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A NEW PRECISION MEASUREMENT SYSTEM FOR BEAM TRANSPORT TYPE MAGNETS*

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I. INTRODUCTION

The purpose of this paper is to present a precision magnetic measurement system based upon magnetic spectroscopy. The technique of spectroscopy has been used for rapid determination of multipole quality,[1,2] but has not seen application in generalized magnetic field measurement.

In transport type magnets the integral of induction over length is the important quantity from the beam behavior point of view. In a dipole magnet the well-known relationship of bending angle, beam energy and $\int B \cdot dz$ is

$$\int_{-\infty}^{\infty} B(z) \cdot dz = \frac{\alpha E}{0.3}$$

where

α is the beam bending angle in radians,
E is the beam momentum in GeV/c, and
$\int B \cdot dz$ is in webers per meter.

In higher multipoles the important quantity is not only B but in addition the derivatives of B with respect to the radius r, thus:

for quadrupole fields, $$\int_{-\infty}^{\infty} \frac{\partial B(z)}{\partial r}\, dz = \frac{\alpha E r}{0.3}$$

for sextupole fields, $$\int_{-\infty}^{\infty} \frac{\partial^2 B(z)}{\partial r^2}\, dz = \frac{\alpha E r^2}{0.3}$$

and for 2n-pole fields, $$\int_{-\infty}^{\infty} \frac{\partial^{(n-1)} B(z)}{\partial r^{(n-1)}}\, dz = \frac{\alpha E r^{(n-1)}}{0.3}$$

Any practical magnet is not a pure multipole but rather a mixture of multipoles. It is also important to note that the individual multipoles may not be oriented in the same angle plane and thus some multipoles may be skewed with respect to other multipoles. Thus in the two-dimensional case the field may be considered a sum of spatial harmonics of the form:

$$B_{nr}(z) = r^{n-1} B_n(z) \sin(n\theta + \alpha_n) \tag{1}$$

and

$$B_{n\theta}(z) = r^{n-1} B_n(z) \cos(n\theta + \alpha_n) \tag{2}$$

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Therefore, the effects of a generalized magnetic field on a particle may be considered a series sum of terms:

$$\sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \frac{\partial^{n-1}\vec{B}_n}{\partial r^{n-1}} dz = \int_{-\infty}^{\infty} \vec{B}_1 dz + \int_{-\infty}^{\infty} \frac{\partial \vec{B}_2}{\partial r} dz + \int_{-\infty}^{\infty} \frac{\partial^2 \vec{B}_3}{\partial r^2} dz \ldots + \int_{-\infty}^{\infty} \frac{\partial^{n-1}\vec{B}_n}{\partial r^{n-1}} dz$$

$$= \vec{\theta} \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \left[B_n(z)\, r^{n-1} \sin (n\theta + \alpha_n) \right] dz$$

$$+ \vec{r} \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \left[B_n(z)\, r^{n-1} \cos (n\theta + \alpha_n) \right] dz \quad . \qquad (3)$$

Define

$$C_n = \int_{-\infty}^{\infty} B_n(z)\, dz \quad .$$

A measurement system based upon this approach is therefore independent of the type of transport magnet being measured, and is equally valid for dipoles, quadrupoles, etc. If one constructs a long loop coil in such a way that one side of the loop is rotated about the magnet centerline and the other side of the loop is on the centerline of the magnet (Fig. 1) then rotation of the loop results in an induced voltage which is determined only by the integrated B_r harmonic components at the radius r. If the coil is long in extent compared with the magnet length, then the output voltage waveform will be proportional to the integral of B_r over the magnet length.

Although such a generator is sensitive only to integrated B_r components of flux, the B_θ component can be deduced using Eq. (1) and Eq. (2).

II. INSTRUMENTATION TECHNIQUES

II.1 Analog Harmonic Analysis

In the conventional analog harmonic analysis method[1, 2] the amplitude and phase angle of each harmonic is measured with a wave analyzer augmented by an oscilloscope for phase measurement. (Figure 2.) The rotational frequency of the coil is taken as the fundamental frequency and integer multiples are the sought harmonics. This method of harmonic analysis yields the values of the constants C_n and α_n for each n and can be used in Eq. (1) and Eq. (2) to calculate $\int B_r\, dz$ and $\int B_\theta\, dz$.

The values of C_n and α_n can be used as inputs to a beam transport problem or in themselves can be used as a measure of quality of the magnet being measured. If the measurement is repeated at several excitation levels, I_{ex}, then the additional data may be used as a quantitative description of the magnet as a function of excitation level.

There are two limitations in the analog method. First, the absolute amplitude accuracy of such an analyzer is typically several percent (± 5% for the Hewlett-Packard 302A). Second, there is usually no accuracy claimed for the phase. Both factors are important in measuring the field.

II. 2 Digital Harmonic Analysis

If the analog signal from the rotating coil is sampled at high speed and digitized by an analog to digital converter, one can use this data to calculate the equivalent Fourier series coefficients using the Discrete Fourier Transform.[3] A discussion of Fourier series as applied to this problem is found in the Appendix I.

The advantage of such an approach is that the accuracy is now limited primarily by system noise and the waveform measurement device (analog to digital converter) since the digital analysis can be performed to almost any accuracy required. Such converters are currently available with an accuracy in the region of ± .01% so that an overall accuracy of ± .01% to ± 0.1% is possible. Using the digital harmonic analysis approach the waveform is sampled at M points in one cycle (360°) where M ≥ twice the highest harmonic present. The analysis is then used to calculate M coefficients in the Fourier expansion.

$$e(t) = \frac{a_0}{2} + a_1 \cos \omega t + a_2 r \cos 2\omega t \ldots + a_{M/2}\, r^{n-1} \cos \frac{M}{2}\, \omega t$$

$$+ b_1 \sin \omega t + b_2 r \sin 2\omega t \ldots + b_{(M/2^{-1})}\, r^{n-1} \sin \left(\frac{M}{2} - 1\right) \omega t$$

The coefficients C_n and α_n are related to the Fourier coefficients a_n and b_n by

$$C_n = (a_n^2 + b_n^2)^{1/2}$$

$$\alpha_n = \tan^{-1} \frac{a_n}{b_n}$$

II. 3 Incremental Method

A third method is described here which is based upon an incremental coil motion instead of continuous rotation. If the coil is driven in small discrete steps by a stepping motor, and the coil output integrated for each step by an integrating digital voltmeter, the resulting data can be used to compute the spatial harmonics using the Discrete Fourier Transform as before.

However, since each point is an integration of flux lines cut during a movement of $\Delta\theta$ instead of a differentiation, the analysis is slightly different from the methods discussed earlier. In this case each data point consists of the integrated volt-seconds resulting from a movement of $\Delta\theta$ through 360°. The resulting series of data points forms a set of data representing values of $\int B_r d\theta$ at incremental angular positions of the coil. Figure 3 shows typical components of induction in a quadrupole magnet.

$$\int_{\theta=0}^{\theta=\theta_m} e\,dt = -C_1 \cos(\theta_m + \alpha_1) - \frac{C_2 r}{2} \cos(2\theta_m + \alpha_2) \ldots$$

$$- \frac{C_n}{n} r^{n-1} \cos(n\theta_m + \alpha_n) - \text{Constant} \qquad (4)$$

Now if the number of increments M is at least twice the highest spatial harmonic present then the data can be transformed using the Discrete Fourier Transform into a series of M a_n and b_n coefficients, equivalent to M/2 C_n and α_n, or M/2 harmonics. It is instructive to consider the case of an ideal dipole field where the coil flux is given by

$$\phi = AB \sin \theta$$

in which A is the coil area, B the uniform field intensity, so that for uniform rotation

$$e(t) = NAB\, \omega \cos \omega t \, .$$

Knowing N, ω, and A, one easily finds the absolute magnitude and direction of the field. However, in the case of the incremental integrated output

$$\int_{\theta=\theta_0}^{\theta_m} e \, dt = NAB\,(\sin \theta_m - \sin \theta_0)$$

The data contains a constant depending upon the starting point, such as shown in Fig. 3(c), and Eq. (4).

It is significant to point out that in continuous rotation the coil voltage represents the integrated B_r component of the flux distribution in space, and the higher harmonics are not attenuated. However, as can be seen from Eq. (4), the incremental approach attenuates the integrated volt-seconds of the nth harmonic by the factor n. Since the amplitudes of the higher harmonics of B typically decrease rapidly with n, this attenuation makes the measurement more difficult.

III. FEASIBILITY TEST

In order to test this approach to harmonic analysis, a simplified incremental system, consisting of commercial instruments and an existing control-data logging system, was implemented. A block diagram of the system as used in this test is shown in Fig. 4.

An HP 2401C integrating digital voltmeter is used to integrate the coil voltage as the coil is stepped in 1.8° increments over a 360° interval. After each step the digital data is recorded onto a Kennedy Model 1400 incremental tape recorder via a digital multiplexer and a control system built of commercial digital modules. Only a portion of the multiplexer and control systems are used in this test. In running the test, each step takes approximately one second, limited primarily by the mechanical settling time, so that an entire measurement cycle through 360° and return takes approximately seven minutes.

One of the important considerations is the precise nature of the mechanical motion – in particular the starting and stopping transients. Since the coil voltage is the time derivative of the flux, speed-acceleration characteristics directly influence the coil signal, the setting of the input voltage scale factor of the DVM, the resolution and the noise performance. As mentioned before, oscillations after the drive has stopped slows the entire process since a settling time must be added before recording the data. It is planned to use a damped stepping motor in the future in order to improve the total measurement and performance.

Using the test system described above, a measurement of a quadrupole magnet was made and the results obtained were compared with the results obtained using the HP302A wave analyzer. The results are shown in Fig. 5. In this case the results were normalized so that both tests resulted in an amplitude of 1.0 for the first harmonic. As can be seen, the agreement of C_n is within the absolute amplitude accuracy of the wave analyzer.

IV. COMPUTER CONTROLLED HARMONIC ANALYZER

A proposed computer system is shown in Fig. 6; except for the mechanical system, the magnet current control, and a minor part of the electronics, all system components are standard commercial items well within the state of the art.

The mechanical system has the function of rotating the coil assembly without irregularities at a speed determined by a ± 0.01% crystal controlled ac power source which drives a 1/8 HP hysteresis-synchronous motor. A speed of 19 revolutions per second has been selected arbitrarily, based on hardware limitations, and in order to be nonsynchronous with 60 CPS. The mechanical system includes provisions for accurate alignment of the coil axis to the magnet axis.

The electrical signal is transmitted from the coil via a slip ring assembly. Previous experience with rotating coil systems at SLAC indicates that with the proper care and with high enough coil voltages, sufficiently accurate transmission may be achieved. A high speed analog-to-digital converter of ± 0.01% to ± 0.02% accuracy (14 bits) samples and digitizes the signal at a continuous rate of 4064 samples per second or 256 samples per revolution (see Appendix I for a discussion of selection of sampling rate, etc.). The timing of the samples is accurately determined by a photoelectric incremental shaft encoder which accurately relates sample time to shaft position. A reference pulse from the encoder for each revolution relates the sample number to the magnet reference axis. By the use of the optical shaft encoder, waveform samples are obtained which can be directly used in calculating the Discrete Fourier Transform. Without the synchronization of the shaft encoder it would be necessary to perform a full spectrum analysis, requiring far more computation and resulting in reduced accuracy. In addition the synchronization permits multiple cycle digital averaging to improve the signal-to-noise ratio, should this prove beneficial in special circumstances. It is expected that an overall accuracy of ± 0.02% can be achieved through the combination of these techniques.

Closed loop magnet control included in the system permits step-by-step control of the magnet current in 8000 discrete steps by using a stepping motor which drives a 40-turn potentiometer in the magnet current power supply. Both the amount and the rate of current change are under direct control of the computer software. This flexibility is accomplished by using the computer to individually generate "up-pulses" or "down-pulses" as required over a range of pulse rates. An internal computer clock assists in pacing the operation to carefully control field charge rate and the resulting eddy current effects. A current shunt monitoring the magnet current is measured by a ± 0.01% integrating digital voltmeter which is interfaced to the computer. Thus, with direct control of the current, and subsequent readout to the computer, the control loop may be closed by software. An alternate manual controller permits operator setting of the magnet current utilizing the visual readout on the digital voltmeter.

Several conventional computer peripherals are included for program entry, display, and data storage. The teletype for example serves for basic computer-operator communication, data logging (printed records) and paper tape punching and reading for program compilation. The high speed (300 characters/sec) paper tape reader facilitates program loading, being considerably faster than the teletype paper tape input. The IBM compatible magnetic tape is the permanent data storage medium, and also serves as the link to a computation center where more sophisticated data processing may be performed. For quick-look evaluation of measurements and for condensed form of data storage, an x-y plotter is included. It is expected that excitation curves of each of the harmonics of interest would be the basic output of the plotter.

The computer is a small, general purpose, 16 bit binary machine. For 0.01% measurements a 16 bit word size is considered mandatory, since the resolution of one part in 32000 will allow single precision arithmetic in many of the operations. Considering the sampling rate and the possible on-line processing load, the core cycle time should be on the order of 2 μsec. For Fourier analysis, a fast multiply time is desirable. For example, with a 25 μsec multiply time, a 256 data point analysis for 14 harmonics takes approximately one second. In order to use a Fortran compiler an 8 K core size is sufficient for the computational and control load envisioned here.

In a system of this complexity, the use of visual monitors is invaluable to assure validity of the data. Two monitors are especially appropriate here: One, a conventional oscilloscope monitor connected to the rotating coil which indicates the qualitative nature of the field, integrity of the coil, quality of the rotation and slip rings, and indirectly, the magnet current; second, a frequency counter to monitor the rotational speed to provide backup assurance on the absolute calibration of the system.

A typical operating sequence with the system is described as follows. First the coil is accurately aligned in the magnet and the reference pulse adjusted to the magnetic axis using stroboscopic techniques. After initial checks to verify the electronics calibration, rotational speed, etc., the system is ready for a measurement. The magnet current is adjusted by the computer in accordance with a prescribed time schedule. The program then reads in 256 samples (one cycle) of the waveform* and performs a Fourier analysis on as many harmonics as desired. For our work at SLAC, 14 harmonics are normally considered sufficient. This procedure (measurement and computation) may be repeated if desired for verifying the results. The Fourier coefficients are then stored on the magnetic tape, after which the magnet current is stepped to the next value. Again the measurement and calculation are repeated and the results stored. After a full sequence of currents leading up to a peak and returning to zero the tape may have stored 560 coefficients (10 current settings going up and 10 going down). Excitation curves for each harmonic can now be plotted, and fitted to a polynomial expansion if desired.

It emphasized that this procedure is presented only to illustrate a typical operational philosophy, since the sequencing, computation, and control are completely determined by the computer software.

V. SUMMARY

The applicability of magnetic spectroscopy to the general measurement of magnets, particularly beam transport magnets, has been discussed. It has been shown that the multipole strengths available from spectroscopy can be used directly in beam transport equations.

Several systems for instrumenting magnetic spectroscopy are described. First, the conventional wave analyzer which is limited in accuracy and difficult to automate. Second, an incremental integrating approach which is appropriate for off-line Fourier analysis by computer; this approach has been tested. Last, a proposed on-line computer system is detailed which has the additional capabilities for on-line data processing and closed-loop current control. Such a system has flexibility and potential considerably beyond those of the other systems. For example, by employing the Fast Fourier Transform it is possible to measure the time history of the multipole factors as the current is stepped through a current ramp.

The cost for the computer-oriented system described here is estimated at $60,000, whereas the investment in the incremental integrating system is estimated at $20,000.

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*Multiple cycle digital averaging may be performed at this time should this prove worthwhile. The subsequent Fourier analysis is the same in this case.

APPENDIX A

FOURIER ANALYSIS TECHNIQUES FOR DIGITAL HARMONIC ANALYSIS

In Fourier theory it is proven that any periodic time function (waveform) can be represented by an infinite series of sinusoidal components whose frequencies are integral multiples of the lowest one (fundamental, or interval of repetition). Such a harmonic series is expressed as

$$e(t) = \frac{a_0}{2} + a_1 \cos \omega t + a_2 \cos 2\omega t \ldots + a_n \cos n\omega t \qquad (A.1)$$

$$+ b_1 \sin \omega t + b_2 \sin 2\omega t \ldots + b_n \sin n\omega t$$

or

$$e(t) = \frac{a_0}{2} + c_1 \sin(\omega t + \alpha_1) + c_2 \sin(2\omega t + \alpha_2) \ldots + c_n \sin(n\omega t + \alpha_n) \qquad (A.2)$$

where ω is the fundamental angular frequency, a_n, b_n, c_n are the amplitude coefficients, and α_n the relative angle of the particular harmonic, n.

Given a particular continuous repetitive waveform the Fourier series parameters can be calculated as follows:

$$a_0 = \frac{2}{T} \int_0^T e(t)\, dt \qquad (A.3)$$

$$b_n = \frac{2}{T} \int_0^T e(t) \sin n\omega t\, dt \qquad n \geq 1 \qquad (A.4)$$

$$a_n = \frac{2}{T} \int_0^T e(t) \cos n\omega t\, dt \qquad n \geq 1 \qquad (A.5)$$

$$c_n = \sqrt{a_n^2 + b_n^2} \qquad (A.6)$$

$$\alpha_n = \text{arc} \tan^{-1} \frac{a_n}{b_n} \qquad (A.7)$$

where T is the period of repetition: that is

$$e(t) = e(t + T) \qquad (A.8)$$

In the general case, however, the waveform function is not known analytically, and may be known only at uniform discrete points (samples). In this case the theory is modified by sampling theory, and the analysis becomes Discrete Fourier Analysis. Sampling theory shows that a waveform limited in frequency to W cycles per second is uniquely determined if it is discretely sampled at a rate of 2W samples per second,[4] known as the Nyquist rate. Stated in another way, the Fourier coefficients can be uniquely determined if the waveform is sampled at a rate at least twice the highest frequency component present in the waveform. Presence of higher harmonics may cause errors. Hence the highest sampling rate within the limitations of computational load should be chosen. For the system described in this report a rate of 256 samples per cycle, permitting up to 128 harmonics, has been selected.

The Discrete Fourier Transform is defined in terms of a series sum instead of the integral:

$$K_n = \frac{1}{M} \sum_{m=0}^{M-1} e_m \exp^{-2\pi jmn/M} \qquad \text{(A.9)}$$

where K_n is the complex form of Fourier coefficient of the nth harmonic, e_m is the waveform sample at the mth point out of a total of M points, and $j = \sqrt{-1}$. The exponential form above can be written alternatively in terms of sine and cosine series since the exponential can be so decomposed.

It can be shown that if the waveform is band-limited and the sampling rate is at least twice the highest frequency component, then the Discrete Fourier Transform is mathematically identical to the continuous Fourier Transform or,

$$\mathrm{Re}\left\{K_n\right\} = a_n \qquad \text{(A.10)}$$

$$\mathrm{Im}\left\{K_n\right\} = b_n \qquad \text{(A.11)}$$

where a_n and b_n are as previously defined. The validity and precision of this identity has been verified in a computer simulation study at SLAC for m up to 1000 points with harmonic ratios of up to 100:1.

The discrete Fourier Transform permits the use of digital computers to accurately compute Fourier coefficients for band-limited waveforms. Moreover, a new computational algorithm is available, termed the Fast Fourier Transform, which greatly reduces the amount of computation required for the generation of a complete spectrum as indicated by Eq. (A.9) when m is a power of 2. However, preliminary study shows that for the particular analysis programs and parameters chosen in this report the use of the Fast Fourier Transform is not necessary. However, this software option remains open, since m = 256, an integer power of 2, has been selected for the number of samples.

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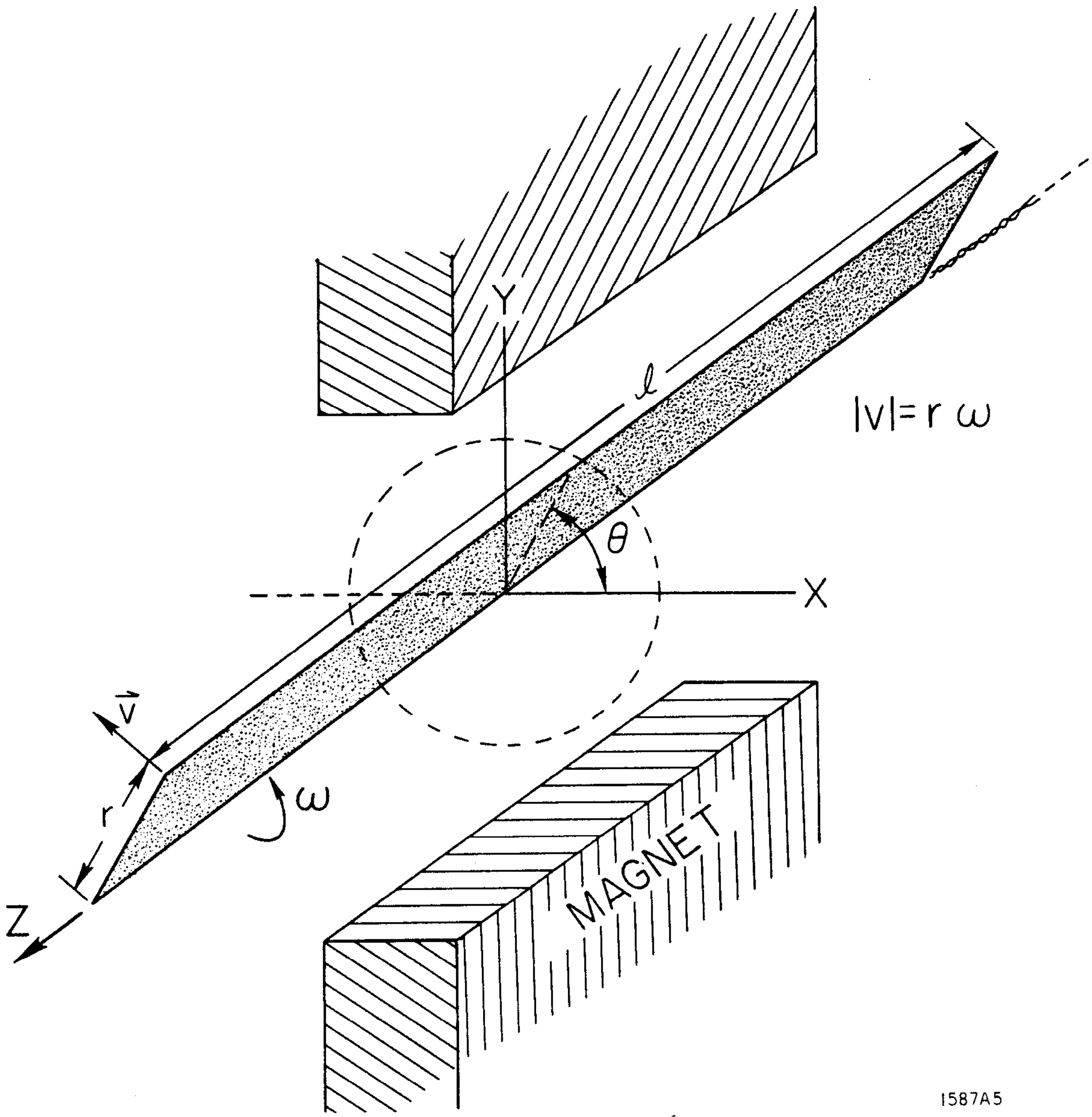

FIG. 1--Rotating loop coil and coordinate system.

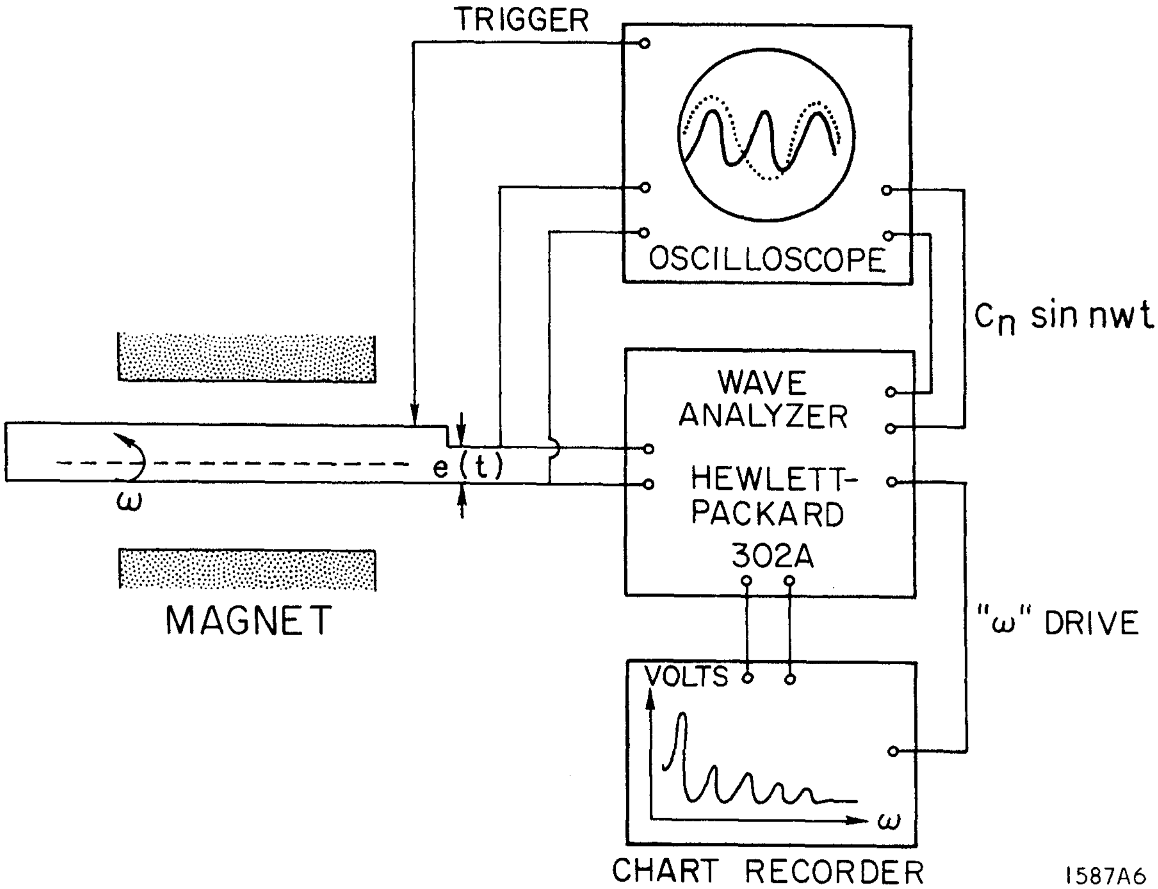

FIG. 2--Block diagram of analog system.

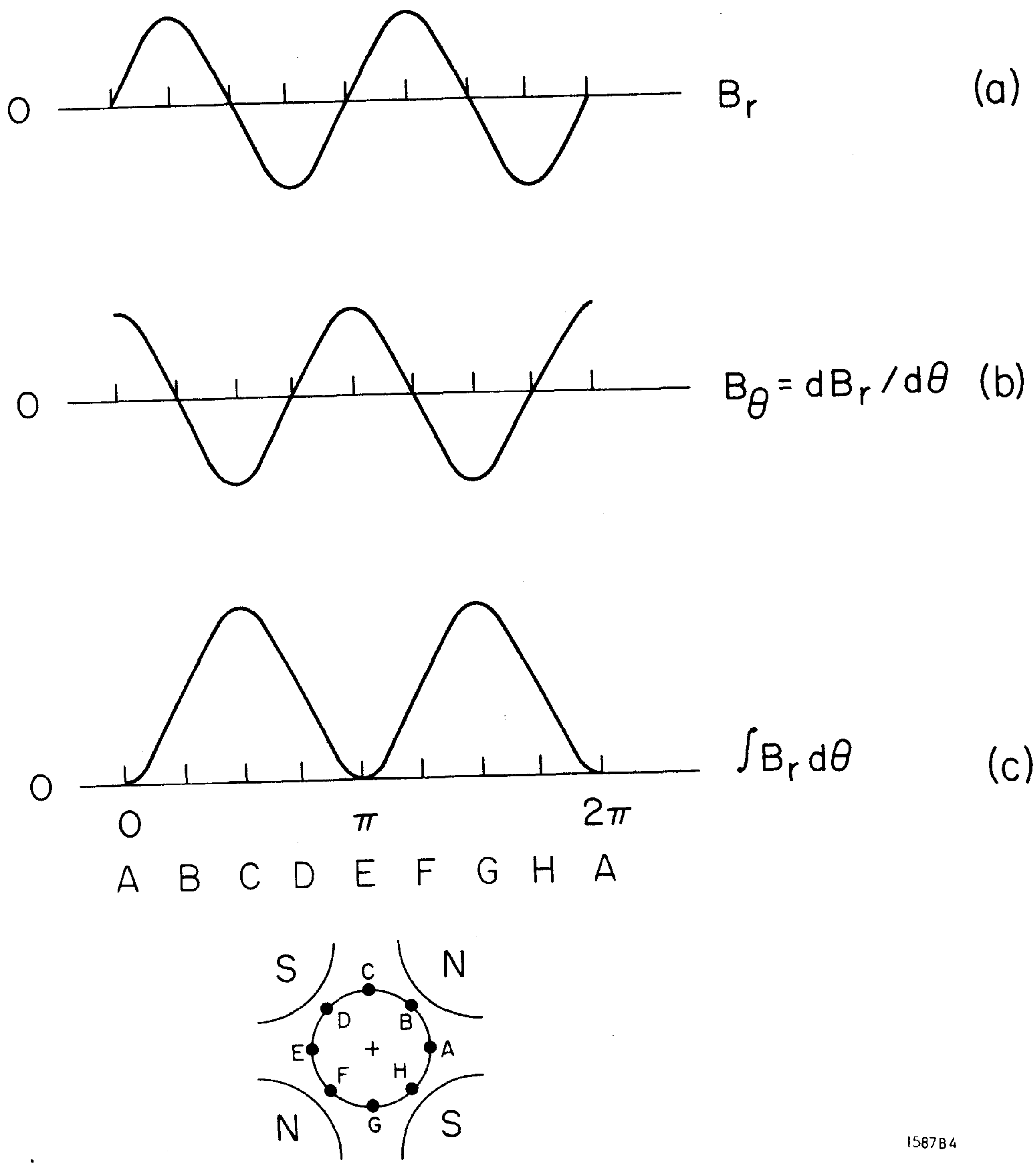

FIG. 3--Typical components of induction in a quadrupole magnet.

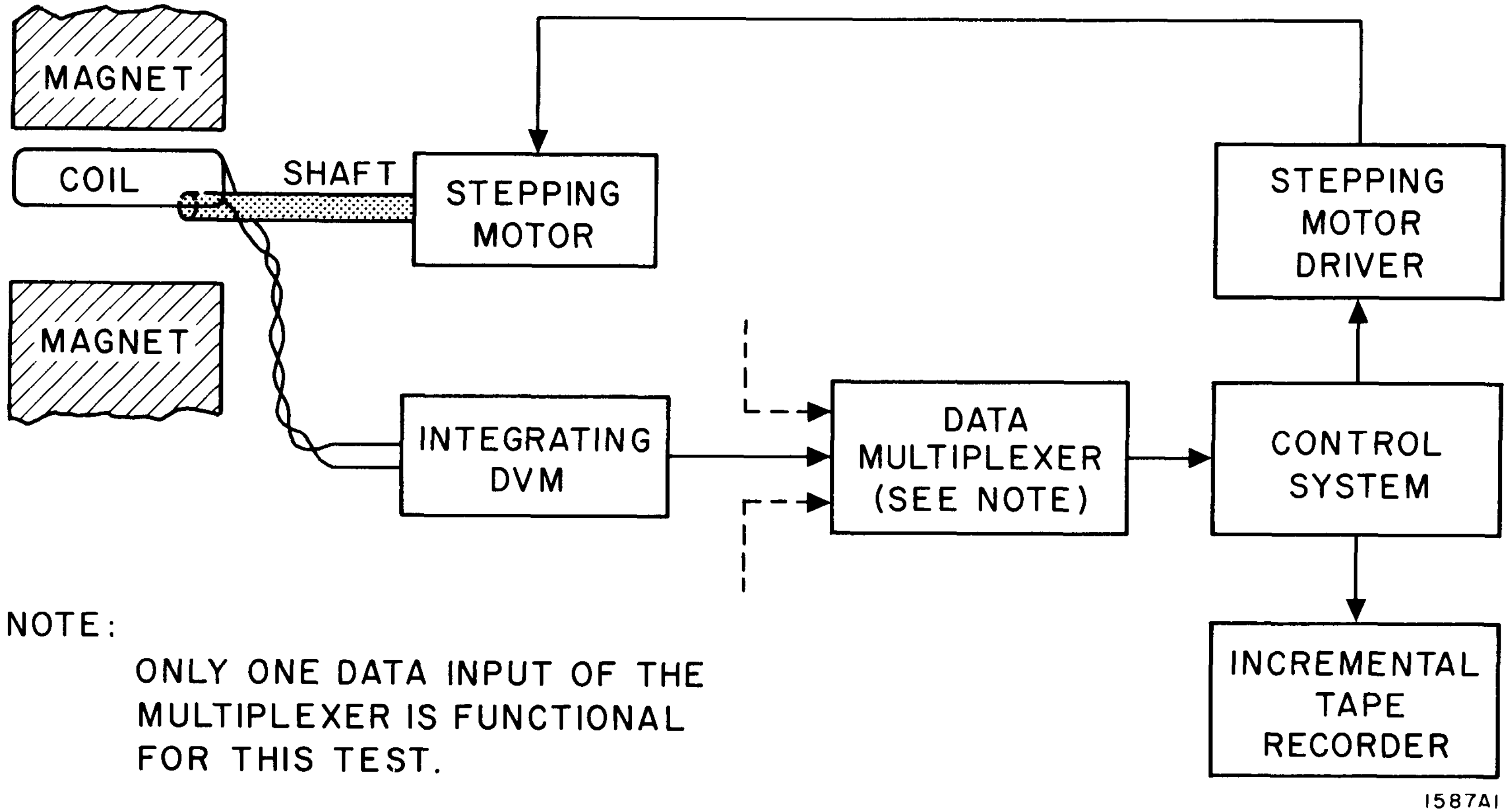

FIG. 4--Block diagram of incremental system.

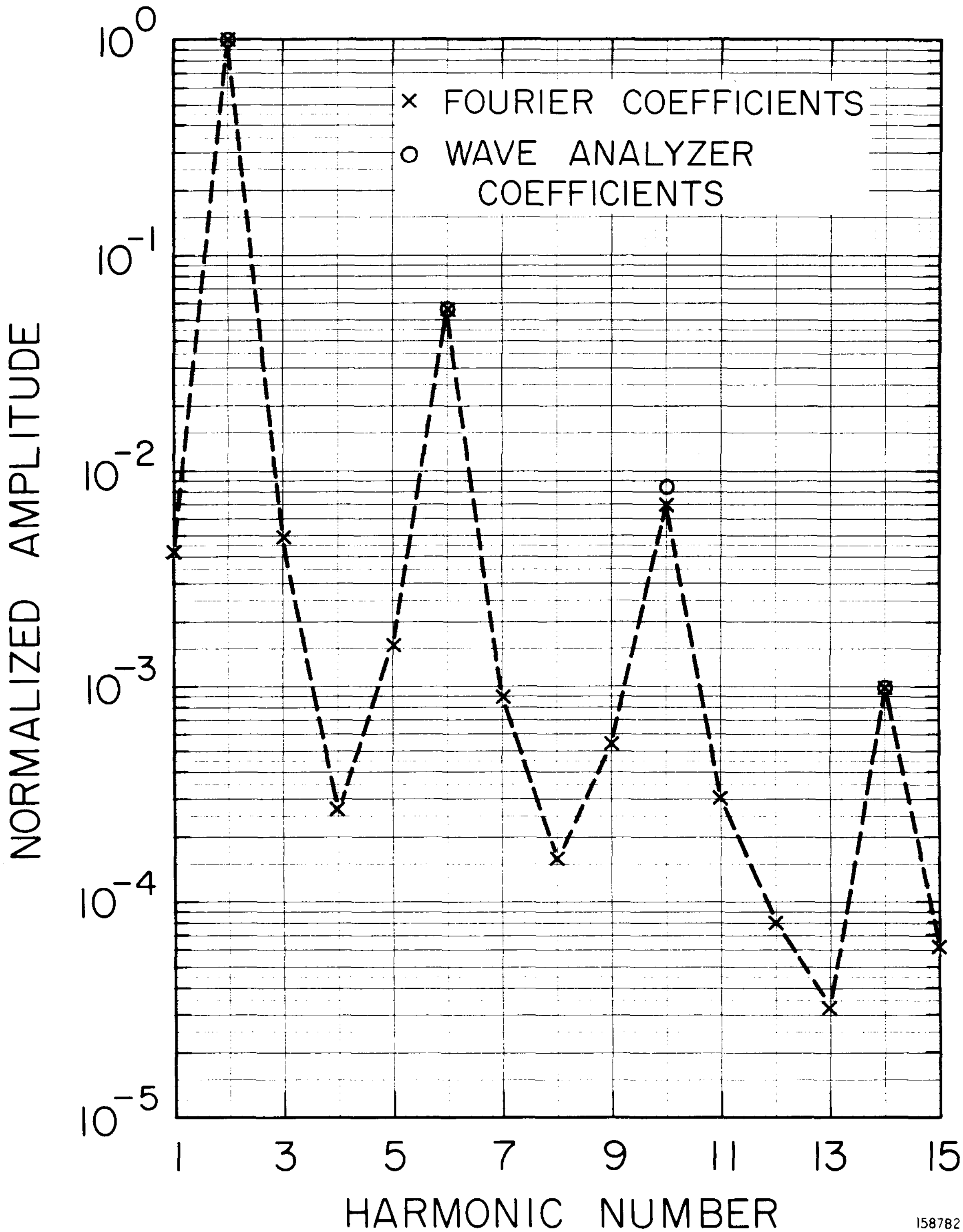

FIG. 5--Comparison of harmonic amplitudes from tests.

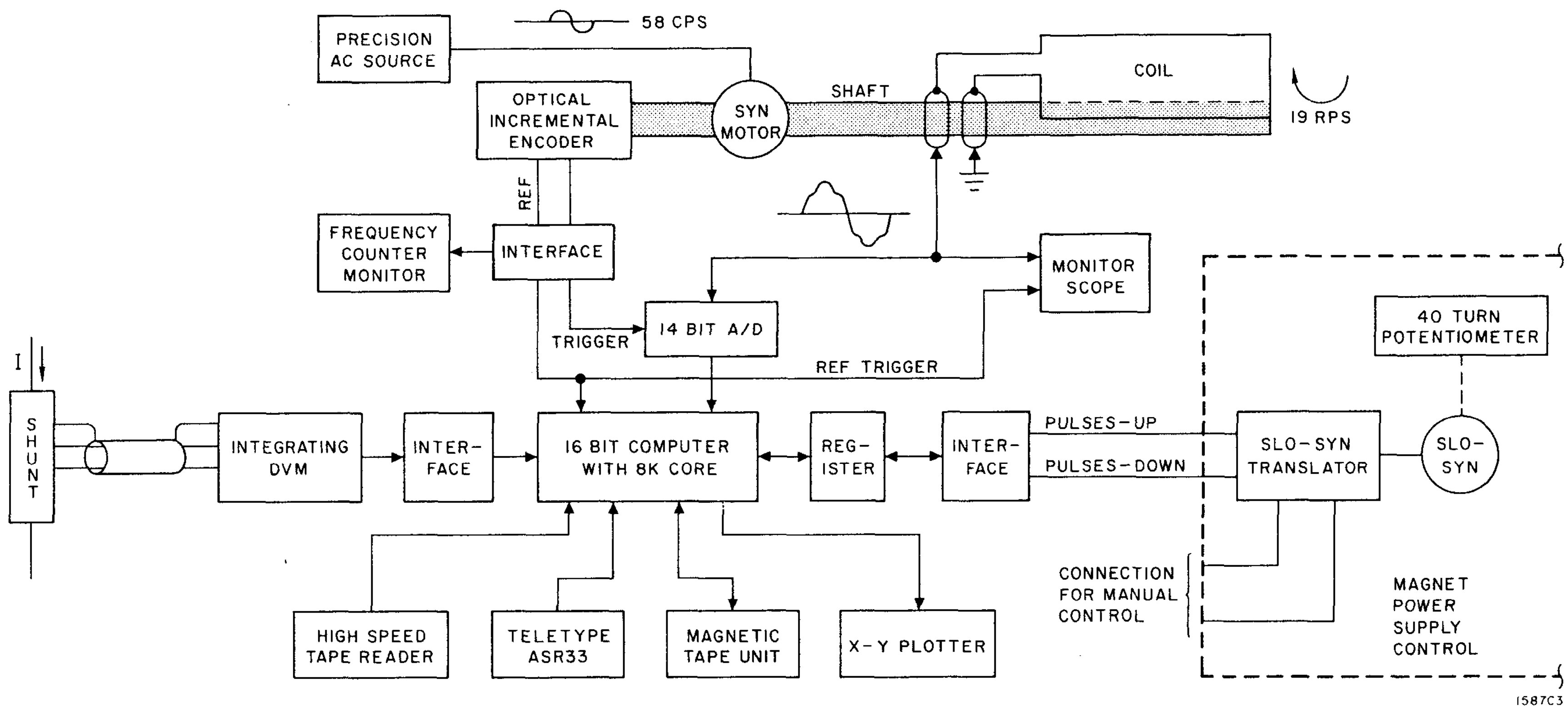

FIG. 6--Computer system block diagram.