \int dk \sum_k \frac{\langle\alpha|V|k, j+\rangle\langle +k, j|V|\beta\rangle}{E - E(k) + i\epsilon} \tag{2}$$

The $M^o_{\alpha\beta}$ are the matrix elements of H^o before V is turned on, i.e., the original energies. The $|k, j\rangle$ states are the continuum states in the spectrum of H^o, the "final states" of the decay with momentum k in channel j. The + symbol takes care of any final state interaction between these states. The integral has a principal value part which is real and corresponds to simply a real level shift; we combine this with M^o to get a new M^o. The δ function contribution at $E(k)=E$ brings in a factor of i and gives us a width matrix Γ. We have then

$$M_{\alpha\beta} = M^o_{\alpha\beta} - \frac{i}{2}\,\Gamma_{\alpha\beta} \tag{3}$$

with

$$\Gamma_{\alpha\beta} = \sum_k 2\pi \langle\alpha|V|j,k+\rangle\langle +k,j|V|\beta\rangle\,\delta(E-E(k)) \,, \tag{4}$$

i.e., the "Golden Rule" in matrix form. While both M and Γ are Hermitian since the numerator in the integral in Eq. (2) is Hermitian, M can be anything, obviously.

With time reversal invariance and the usual choice of phases, however, M is symmetric

$$\widetilde{M} = M \tag{5}$$

(using time reversal)

and M and Γ are real. Thus looking at the integral in Eq. (2), we have the usual correspondence: virtual transitions, with $E(k)\neq E$ give contributions to the real part of M; physical transitions, with $E=E(k)$, give contributions to the imaginary part of M. This means that the elements of Γ may be found or inferred from experimental data on the decays of the states. M on the other hand is more difficult and we must rely on models or symmetry statements. Although we henceforth assume time reversal invariance, note that this does not simplify things especially. Eigenvectors are still not necessarily orthogonal and a symmetric matrix need not even be diagonalizable, e.g., $\begin{pmatrix} 0 & 1 \\ 1 & 2i \end{pmatrix}$.

MATHEMATICS

The relevant mathematics is the following. By substitution of the symbols $+ \rightarrow \sim$ the usual proof for the orthogonality of the eigenvectors of a Hermitian matrix becomes for a symmetric matrix

$$\widetilde{U}_n U_m = 0, \qquad n\neq m \tag{6}$$

where U_m is the m^{th} eigenvector. The lack of the usual complex conjugation in Eq. (6) means that bizarre things can happen, for example, $\binom{1}{i}$, the eigenvector of the matrix mentioned above, is "orthogonal" to itself in the sense of Eq. (6).

If, however, the eigenvalues of M are nondegenerate or more generally if its eigenvectors span the space such things do not happen.[2] M can then be diagonalized and has as many distinct eigenvectors as it has dimensions. But if there should be degenerate eigenvalues then M may not be diagonalizable. In this case it will have fewer eigenvectors than dimensions. This later situation may be recognized by the fact that although the matrix (or relevant submatrix for larger than 2×2) has degenerate eigenvalues, it is not proportional the unit matrix as it would necessarily be if its eigenvectors were distinct. Physically, matrices approaching this nondiagonalizable condition correspond to the appearance of the "dipole" phenomenon. We can change Eq. (6) to the usual bra-ket notation by using the time reversal operator T to introduce a complex conjugation. That is, since $|\alpha\rangle$, $|\beta\rangle$, ... are taken to be eigenstates under T we have

$$T(a|\alpha\rangle + b|\beta\rangle, \ldots) = a^*|\alpha\rangle + b^*|\beta\rangle, \ldots$$

so that Eq. (6) is

$$\langle Tn|m\rangle = 0, \qquad n \neq m \tag{7}$$

Our choice of phases is the conventional one; for example if $|\alpha\rangle$ is ρ_0 meson (of pure isospin 1), and $|j\rangle$ is the 2π state, then in the matrix element $\langle\alpha|V|k,j\rangle$ is essentially the real coupling constant $f_{\rho\pi\pi}$. On the other hand, the $|n\rangle$, $|m\rangle$... are expressed in terms of the $|\alpha\rangle$, $|\beta\rangle$... with complex coefficients, so that the $\langle n|V|k,j\rangle$ are not necessarily real.

It is important to notice that the usual expansion of the unit matrix as $I = \sum_\alpha |\alpha\rangle\langle\alpha|$ does not hold for the $|n\rangle$, $|m\rangle$, due to their nonorthogonality. Instead, if we supplement Eq. (7) with a normalization condition so that

$$\langle Tn|m\rangle = \delta_{nm} \tag{8}$$

then we can write

$$I = \sum_n |n\rangle\langle Tn| = \sum_n |Tn\rangle\langle n| \tag{9}$$

The normalization chosen in Eq. (8) is not the usual one and requires special handling as $\langle Tn|n\rangle \rightarrow 0$. This only happens, however, in the case that M becomes nondiagonalizable, otherwise it is diagonalized by an orthogonal matrix[3] so that the product is preserved, and cannot be zero. A final point follows from the definition of $|n\rangle$, $|m\rangle$ as eigenvectors with (complex) eigenvalues $E_n, E_m, \ldots$. Applying this and subtracting we get

$$\langle n|M - M^{+}|m\rangle = (E_m - E_n^*)\ \langle n|m\rangle$$

or from Eq. (3) (10)

$$\langle n|\Gamma|m\rangle = i\ (E_m - E_n^*)\ \langle n|m\rangle$$

For $n \neq m$ this is known as the Bell-Steinberger relation;[4] for n=m it gives the width of $|n\rangle$, since $E_n - E_n^* = -i\Gamma_n$.

FORMATION AND PRODUCTION

For "formation" reactions in which the resonances are formed directly from the incoming particles, as in $e^+e^- \rightarrow (\rho, \omega) \rightarrow 2\pi$, the scattering amplitude can be solved for directly to give

$$T_{+i} = \langle f, k_f|V|\alpha\rangle \left[\frac{1}{M-E}\right]_{\alpha\beta} \langle\beta|V|k_i, i\rangle\ . \tag{11}$$

If "final state interactions" with the same quantum numbers are present other than that already accounted for by the resonances, they can be taken care of by simply adding to (11) the scattering they would produce in isolation and then by putting appropriate + and - symbols in the initial and final states.[1]

By inserting Eq. (9), Eq. (11) can be brought to the form of a sum of distinct Breit-Wigner terms:

$$T_{fi} = \sum_n \frac{\langle f, k_f|V|n\rangle \langle Tn|V|k_{ij}\, i\rangle}{E_n - E}$$

and using the time reversal invariance of V

$$T_{fi} = \sum_n \frac{\langle f, k_f|V|n\rangle \langle i, k_i|V|n\rangle}{E_n - E} \tag{12}$$

Note the explicit symmetry in the indices i and f, exhibiting the time reversal invariance. In general $\langle f, k_f|V|n\rangle$ will depend weakly on the momentum and if we take out an explicit spin function describing the angular distributions, we can write for the reduced matrix element

$$\langle i|V|n\rangle = f_{in} \quad . \tag{13}$$

These f_{in} are the coupling constants from the physical resonance states to the initial and final states. Then we have for the resonant cross section into channel f

$$\sigma_f \sim \overline{\sum_i} |T_{if}|^2 \sim \overline{\sum_i} \left| \sum_n \frac{f_{fn}\, f_{in}}{E_n - E} \right|^2 \tag{14}$$

The f_{in} can be expressed in terms of the couplings $f_{i\alpha}$ to the basic states $|\alpha\rangle$, $|\beta\rangle$ .... It is the $f_{i\alpha}$ that are real and that are usually known or guessed in terms of SU_3, the quark model, etc. So if we have

$$|n\rangle = a|\alpha\rangle + b|\beta\rangle + \ldots \tag{15}$$

then Eq. (13) becomes

$$f_{in} = a\, f_{i\alpha} + b\, f_{i\beta} + \ldots \quad . \tag{16}$$

In general M(E) is a function of energy; in addition to the explicit E dependence in the dispersion integral in Eq. (2), we expect phase space and kinematic factors to vary with energy. This adds no fundamental complication since everything we have done can be done at each energy separately. If

M varies somewhat across the resonance region, it simply means that the E_n in Eq. (14) and the a's and b's in Eqs. (15) and (16) may be slowly energy varying. Various models may be used for this energy dependence, for example the usual assumption that widths vary as $\Gamma \sim k^{2\ell+1}$ due to phase space factors might suggest using $\Gamma_{nm} \sim k^{2\ell+1}$ since the same phase space sums appear. In the present phenomenological context the "current-mixing" formalism, which has to do with taking into account threshold constraints on S=1 meson resonances to presumably coupled to conserved currents, simply amounts to a certain model for treating this energy variation. In any case, even for rather broad resonances, such as in the ω-ρ system, such effects are small.

If we now consider "production" reactions like $\pi N \rightarrow (\rho^0, \omega) N$, where the resonant system is produced together with other particles in the final state, we cannot, due to the complexities of the more than two-body problem give a complete derivation of the scattering amplitude from first principles. If we assume, however, that the resonant system, once produced by a production amplitude A, does not further interact with the other particles in the final state, then the obvious generalization of Eq. (11) is, for decay into channel f

$$T_{fi} = \langle f, k_f | V | \alpha \rangle \left[\frac{1}{M - E}\right]_{\alpha\beta} A_\beta(i) \tag{17}$$

The symbol i represents the ensemble of quantum numbers characterizing the initial state of the reaction as well as those of the other particles in the final state. The cross section into channel f is found analogously to Eq. (14), by inserting Eq. (9) so that

$$T_{fi} = \sum_n \langle f, k_f | V | n \rangle \frac{1}{E_n - E} A_{Tn}(i)$$

and

$$\sigma_f \sim \overline{\sum_i} \left| \sum_n \frac{f_{fn} A_{Tn}(i)}{E_n - E} \right|^2 \tag{18}$$

The notation A_{Tn} means that the production amplitude A is to be evaluated onto the state $\langle Tn|$ related to Eq. (15) by

$$\langle Tn| = \langle \alpha| a + \langle \beta| b + \ldots \tag{19}$$

THE NONORTHOGONALITY

When we imagine "turning on" the V which couples the stable states to the continuum it may be that V respects some quantum number which serves to distinguish the states. In that case there remain essentially uncoupled states that have different quantum numbers and a unitary transformation will diagonalize M. Then we have the conventional situation where the eigenstates are orthogonal, $\langle n|m\rangle = 0$. This is the case in the K^0 system without CP violation or the ρ, ω system without isospin violation. The further weak violation of the symmetry here will lead to a small nonorthogonality of the eigenstates. Or it can simply happen, as may be the case for the A_2 splitting that there is no quantum number involved and the coupling between the states immediately produces a large nonorthogonality.

In any case it is interesting to note that the nonorthogonality of the states is effectively measured in the "missing-mass" type of experiment. The size of the interference term needed in addition to the simple sum of incoherent Breit-Wigner intensities gives this nonorthogonality. This is because in the missing-mass experiment we sum over all final states f for the decay of the resonant system and if the states involved are orthogonal the interference dips and peaks in different f cancel each other out. This follows from formal manipulation of Eq. (17), since we can put

$$\sum \sigma_f = \sum_f |T_{fi}|^2 = \sum_f A^*_\gamma(i) \left[\frac{1}{M^\dagger - E}\right]_{\gamma\delta} \langle \delta|V|k_f, f\rangle\langle f, k_f|V|\alpha\rangle \left[\frac{1}{M-E}\right]_{\alpha\beta} A_\beta(i) \tag{20}$$

Now if we carry out the $\sum_f$ over final decay states we see that the bilinear form $V^\dagger V$ in Eq. (20) is by Eq. (4), just the width matrix Γ so in symbolic notation

$$\sum \sigma_f \sim A^\dagger(i) \frac{1}{M^\dagger - E} \Gamma \frac{1}{M - E} A(i) \tag{21}$$

Now using Eqs. (8) and (9) appropriately

$$\sum \sigma_f \sim \sum A^*_{Tn}(i) \; \frac{1}{E^*_n - E} \; \langle n \,|\, \Gamma \,|\, m \rangle \; \frac{1}{E_m - E} \; A_{Tm}(i) \quad . \tag{22}$$

If Γ does not connect $|n\rangle$ and $|m\rangle$ there is no interference between different Breit-Wigner terms. But $\langle n|\Gamma|m\rangle$ is related to the nonorthogonality of $|n\rangle$ and $|m\rangle$ by Eq. (10), $\left(\langle n|\Gamma|m\rangle = i\,(E_m - E^*_n)\,\langle n|m\rangle\right)$, so if $\langle n|m\rangle = 0$ there is no interference in $\sum\sigma_f$. Hence the need for interference terms beyond the simple sum of Breit-Wigner intensities in experiments where decay modes are not selected is *prima facie* evidence for the nonorthogonality of the underlying wavefunctions. In principle the $\langle n|\Gamma|m\rangle$ can be found by summing over i and fitting Eq. (22) to the data. The utility of knowing $\langle n|\Gamma|m\rangle$, or $\langle n|m\rangle$ is that it is a global parameter characterizing the entire system. Thus in the theory of the ρ-ω system[5] we have a definite prediction $\langle \rho|\omega\rangle = -i2 \text{ Im } \epsilon \cong -i\,(0.12)$, while in the A_2 problem the inability to fit with incoherent Breit-Wigner automatically indicates a large nonorthogonality of the states.

THE DIPOLE AND THE ZOMBIE

What happens if the two eigenvalues of a 2×2 matrix, in both their real and imaginary parts, approach each other? Either we get $M \sim I$ or M approaches a nondiagonalizable matrix. When this happens the two eigenvectors approach each other and become parallel; the nonorthogonality discussed above becomes maximal, and the dipole can result.

For 2×2, this matrix is essentially unique up to addition and multiplication by constants. The degenerate eigenvalue E^d fixes one of these constants. That is

$$E_d I - d \begin{pmatrix} -i & \pm 1 \\ \pm 1 & i \end{pmatrix} \tag{23}$$

is the general symmetric nondiagonalizable matrix, with eigenvalue $E_d = E^o_d - i\,(\Gamma_d/2)$. Furthermore let us rotate to the basis in which the width matrix Γ is diagonal. Since by time reversal Γ is real and symmetric, this can be accomplished by an orthogonal transformation and M remains symmetric. Thus (23) still applies but d must now be real, and since widths must be positive, $-\Gamma/2 \leq d \leq \Gamma/2$. The connection with non-exponential decay is most seen easily in terms of conventional basis formed of the eigenvector of (23), $|u\rangle$, and $|Tu\rangle$ forming a conventional orthonormal basis since $\langle Tu|u\rangle = 0$. Here M_d is in reduced form

$$M = \begin{pmatrix} E_d & +2id \\ 0 & E_d \end{pmatrix}$$

and the solutions to $i d\psi/dt = M\psi$ are $\binom{1}{0} e^{-iE_d t}$ and $\binom{2idt}{1} e^{-iE_d t}$.

For analyzing scattering, it is helpful to have an explicit representation of $1/M\text{-}E$. The reader may verify by application to a linear combination of eigenvectors that

$$\frac{1}{M-E} = \frac{I(E_n + E_m - E) - M}{(E_n - E)(E_m - E)} \equiv \frac{N}{(E_n - E)(E_m - E)} \tag{24}$$

where E_n and E_m are the two eigenvalues of M. Then the amplitude for production and decay is Eq. (17) or

$$T = \frac{V\ NA}{(E_n - E)(E_m - E)} \tag{25}$$

As long as M can be diagonalized, Eq. (24) breaks up into two Breit-Wigner terms. Otherwise we can simply insert M and $E_n \rightarrow E_m \rightarrow E_d$ to find N. As M approaches (23) then N becomes

$$N_d = I(E_d - E) + d\begin{pmatrix} -i & \pm 1 \\ \pm 1 & i \end{pmatrix}, \tag{26}$$

and we have

$$T = \frac{(E_d - E)\, C_1 + C_2}{(E_d - E)^2} \tag{27}$$

with $C_1 = VA$, $C_2 = d\,V\begin{pmatrix} -i & \pm 1 \\ \pm 1 & i \end{pmatrix} A$ as the general degenerate form. In general C_1 and C_2 will vary with the decay mode and the production situation as V and A vary.

A simplification results if one of the elements of our diagonalized width matrix is zero. This always happens if there is only one decay mode f for the two states, for then there is a linear combination $a|\alpha\rangle + b|\beta\rangle$ which decouples from the decay operator V. This then corresponds to $d = \Gamma_d/2$ in Eq. (23), with $\Gamma = 2\Gamma_d$ being the width of the coupled state. Then

$$N_D = I(E_d^o - E) + \Gamma/2 \begin{pmatrix} -2i & \pm 1 \\ \pm 1 & 0 \end{pmatrix} \tag{28}$$

where we have taken the state $|1\rangle$ as the decoupled one. If in addition, state $|1\rangle$ is also decoupled from the production amplitude A, then only the N_{22} element enters and

$$T_{fi} \sim V_{f2}\, A_{2i}\, \frac{(E_d^o - E)}{(E_d - E)^2}\ , \tag{29}$$

which is the "dipole". In the original physical example suggested by Lassila and Reiuskanen;[6] they consider crossing of S and P levels in the hydrogen atom with an electric field to provide the off-diagonal coupling. Here only the P state can decay or be excited. The situation is then exactly analogous to the one just discussed. As further suggested by these authors, it is perhaps not wildly improbable that the same situation approximately obtains for the A_2 since there is a strongly dominant decay mode $(\pi\rho)$ and the production might also proceed peripherally through a $\pi\rho$ interaction. Then production and decay would couple to the same state and granting that the splittings of the original states had been properly arranged to give Eq. (28) (namely original states $(|1\rangle \pm |2\rangle)/\sqrt{2}$ split by $\Gamma/2$), Eq. (29) could result.

If all this is true, however, the situation for a rare decay mode, f', is interesting. For there is no reason to suppose that f' couples in the same way as the dominant decay mode and there should be a $V_{f'1}$ coupling. Then under the same conditions as before, since the weak coupling of f' should not change N_D very much

$$T_{f'i} \sim \frac{V_{f'2}(E_d^o - E) \pm \frac{\Gamma}{2} V_{f'1}}{(E_d - E)^2} A_{2i} \quad . \tag{30}$$

We would expect the dip to be shifted off center and asymmetric. If, as reported at this conference, by the CERN Boson Spectrometer Group (CBS), the other decays have the same shape as $\pi\rho$, then all decay modes are decoupled from $|1\rangle$.

To summarize, then, to get a "dipole" by this mechanism we need

1. initial states $(|1\rangle \pm |2\rangle)/\sqrt{2}$ split by an amount Δ
2. the state $|1\rangle$ decoupled from decay and production, at least for the dominant channel
3. the state $|2\rangle$ has a width $\Gamma = 2\Delta$

This will give a "dipole" for the dominant decay mode, $(E_0-E)^2/\left[E_0^2-(\Gamma/4)^2\right]^2$. If in addition we demand symmetric "dipoles" for the rare decay modes then

4. the state $|1\rangle$ is decoupled from all final states, and becomes a kind of "Zombie" state which is more dead than alive and only sees the light of day through its virtual interaction with $|2\rangle$ which gives rise to Δ.

If we are willing to go so far as to consider the possibility of these "Zombie" components to the wavefunction, perhaps it is worth dropping the dipole to see what the "Zombie" by itself will do.

Even though now with $E_n \neq E_m$ the amplitude has a conventional two-Breit-Wigner representation, it is convenient to still use Eq. (24) and Eq. (25) in the basis with Γ diagonal. Then since the Zombie width is $\simeq 0$

$$N = I(E_n + E_m - E) - \begin{pmatrix} M_{11} & M_{12} \\ M_{12} & M_{22} - i\Gamma \end{pmatrix} \tag{31}$$

and using the invariance of the trace

$$N_{22} = E_n + E_m - M_{22} + i\Gamma - E = M_{11} - E$$

Since the Zombie, $|1\rangle$, is not coupled to production or decay only N_{22} enters and for all decay modes

$$T_{fi} = \frac{M_{11} - E}{E_n - E} \; \frac{1}{E_m - E} \; V_{f2} \; A_{2i} \tag{32}$$

is the "general Zombie form". Now if we multiply through by $i\Gamma$, we see this is proportional to (Im $\det|M-E|$)/($\det|M-E|$), giving two peaks of equal height when Re $\det|M-E| = 0$, and a zero in-between at M_{11}.

For elastic scattering this leads simply to $S \sim [(E_n-E)(E_m-E)]^*/[(E_n-E)(E_m-E)]$. Near the Zombie the amplitude has in quick succession a peak followed by a zero, the width of the effect being given by the width in the Zombie eigenvalue E_n. This width is mainly acquired by the Zombie's proximity to a normal resonance; if the initial Zombie mass M_{11} should fall far from a normal resonance with the same quantum numbers then the real mixing potential M_{12} will be ineffective in mixing the states and the Zombie width reverts to its usual (very small) value. Thus the missing-mass landscape could be littered with Zombies and we would never know it unless one happened to drop on a "normal" mate. A symmetric effect results if the mixing is sufficiently strong to make the widths in E_n and E_m equal, an exact "dipole" is not necessary. To get such a mixing, without a large M_{12} which would give a strong repulsion of the levels, does mean that the Zombie has to land rather near the center of the other resonance, however. Dalitz[7] has discussed the possible nature of the states involved and considered the problem from the point of view of a K matrix with two poles. This and Eq. (32) lead essentially to the same structures since the K matrix leads to a 2nd order polynomial in E where we have $(E_n-E)(E_m-E)$ in S. Should it turn out that the A_2 effect is not the same independent of the mode of production and decay and so Zombies are not relevant, we re-emphasize that Eq. (25) applies for any production and decay configuration.

The Zombie-A_2 coupling is small; to get a mixed Zombie width of 20 MeV with an A_2 width of 60 MeV we need $M_{12} \sim 10$ MeV. Since this is almost an order of magnitude less than normal mass splitting forces, we should expect the production and widths of the isolated Zombie to be down with respect to normal states by perhaps one or two orders of magnitude. Could it be found in isolation by high resolution experiments?

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