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# FINAL PARTICLE CORRELATIONS IN DEEP INELASTIC LEPTON PROCESSES*

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A study is made of the additional information learned from measurements of the correlation of two (or more) detected particles in the final states of deep inelastic lepton processes. Generalized Bjorken limits are derived in which the new structure functions depend only on the ratios of the kinematical variables available, just as do the more familiar structure functions $W_1$ and $\nu W_2$ for electron-proton scattering in the deep inelastic region. Experimental implications are discussed.

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The deep inelastic lepton-hadron processes of

*i*) $e + p \rightarrow e' +$ "anything"

*ii*) $e + \bar{e} \rightarrow$ hadron + "anything"

*iii*) $\nu + p \rightarrow e +$ "anything"

have been studied using the formalism of canonical field theory.[1] Within this framework the "parton" model[2] has been derived and the Bjorken limiting[3] behavior established for the invariant structure functions. A simple physical picture emerges for these processes in which only one final particle is detected by making full use of unitarity and summation over all unobserved final states. For instance, when viewed in an infinite momentum frame of the target, *i*) appears as an incoherent superposition of <u>elastic</u> scatterings from the virtual constituents of the proton. These constituents behave <u>as if</u> they were point-like, structureless particles and the strong interaction dynamics is isolated soley in the description of the proton structure in terms of these constituents. The particles present immediately after the scattering propagate freely and independently <u>as if</u> there is no interaction among

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them. This is the so-called parton model and is illustrated in Fig. 1(a). In actuality, of course, interactions do occur among the particles after the elementary scattering act of the virtual photon. For a given set of particles in the final state an actual scattering event looks like the one shown in Fig. 1(b). The blobs in the figure represent all possible interactions. However, the two groups of particles, labelled (A) and (B) in Fig. 1(b), do not interact because they are separated by an asymptotically large transverse momentum of magnitude $Q^2$, where $-Q^2$ is the invariant momentum transfer squared from the virtual photon. Completeness of the final states and unitarity make it possible to obtain from the true picture, as given in Fig. 1(b), the simplified overall picture of the parton model as represented in Fig. 1(a).

Similar results can be derived for *ii*) and *iii*) and the three processes can be interrelated through the formal field theory framework. A basic ingredient in this work is the assumption that there exists an asymptotic region of large $Q^2$ relative to the transverse momenta of the hadron constituents, virtual or real, when viewed in the infinite momentum frame. A momentum cut-off was introduced to ensure existence of this asymptotic region as discussed fully in earlier papers.[1]

In this letter we report results of a study of the additional information learned from measurements of the correlation of two (or more) detected particles in the final states. We find that there exists a generalized Bjorken limit in which the new structure functions depend only on the ratios of the kinematical variables available, just as do the more familiar structure functions $W_1$ and $\nu W_2$ for electron-proton scattering in the deep inelastic region. Characteristics of the angular correlation between the detected final particles are also discussed. Since these results are subject to direct experimental test we report them here reserving details for a more complete publication.

Present experimental data[4] indicate that for $Q^2 > 0.5\ (\mathrm{GeV}/c)^2$ $W_1$ and $\nu W_2$ are consistent with the Bjorken's limiting behavior.[3] It is of extreme theoretical interest to see if the generalized Bjorken limit discussed in this Letter will also be reached at such low values of $Q^2$.

For inelastic electron scattering with detection of the final electron plus one hadron there are four invariant variables to describe the hadron structure:

$$Q^2 = -q^2 > 0; \ \nu = \frac{P \cdot q}{M} \ ; \ \nu_1 = \frac{P_1 \cdot q}{M_1} \ ; \ \kappa_1 = \frac{P \cdot P_1}{M} \tag{1}$$

where $P_\mu$, M and $P_{1\mu}$, $M_1$ are the four-momentum and mass of the initial proton and the detected hadron respectively. The two new variables are $\nu_1$, the energy loss of the electron in the rest frame of the detected hadron, and $\kappa_1$, the energy of the detected hadron in the laboratory system. Averaging over all spins, there are four structure functions for the hadrons. However we shall immediately simplify to a study of the differential cross section as a function of the four variables (1) by integrating over the extra azimuthal angle associated with $d^3P_1$. This gives

$$\frac{d^4\sigma}{dQ^2 d\nu \, d\kappa_1 d\nu_1} = \frac{4\pi\alpha^2}{\left(Q^2\right)^2} \left(\frac{\epsilon'}{\epsilon}\right) \left[\mathscr{W}_2 \cos^2\frac{\theta}{2} + 2\mathscr{W}_1 \sin^2\frac{\theta}{2}\right] \tag{2}$$

where $\epsilon$ is the incident energy and $\epsilon'$, $\theta$ are the energy and angle of the scattered electron in the laboratory system. Initial spins are averaged and final ones summed over. The two structure functions $\mathscr{W}_{1,2}$ are defined by

$$\mathscr{W}_{\mu\nu} = 4\pi^2 \frac{E_P}{M} \sum_n \int d^3P_1 \ \delta\left(\kappa_1 - \frac{P \cdot P_1}{M}\right) \delta\left(\nu_1 - \frac{P_1 \cdot q}{M_1}\right) \langle P | J_\mu(0) | P_1 n\rangle$$

$$\langle nP_1 | J_\nu(0) | P\rangle \times (2\pi)^4 \, \delta^4\left(q + P - P_1 - P_n\right) \tag{3}$$

$$= -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\mathscr{W}_1\left(q^2, \nu, \kappa_1, \nu_1\right) + \frac{1}{M^2}\left(P_\mu - \frac{P \cdot q}{q^2} \ q_\mu\right) \times$$

$$\left(P_\nu - \frac{P \cdot q}{q^2} q_\nu\right)\mathscr{W}_2\left(q^2, \nu, \kappa_1, \nu_1\right)$$

where $J_\mu$ is the hadronic electromagnetic current operator; $|P>$ is a one-proton state and $|P_1 n>$ is a state of the one hadron being detected plus all possible others with quantum numbers summarized by n; and spin averages are implicit in this definition. Eq. (2) has the same form as the corresponding expression $\frac{d^2\sigma}{dQ^2 d\nu}$ for $i$) because there are no additional tensors from which to construct the current conserving tensor $\mathcal{W}_{\mu\nu}$ after we perform the azimuthal average indicated. There are now four scalar variables $q^2$, $\nu$, $\kappa_1$, and $\nu_1$ which satisfy the kinematical constraints

$$\begin{aligned} &(q+P)^2 > M^2 \quad \text{or} \quad 2M\nu - Q^2 > 0 \\ &(q+P-P_1)^2 > 0 \quad \text{or} \quad 2M\nu - Q^2 - 2M\kappa_1 - 2M_1\nu_1 + M^2 + M_1^2 > 0. \end{aligned} \tag{4}$$

As before we analyze (3) in the infinite momentum center of mass frame of the initial electron and the initial proton, "undressing" the current by the unitary transformation U(t): $J_\mu(t) = U^{-1}(t)\, j_\mu(t)\, U(t)$. Only the good components ($\mu = 0, 3$) of the hadronic electromagnetic current are considered so that the earlier discussion in terms of good and bad vertices can be imitated <u>in toto</u>. The key to the analysis here as in Ref. 1 leading to a parton model is this: the final particles are divided into two well separated well identified and non-interacting groups, (A) and (B) as labelled in Fig. 1(b). Moreover the particles in each of the two groups move close to each other, due to our assumed transverse momentum cut-off. Therefore the invariant masses of the two groups (A) and (B) are separately finite and negligible as compared with $Q^2$ or $M\nu$ and hence energy as well as momentum is effectively conserved in the Bjorken limit across the point-like electromagnetic vertex $j_\mu(x)$. From this discussion we come to a clear specific experimental prediction already made in Ref. 1: <u>In the laboratory system any observed final hadron will emerge within a cone of width $k_{1\,max} \approx 400$ MeV along the momentum transfer direction in group (B) of Fig. 1 and with a finite fraction of the large energy $\nu$, or it will be "left behind" and</u>

emerge in group (A) with low energy and momentum $\lesssim k_{1\,max} \approx 400$ MeV. Concerning the dependence of the structure functions on the scalar variables (1) we must consider these two possibilities separately.

Consider first the case in which the detected hadron originates from group (B) and emerges along the direction of $\underset{\sim}{q}$ in the lab. Then the two new variables $\nu_1$ and $\kappa_1$ in (1) are not independent but are kinematically related in the Bjorken limit by $\omega = \frac{2M\nu}{Q^2} = -\frac{M\kappa_1}{M_1\nu_1}$. This relation is readily derived by introducing $\underset{\sim}{P}_B$ the momentum immediately after scattering of the charged constituent which interacts with the virtual photon. Neglecting the bounded, finite transverse momentum relative to $\underset{\sim}{q}_\perp$, we have $P_B^\mu = \frac{1}{\omega} P^\mu + q^\mu$ in the infinite momentum frame where $\frac{1}{\omega}P$ is the longitudinal momentum of the charged constituent before scattering. This just states that both energy and momentum are conserved to leading order in $Q^2$ and $M\nu$ across the electromagnetic vertex. If $\eta_1$ denotes the fraction of the longitudinal momentum of the detected hadron with respect to the scattered charged constituent, we can write $\underset{\sim}{P}_1 = \eta_1 \underset{\sim}{P}_B + \underset{\sim}{k}_\perp$ ; $\underset{\sim}{k}_\perp \cdot \underset{\sim}{P}_B = 0$, $1 > \eta_1 > 0$. The same approximation as above gives $P_1^\mu = \eta_1 P_B^\mu = \eta_1\left(\frac{1}{\omega}P^\mu + q^\mu\right)$. From this and definition (1) it follows to leading order that $M\kappa_1 = P_1 \cdot P = \eta_1 M\nu$ ; $M_1\nu_1 = P_1 \cdot q = -\frac{1}{2}\eta_1 Q^2$ or

$$\eta_1 = \frac{\kappa_1}{\nu} \equiv \frac{1}{\omega_1};\ M_1\nu_1 = -\frac{1}{\omega} M\kappa_1 \tag{5}$$

Thus as $\kappa_1$ and $\nu_1$ individually become infinite their ratio is finite and determined, as we claimed above. Furthermore, it follows from (4) that $0 < \frac{1}{\omega_1} < 1$. For later convenience we also define

$$u_1 \equiv \frac{M_1\nu_1}{M\nu} = -\frac{1}{\omega\omega_1} \quad . \tag{6}$$

Next we turn to establishing the analogue of Bjorken scaling behavior for the structure functions. Inserting (5) and (6) and "undressing" the current as in Ref. 1 we obtain from (3)

$$\mathscr{W}_{\mu\nu} = 4\pi^2 \frac{E_P}{M} \frac{M_1}{M} \frac{1}{\nu^2} \delta\left(u_1 + \frac{1}{\omega\omega_1}\right) \sum_n \int (dx)\, e^{iq\cdot x} d^3P_1\, \delta\left(\eta_1 - \frac{1}{\omega_1}\right) \times$$

$$< UP \mid j_\mu (x) U \mid P_1 n > < n P_1 \mid U^{-1} j_\nu (0) \mid UP > \quad . \tag{7}$$

Since the U-matrix acts on the two groups of final particles (A) and (B) independently and separately it can be removed from group (A) because we sum over all possible states in group (A) and the total probability for anything to happen is unity. Also, taking into account the fact that the undressed current $j_\mu(x)$ is a one-body operator which scatters a single charged constituent denoted by $\lambda_a$ to momentum $\underline{P}_B$ as defined earlier we write

$$\mathscr{W}_{\mu\nu} = 4\pi^2 \frac{E_P}{M} \frac{M_1}{M} \frac{1}{\nu^2} \delta\left(u_1 + \frac{1}{\omega\omega_1}\right) \int (dx) e^{iqx} \sum_{n_A, n_B \lambda_a, s, s'} \int d^3P_1$$

$$\delta\left(\eta_1 - \frac{1}{\omega_1}\right) \times \; < UP \mid j_\mu(x) \mid n_A + \lambda_a, s > < P_B, \lambda_a s \mid U\left(P_1 n_B\right) > \tag{8}$$

$$< \left(n_B P_1\right) U^{-1} \mid P_B, \lambda_a, s' > < n_A + \lambda_a, s' \mid j_\nu (0) \mid UP >$$

where $n_A$ and $n_B$ denote the two distinct subsets of states in Fig. 1(b) over which we sum and a complete set of such states has been introduced.

We can now perform the final state integral formally, writing

$$\frac{M_1}{M} \int d^3P_1\, \delta\left(\eta_1 - \frac{1}{\omega_1}\right) \sum_{n_B} < P_B, \lambda_a, s \mid U\left(P_1 n_B\right) > < \left(n_B P_1\right) U^{-1} \mid P_B, \lambda_a, s' >$$

$$= \chi_s^{*} f_{\lambda_a}\left(\omega_1, \sigma\right) \chi_{s'} = \delta_{ss'} f_{\lambda_a}\left(\omega_1\right) \tag{9}$$

where $\chi$ is a two-component Pauli spinor. Since $\underset{\sim}{P}_B$ is the only preferred direction, $f_{\lambda_a}(\omega_1, \sigma)$ can depend on spin only through the combination $(\underset{\sim}{P}_B \cdot \underset{\sim}{\sigma})^2 = P_B^2$, as dictated by rotational invariance and parity conservation. We conclude therefore $f_{\lambda_a}$ is spin independent and s = s', which leads to the second form. Furthmore, the left-hand side of (9) is independent of the scale of the infinite momentum $P_B$, so $f_{\lambda_a}$ is a function of the ratio $\omega_1$ only. This gives finally

$$\mathscr{W}_{\mu\nu} = \frac{1}{\nu^2}\,\delta\left(u_1 + \frac{1}{\omega\omega_1}\right)\sum_{\lambda_a} W_{\mu\nu}(\lambda_a)\, f_{\lambda_a}(\omega_1) \qquad \text{where}$$
$$W_{\mu\nu}(\lambda_a) = 4\pi^2\,\frac{E_P}{M}\int (dx)\; e^{iqx} < UP \mid j_\mu(x)\, j_\nu(0) \mid UP >_{\lambda_a} \tag{10}$$

is the total contribution of $W_{\mu\nu}$ for process $i$) from a particular type of charged[5] constituent $\lambda_a$. Eq. (10) in conjunction with the scaling results already derived for $W_{\mu\nu}(\lambda_a)$ shows that $M\nu^2\mathscr{W}_1$ and $\nu^3\mathscr{W}_2$ are functions of the ratios $\omega$ and $\omega_1$ in the Bjorken limit:

$$\mathrm{Lim}_{bj}\, M\nu^2\mathscr{W}_1 = \mathscr{F}_1(\omega, \omega_1, u_1) = \delta\left(u_1 + \frac{1}{\omega\omega_1}\right)\sum_{\lambda} F_{1\lambda}(\omega)\, f_\lambda(\omega_1)$$
$$\mathrm{Lim}_{bj}\, \nu^3\mathscr{W}_2 = \mathscr{F}_2(\omega, \omega_1, u_1) = \delta\left(u_1 + \frac{1}{\omega\omega_1}\right)\sum_{\lambda} F_{2\lambda}(\omega)\, f_\lambda(\omega_1) \tag{11}$$

For spin 1/2 current contributions $\mathscr{F}_1$ and $\mathscr{F}_2$ have a fixed ratio independent of $\omega_1$

$$\mathscr{F}_1(\omega, \omega_1, u_1) = \frac{\omega}{2}\,\mathscr{F}_2(\omega, \omega_1, u_1) \tag{12}$$

and for spin 0 current contributions $\mathscr{F}_1$ vanishes

$$\mathscr{F}_1(\omega, \omega_1, u_1) = 0 \tag{13}$$

Eqs. (11), (12), and (13) are our central result of generalized scaling. This is a non-vanishing and hence non-trivial result because a sum over all charged constituents and kinematic values of $u_1$, $\omega_1$, and $\omega$ gives the total inelastic cross section as a lower bound. According to present experimental indications[4] the spin 1/2 current contribution is dominant. Hence the group of energetic recoiling particles in (B) should include a baryon or anti-baryon; one of the octet according to our model. We have no prediction in our model on the ratio of $\rho$' s to $\pi$'s appearing, plus "anything else," in the final state. Moreover any individual channel will have a rapidly decreasing production cross section, and it is only the aggregate sum of all possible channels (inclusive[2] measurements) that survive in the Bjorken limit. This is a crucial prediction of our model that can be checked. It is very different from Harari's[6] prediction of large "diffraction" production of single $\rho^o$'s only by very virtual photons.

For the second case when the detected hadron originates from group (A) one learns little more than

a) It moves with finite momentum in the lab system, i.e. it is one of the constituents "left behind" after the impact by the virtual photon. In this case therefore the new ratio $\frac{\kappa_1}{\nu}$ vanishes in the Bjorken limit.

b) As in the previous case $\frac{M\nu\mathcal{W}_1}{\nu^2\mathcal{W}_2} = \frac{\omega}{2}$ for a spin 1/2 current playing the dominant role and $\nu\mathcal{W}_1 = 0$ for a spin zero current.

Finally we turn to the annihilation process $e + \overline{e} \rightarrow H_1 + H_2$ + "anything," where H denotes an arbitrary (anti-) hadron. Defining the variables as in (1) and doing the angular average over $P_1$ with fixed $\nu_1$ and $\kappa_1$ we find the differential cross section, similar in form to $\frac{d\sigma}{dE d\cos\theta}$ for *ii*)

$$\frac{d\sigma}{dE\, d\cos\theta\, d\kappa_1 d\nu} = \frac{4\pi\alpha^2}{\left(q^2\right)^2} \frac{M^2\nu}{\sqrt{q^2}} \sqrt{1-\frac{q^2}{\nu^2}} \left[2\overline{\mathcal{W}}_1 + \frac{2M\nu}{q^2}\left(1-\frac{q^2}{\nu^2}\right)\frac{\nu\overline{\mathcal{W}}_2}{2M}\sin^2\theta\right] \tag{14}$$

where E and $\theta$ are the energy and angle with respect to the $e^+e^-$ colliding beam of the hadron with four momentum $P_\mu$. The relation of $\overline{\mathcal{W}}$ to $\mathcal{W}$ in (3) is similar to that between W and $\overline{W}$ in Ref. 1 for integrated cross sections. Although the four-momentum $P_\mu$ of the first hadron is fully specified, only the two invariant combinations $\kappa_1$ and $\nu_1$

for the second hadron are given. No information is lost in this angular averaging over the second hadron, in the absence of spin or polarization information, because in the Bjorken limit the two hadrons are approximately parallel or antiparallel to one another.

For the case that the second hadron emerges predominantly back to back relative to the first hadron, an analysis similar to the one leading to (11), (12), and (13) shows that for the annihilation process we have

$$
\begin{aligned}
\mathrm{Lim}_{bj}(-)M\nu^2\overline{\mathscr{W}_1} &= \overline{\mathscr{F}}_1(\omega,\omega_1,u_1) = \delta\left(u_1 - \frac{1}{\omega\omega_1}\right)\sum_\lambda \overline{F}_{1\lambda}(\omega)\, f_\lambda(\omega_1) \\
\mathrm{Lim}_{bj}\,\nu^3\overline{\mathscr{W}_2} &= \overline{\mathscr{F}_2}(\omega,\omega_1,u_1) = \delta\left(u_1 - \frac{1}{\omega\omega_1}\right)\sum_\lambda \overline{F}_{2\lambda}(\omega) f_\lambda(\omega_1)
\end{aligned}
\tag{15}
$$

where $u_1$ and $\omega_1$ are defined as in (5) and (6), and

$$
\begin{aligned}
\overline{\mathscr{F}_1}(\omega,\omega_1,u_1) &= \frac{\omega}{2}\,\overline{\mathscr{F}_2}(\omega,\omega_1,u_1) \qquad &\text{(spin 1/ 2 current)} \\
\overline{\mathscr{F}_1}(\omega,\omega_1,u_1) &= 0 &\text{(spin 0 current)}
\end{aligned}
\tag{16}
$$

In (15) $\overline{F}_{1,2\lambda}(\omega)$ is the total contribution to $\overline{F}_{1,2}(\omega)$ in *ii*) from a charged constituent of type $\lambda$. It has been verified explicitly[1] in III for our model that $\overline{F}_{1,2}(\omega)$ is the same function of $\omega$ as $F_{1,2}(\omega)$ continued from $\omega > 1$ to $\omega < 1$. This applies separately for each $\lambda$, i.e. $\overline{F}_{1\lambda}(\omega) = F_{1\lambda}(\omega)$; $\overline{F_{2\lambda}}(\omega) = F_{2\lambda}(\omega)$. Furthermore, $f_\lambda(\omega_1)$ in (15) is the same function as the one which appeared in (11). No continuation is necessary here, since $\omega_1$ in both cases varies between the same range 0 and 1. Eqs. (15), (16), and (11) imply

$$
\overline{\mathscr{F}_1}(\omega,\omega_1,u_1) = \mathscr{F}_1(\omega,\omega_1,-u_1)\,;\ \overline{\mathscr{F}_2}(\omega,\omega_1,u_1) = \mathscr{F}_2(\omega,\omega_1,-u_1) \tag{17}
$$

On the right-hand side, the functions $\mathscr{F}_{1,2}(\omega,\omega_1,-u_1)$ are continued from $\omega > 1$ to $\omega < 1$. The continuation in the variable $u_1$ is trivial since the $u_1$ dependence on both sides is explicitly known in (15) and (11). Similar rules can be derived for the second case of parallel hadrons.

When more than two final particles are detected the cross sections have the same form as in (2) and (14), assuming all azimuthal averages are taken, and the scaling properties (11) and (15) can be generalized in terms of the scalars $\nu_i$ and $\kappa_i$ for all detected particles. A more detailed discussion of these results will be published elsewhere.

## REFERENCES


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## FIGURE CAPTION

Fig. 1 - Deep inelastic scattering of an incident proton by a current interacting at X. All the initial and final state interactions illustrated by the blobs in diagram (b) in the Bjorken limit reduce to (a) in the parton model.

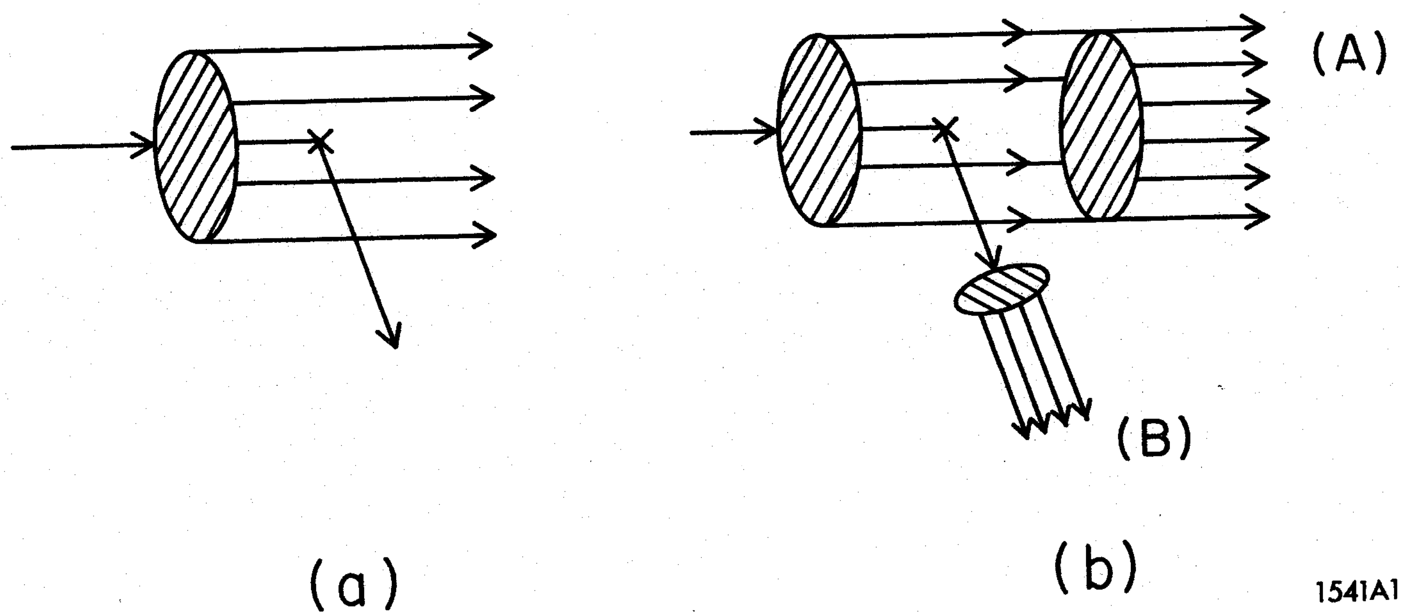


Fig. 1