# IS THERE ANY "SCALING" IN ELASTIC PROTON-PROTON SCATTERING? ${ }^{\dagger}$ Fritjof Capra 

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#### Abstract

Using the elastic large-angle proton-proton data obtained by the CERN group in 1968, an analysis of the fits proposed by Orear and Kirsch is made. It is seen that these fits do not reflect the fact that the differential cross section becomes a function of a single variable in the limit of high energies and large angles, but can only be taken as crude approximations to the data.


[^0]The question of "scaling" or "scale invariance" has recently received great attention in connection with inelastic electron-proton scattering ${ }^{1}$. Both expressions refer to the prediction ${ }^{2}$ that at large values of the squared mass $q^{2}$ and the laboratory energy $\nu$ of the exchanged virtual photon, the inelastic form factors should become a so-called "universal function" of the dimensionless variable $q^{2} / 2 \mathrm{M} \nu$, where M is the proton mass. Recent experiments seem to indicate such a behavior ${ }^{3}$, but doubts about the conclusiveness of the analysis of the data have been raised ${ }^{4}$ and the discussion on this matter is still open.

In view of this controversy, it is interesting to note that universal functions have been proposed for the elastic proton-proton cross section at high energies and large angles several years ago. In 1964, Orear ${ }^{5}$ fitted the experimental data available at that time with the curve

$$
\mathrm{s} \frac{\mathrm{~d} \sigma}{\mathrm{~d} \Omega}=\mathrm{C} \mathrm{e}^{-\mathrm{cp} p_{1}}
$$

where $s$ is the c.m. energy, $p_{\perp}=p \sin \theta$ the transverse momentum, and $C$ and c are constants. This curve seemed to fit the data very well and thus encouraged further research along these lines. In the following years, when better data became available, there was an extensive phenomenological effort to find a single variable against which all cross sections could be plotted ${ }^{6}$. The most successful proposal, besides the Orear fit, was the one advanced by $\mathrm{Krisch}^{7,8}$ who fitted all the p-p data by three exponentials of the form ${ }^{9}$

$$
\frac{\mathrm{d} \sigma}{\mathrm{dt}}=\mathrm{B} \mathrm{e}^{-\mathrm{b} \beta^{2} \mathrm{p}_{\perp}^{2}}
$$

where $\beta$ is the c.m. velocity of the protons and B and b are again constants.

In this note, we present a critical analysis of the two fits, using the data taken in 1968 by a CERN group ${ }^{10}$ who measured cross sections at incoming momenta between 7.1 and 12.1 GeV and angles from $40^{\circ}$ to $90^{\circ}$. The purpose of this analysis is to find out whether the elastic p-p cross section becomes indeed a function of a single (though not dimensionless) variable in the limit of high energies and large angles. In analogy to inelastic e-p scattering, we shall use the expression "scaling." for such a behavior.

Our results are presented in Figures 1 and 2. Figure 1 shows an Orear plot of the data of Ref. 10. $\mathrm{s} \frac{\mathrm{d} \sigma}{\mathrm{d} \Omega}$ is plotted against $p_{\perp}$ and points corresponding to equal scattering angles are connected. One can see immediately that there is a clear correlation between these points. The slopes of the connecting lines decrease with increasing scattering angle and thus the points around $60^{\circ}$, and not those at larger angles, will satisfy an overall straight-line fit best. This shows that there is no evidence for "scaling".

Figure 2 shows the corresponding plot for the Krisch parametrization. The features of the two plots are very similar which is rather surprising, since the two parametrizations seem to be quite different. For high energies, they can be written

$$
\begin{align*}
& \frac{d \sigma}{d t} \sim q^{-6}\left(q^{2}-M^{2}\right) e^{-c q \sin \theta}  \tag{Orear}\\
& \frac{d \sigma}{d t} \sim e^{-b\left(q^{2}-M^{2}\right) \sin ^{2} \theta} \tag{Krisch}
\end{align*}
$$

where q is the $\mathrm{c} . \mathrm{m}$. momentum and M is the proton mass.

Summarizing, we have seen that the fits proposed by Orear and Krisch do not reflect any "scaling" property (in the sense defined above) of the elastic p-p cross section in the energy range under consideration, but can only be taken as crude approximations to the data. It should not be too surprising if a similar situation energed for inelastic e-p scattering, once more experimental data become available.

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Fig. 1
Proton-proton elastic differential cross section as a function of $p_{\perp}$; points with equal c.m. scattering angle are connected.


Fig. 2

Proton-proton elastic differential cross section as a function of $\beta^{2} \mathrm{p}_{\perp}^{2}$; points with equal c.m. scattering angle are connected.


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