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COMMENTS ON LEPTON-INDUCED HADRON PROCESSES AT HIGH ENERGY

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Inelastic lepton-hadron interactions at high energies should provide the high-energy physicist with one of the cleanest cases for studying small-distance nucleon structure. Recent experiments[1] with electrons, muons, and neutrinos[2] indicate a large cross-section for inelastic production, comparable to that for scattering from structureless objects. The simplicity in the character of the data suggests that there should be a simplicity in its theoretical interpretation. However, the theory is too primitive to be able to provide much more than some guidance for a future experimental program.

This report will consider three rather specialized topics. First, we briefly review the data on electroproduction and neutrino interactions and discuss the connection between the two provided by the conserved vector current hypothesis. We find that given the electroproduction measurements and reasonably general assumptions, the neutrino data can be understood semiquantitatively.

For the second topic, we shall consider the connection between these processes and the existence of equal-time current commutators.

Finally, we consider recent arguments by Ioffe[3] concerning the important distances in the current-commutator which governs these processes. Ioffe finds that (in the laboratory frame) the hypothesis that only distances $|\underset{\sim}{x}| \lesssim 2R$, the nucleon diameter, are important in the commutator $\left[J(x), J(0) \right]$ is in contradiction with experiment. This result seems to suggest a "diffractive" interpretation for these processes.

I. Electroproduction, Neutrino Production, and CVC

The kinematics for the process

$$\text{lepton} + \text{nucleon} \longrightarrow \text{lepton} + \text{any hadron state} \tag{1.1}$$

is shown in Fig. 1. For electroproduction, the cross-section can be written at high energy $(\nu >> M_p)$ as follows:

$$\frac{d\sigma}{dQ^2\,d\nu} = \frac{4\pi\alpha^2}{Q^4}\left(\frac{E'}{E}\right) W_2(Q^2,\nu)\left[1+\frac{\nu^2}{2EE'}\;\frac{\sigma_T}{\sigma_T+\sigma_S}\right] \tag{1.2}$$

For neutrino induced processes, the answer is similar

$$\frac{d\sigma}{dQ^2\,d\nu} = \frac{G^2}{2\pi}\;\frac{E'}{E}\,\beta(Q^2,\nu)\left[1+\frac{\nu}{E'}\;\frac{\sigma_L}{\sigma_L+\sigma_R+2\sigma_S}-\frac{\nu}{E}\;\frac{\sigma_R}{\sigma_L+\sigma_R+2\sigma_S}\right] \tag{1.3}$$

If the incident particle is a $\bar{\nu}$, one replaces in (1.3) $\beta \longleftrightarrow \bar{\beta}$ and $R \longleftrightarrow L$. $\sigma_{L,R,S}$ are cross-sections for absorption of a " irtual W" of definite helicity on a nucleon. For electroproduction $\sigma_{T,S}$ are similarly defined as the cross-sections for transversely and longitudinally polarized photons. The important variables upon which the form factors depend are $Q^2 = -q^2 = 4EE'\sin^2\frac{\theta}{2}$ and $\nu = M_p^{-1}(q.P) = E-E'$. Some data[1] on electroproduction are shown in Fig. 2. The remarkable feature is that the dimensionless function νW_2 shows little Q^2 and ν- dependence over large ranges of these variables (specifically $M_p\nu/Q^2 > 2$; $0.5\,\text{GeV}^2 < Q^2 < 4\,\text{GeV}^2$). Where νW_2 does vary, it is only in terms of the ratio ν/Q^2 as would be expected from dimensional analysis, assuming no natural mass scale other than that provided by the kinematical variables. For the case of neutrino-processes, the CERN heavy-liquid bubble chamber group reports[2] that the total cross-section rises linearly with energy up to $E_\nu \sim 10$ GeV, again as expected from dimensional analysis:

$$\sigma_{Total} \sim G^2 s = G^2 (2 M_p E_\nu)$$

Given this scale-invariance of the electroproduction data, it follows that the linear rise of the neutrino cross-section is very reasonable. The vector $\Delta S = 0$ part of the weak structure-function $\beta(\nu, Q^2)$ is related to the isotopic-vector part of the electroproduction function W_2 by the CVC hypothesis:

$$\beta_{V, \Delta S=0} = \frac{1}{2}(\beta_p + \beta_n)_{V, \Delta S=0} = (W_{2p} + W_{2n})_{isovector} \tag{1.4}$$

Because νW_{2p} is scale-invariant, it is necessary that β is bounded below by a scale-invariant function, unless only isoscalar photons contribute to W_2 (a very unlikely case on grounds of SU(3) symmetry). It is natural, in fact, to suppose that $\nu\beta$ is scale-invariant:

$$\nu \beta(Q^2, \nu) \approx F\left(\frac{Q^2}{2M_p\nu}\right) \tag{1.5}$$

Given that assumption alone one finds

$$\sigma_{tot} = \frac{G^2 M_p E}{\pi} \int_0^1 dx\, F(x) \left\{ \frac{1}{2} + \frac{1}{2}\left\langle \frac{\sigma_L}{\sigma_R + \sigma_L + 2\sigma_S} \right\rangle - \frac{1}{6}\left\langle \frac{\sigma_R}{\sigma_R + \sigma_L + 2\sigma_S} \right\rangle \right\} \tag{1.6}$$

The factor in curly brackets must lie between 1/3 and 1, and thus σ_{tot}, on the average, rises linearly with neutrino energy. To get an idea of the numerical magnitudes involved, assume

1) All electroproduction goes through the isovector nucleon current.

2) $W_{2n} \approx W_{2p}$

3) All ν-production goes via $\Delta S = 0$ processes.

4) For the neutrino process, axial absorption = vector absorption; i.e. $\beta_A = \beta_V$.

5) $\sigma_T \gg \sigma_S$. This is established according to the preliminary MIT-SLAC results reported at the Daresbury conference[1]; the secondary muon energy spectra in the neutrino experiments likewise favor this hypothesis.

6) $\sigma_L \neq \sigma_R$.

Then these hypotheses imply $\beta = 4W_{2p}$ and from the measurements of W_{2p}, one finds from (1.6)

$$\sigma_{tot} = \frac{2}{3}\int_0^1 dx\, F(x)\left(\frac{G^2 M_P E}{\pi}\right) \cong 0.48\left(\frac{G^2 M_P E}{\pi}\right) \tag{1.7}$$

to be compared with the experimental

$$\sigma_{tot}^{exp} = (0.6 \pm .15)\,\frac{G^2 M_P E}{\pi} \tag{1.8}$$

This comparison is meant only to be illustrative of the magnitudes involved; in particular a realistic modification of hypothesis (1) will widen the discrepancy between the estimate (1.7) and experiment. However, the above argument does show that almost any reasonable theory of electroproduction will give the right order of magnitude for the neutrino process.

II. Connection with Current Commutation-Relations

The electroproduction cross-section is related by the optical theorem to the imaginary part of the forward amplitude T^* for scattering a "current" (virtual photon or W) from a nucleon. For transversely polarized quanta the kinematical factors are simple, and we have

$$\mathrm{Im}\, T_\perp^*(q^2,\nu) \equiv W_\perp(q^2,\nu) = \frac{P_0}{M}\int \frac{d^4x\, e^{iq.x}}{(2\pi)} \langle P|[J_\perp(x), J_\perp^\dagger(0)]|P\rangle \tag{2.1}$$

where $J_\perp$ is a component of J^μ orthogonal to q and P. For electroproduction, $W_\perp(q^2,\nu)$ is related to the transverse photoabsorption cross-section σ_T :

$$W_\perp(q^2,\nu) = \left(\nu - \frac{Q^2}{2M_p}\right)\frac{\sigma_T(q^2,\nu)}{4\pi^2\alpha} \tag{2.2}$$

In principle $T_\perp^*$ is an observable. If the behavior of $T_\perp^*$ in momentum space is not too singular at ∞, then $T_\perp^*$ is related to a retarded commutator of the currents.

If for spacelike $q^2 < 0$, q^2 fixed, we assume asymptotic behavior of $T_\perp^*$ as $\nu \to \infty$ no worse than that corresponding to exchange of the Pomeranchuk trajectory, then we may write a once-subtracted dispersion relation

$$T_\perp^*(q^2,\nu) = T_\perp^*(q^2,0) + \frac{\nu}{\pi}\int_{-\infty}^{\infty}\frac{d\nu'}{\nu'}\frac{W_\perp(q^2,\nu')}{(\nu'-\nu)} \tag{2.3}$$

Now for fixed P take $q_0 \to i\infty$, $\underset{\sim}{q}$ fixed. In the integrand $M_p^{-1}|q_0P_0| \approx |\nu| \ll |q_0^2/2M_p| \lessgtr |\nu'|$ and we may therefore develop the denominator

$$\begin{aligned} T_\perp^* &= T_\perp^*(q^2,0) + \frac{\nu}{\pi}\int_{-\infty}^{\infty}\frac{d\nu'}{\nu'^2}W_\perp(\nu',q^2) + \frac{\nu^2}{\pi^2}\int_{-\infty}^{\infty}\frac{d\nu'}{\nu'^3}W_\perp(q^2,\nu') + \cdots \\ &= T_\perp^*(q^2,0) + \sum_{k=1}^{\infty}\nu^k I_k(q^2) \end{aligned} \tag{2.4}$$

with

$$I_k(q^2) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{d\nu'}{(\nu')^{k+1}}W_\perp(\nu',q^2) \tag{2.5}$$

From the data, $W_\perp$ appears to be scale-invariant

$$W_\perp \cong W_\perp(x) \quad , \quad x = \frac{Q^2}{2M_p\nu} \tag{2.6}$$

Given this we can deduce the asymptotic behavior of the coefficients $I_k(q^2)$, for q^2 large

$$I_k(q^2) \cong \frac{1}{\pi}\left(\frac{-2M_p}{q^2}\right)^k\int_{-1}^{1}dx\, x^{k-1}\,W_\perp(x) \tag{2.7}$$

If $T_\perp$ can be developed in inverse powers of q_o as $q_o \rightarrow i\infty$ it is easy to show from the Low equation that the coefficients are equal-time commutators of the current with its adjoint and time-derivatives thereof. To determine asymptotic behavior to order q_o^{-n}, we need only a finite number of terms in the series expansion (2.4) corresponding to $2k \lesssim n$. The coefficient of q_o^{-1} is the first term in the sum in (2.4).

$$\frac{P_o q^2}{\pi}\int_{-\infty}^{\infty}\frac{d\nu'}{\nu'^2}W_\perp(q^2,\nu') = \frac{2M_P P_o}{\pi}\int_{-1}^{1}dx\, W_\perp(x) \qquad (2.8)$$

In this case the subtraction term $T_\perp^*(q^2,o)$ has the wrong crossing symmetry and does not contribute . Therefore the <u>empirical</u> behavior of the data on $W_\perp$, along with the assumption of Regge-asymptotics, <u>requires</u> that the integrated equal-time commutators of the space-components of the currents exists. However, this result does not follow for the higher commutators such as $[J,\dot{J}]$, the coefficient of q_o^{-2}. For example, if the subtraction term behaves badly

$$\lim_{q^2\to\infty} q^2\, T_\perp^*(q^2,o) = \infty \qquad (2.9)$$

then the coefficient of q_o^{-2} does not exist. However it can still be true that a piece of this commutator exists. That is, coefficient

$$\text{of } q_o^{-2} = \lim_{q^2\to\infty} q^2\, T_\perp(q^2,o) + \frac{(2M_P P_o)^2}{\pi}\int_{-1}^{1}dx\; x\, W_\perp(x) \qquad (2.10)$$

and the finite second term has a different structure under Lorentz transformation from the possibly infinite part. For higher powers of q_o^{-1}, this feature is still true: the coefficient of q_o^{-n} may not exist, but the term in the formal expression for that coefficient which has the maximum number of powers of P_o does exist, provided there is a sensible mathematical way to extract it. There is a need here for a careful limiting procedure; what exists at present really isn't good enough.

There are arguments[4,5,6] that the equal-time commutators cannot be computed "naively" from the canonical rules of field theory. The operator products are too singular. At present it seems that all that can be done is to postulate the commutators on grounds which are independent of specific equations of motion. An example is the U(6) x U(6) scheme of Feynman, Gell-Mann, and Zweig.[7]

The main purpose of this discussion is to emphasize that the question of existence of equal-time commutators is to a large degree empirical. From data alone, we can infer that the crossing-odd part of $T^*_\perp$ is in fact the Fourier transform of a retarded commutator[8], and that the current commutator, by definition the coefficient of q_0^{-1} for $q_0 \to \infty$, exists.

III. Space-Time Structure of the Commutators

The last topic is a report of the work of Ioffe[3] on the space-time structure of the current commutator appearing in (2.1). He argues that the assumption that only small distances are important is incompatible with experiment. Consider again (2.1) in the nucleon rest frame

$$W_\perp = \int \frac{d^4x}{2\pi}\, e^{iq.x} \langle P | [J_\perp(x), J_\perp^\dagger(0)] | P \rangle \tag{3.1}$$

For ν/Q^2 large, one can write (for $\underset{\sim}{q}$ along the z axis)

$$e^{iq\cdot x} = e^{i\nu t - i\sqrt{\nu^2 + Q^2}\, z} \approx e^{i\nu(t-z) - \frac{iQ^2}{2\nu} z} \tag{3.2}$$

If only distances $z \lesssim 2R$, the nucleon radius, are important in the commutator, then for

$$\frac{Q^2 R}{\nu} < 1 \tag{3.3}$$

the last term in the exponential can be ignored, and $W_\perp$ becomes a function of ν alone. However, choosing R = 0.8f. $\approx$ $4M_p^{-1}$, the electroproduction experiments show

$$W_\perp \sim \frac{\nu}{Q^2} \quad \text{for} \quad \frac{M_p \nu}{Q^2} > 4 \tag{3.4}$$

in contradiction with this hypothesis. Therefore large longitudinal distances contribute to the commutator. Ioffe then goes on to show that the important transverse distances are small. From the causality condition

$$X^2 = (t-z)(t+z) - X_\perp^2 \geqslant 0 \tag{3.5}$$

and the estimates (from (3.2))

$$(t-z) \lesssim \frac{1}{\nu} \qquad z \lesssim \frac{1}{2}(t+z) \sim \frac{2\nu}{Q^2} \tag{3.6}$$

he finds

$$X_\perp^2 \lesssim \frac{2}{Q^2} \tag{3.7}$$

It is not clear what the implications of these results are. Ioffe suggests, tentatively, that

$$\sigma_T \sim \langle X_\perp^2 \rangle \sim \frac{1}{Q^2} \tag{3.8}$$

It is also suggestive that, given large distances to be important, the Pomeranchuk-exchange asymptotic behavior is correct for $M_p\nu/Q^2 > 4$, which indeed covers most of the region where νW_2 is large. This may be considered an argument for the "diffractive" interpretation of the electroproduction process[9,10].

References

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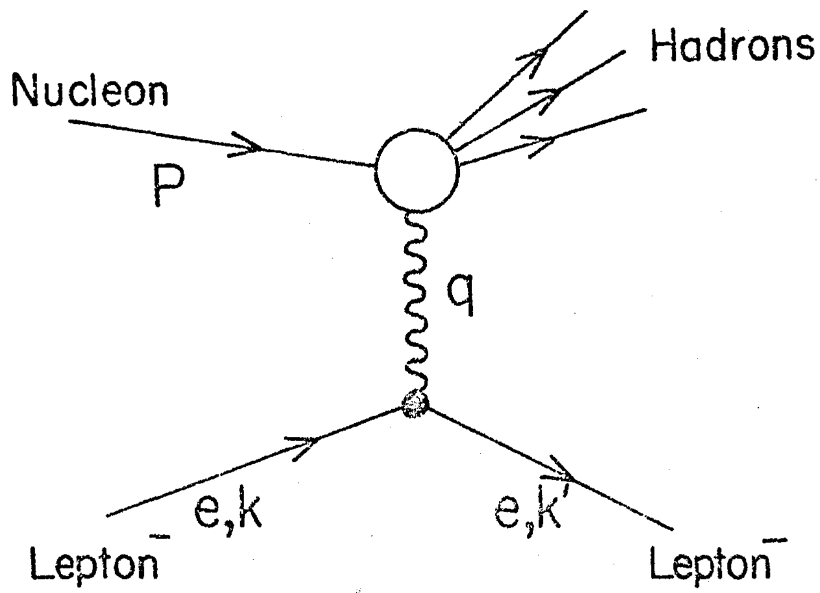

Fig. 1 Kinematics for the electroproduction and semileptonic weak processes.

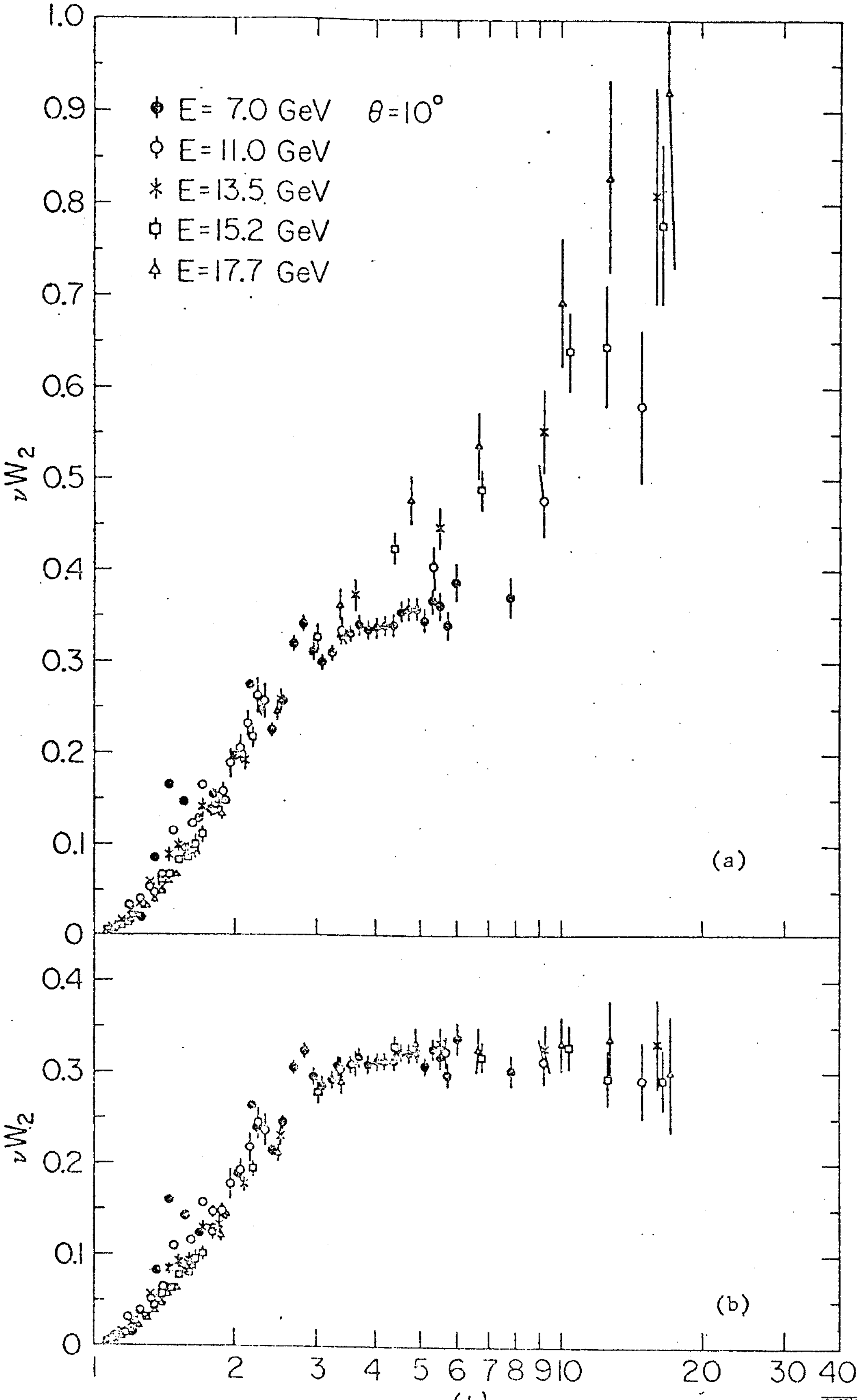

Fig. 2 SLAC-MIT electroproduction data presented at the Daresbury conference: (a) $\sigma_S \gg \sigma_T$ assumed; (b) $\sigma_T \gg \sigma_S$ assumed; this hypothesis is supported by additional measurements.