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MUON INELASTICITY IN UNDERGROUND COSMIC-RAY NEUTRINO INTERACTIONS*

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ABSTRACT

The inelasticity of the secondary muon in high-energy neutrino interactions, defined in a manner appropriate for the deep-mine cosmic-ray experiments, is computed. On the basis of the hypothesis of locality of the lepton current, $\langle E_\mu / E_\nu \rangle = 0.62 \pm 0.12$.

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In order to interpret underground cosmic-ray neutrino experiments,[1,2] it is necessary to estimate the fraction of energy $\epsilon = \langle E_\mu/E_\nu \rangle$ retained by the muon in the interaction which absorbs the neutrino. The purpose of this note is to show that, provided only that the lepton current is of local V-A form, the effective ϵ for the high-energy contribution is given by $\epsilon = 0.62 \pm .12$, where the limits are absolute.

Let E_ν = neutrino energy, E_μ = muon energy at production, and let $\nu = E_\nu - E_\mu$. Assume that the muon ionization loss $dE_\mu/dx \cong$ constant and range fluctuations may be ignored. Assume the incident differential neutrino spectrum can be approximated[3-5,6] by a power law[7] spectrum $\sim E_\nu^{-\gamma}$. Then the rate for detecting muons underground in a given element of solid angle is

$$R = C \int_0^\infty dE_\nu \, E_\nu^{-\gamma} \int_0^{E_\nu} dE_\mu \, E_\mu \left(\frac{d\sigma}{dE_\mu}\right) \tag{1}$$

where the nature of the constant C is not important for the argument here.

Conventionally, in the cosmic-ray literature it is assumed that the muon carries off a unique fraction ϵ of the neutrino energy; i.e.,

$$\frac{d\sigma}{dE_\mu} = \frac{d\sigma}{d\nu} = \sigma(E_\nu) \, \delta(E_\mu - \epsilon E_\nu) \tag{2}$$

where ϵ in principle could vary with E_ν.

With this artifice, the rate underground becomes

$$R = C \int_0^\infty dE_\nu \, E_\nu^{1-\gamma} \, \sigma(E_\nu) \, \epsilon \tag{3}$$

If one assumes that the lepton current is local the cross section for the process

$$\nu_\mu(\bar{\nu}_\mu) + N \rightarrow \mu^-(\mu^+) + \text{hadrons}$$

may be written (upon neglect of the muon mass) as

$$\frac{d\sigma}{dE_\mu} = \sum_{n=0}^{2} \left(\frac{E_\mu}{E_\nu}\right)^n f_n(\nu) \equiv \sum_{n=0}^{2} \frac{d\sigma_n}{dE_\mu} \tag{4}$$

To obtain this form[8] requires an approximation, valid for $\nu \gg 1$ GeV. The correct expression would modify the value of E_μ and E_ν in Eq. (4) by an amount which is at most 0.5 GeV.

The $f_n(\nu)$ in Eq. (4) are non-negative functions, being the contributions from hadronic final states of helicity -1, 0, +1 (+1, 0, -1 for $\bar{\nu}_\mu$ incident) for n = 0, 1, 2 respectively. To protect our approximation,[8] we hereafter take $f_n(\nu) = 0$ for $\nu < \nu_0$ (ν_0 could be taken ~ 5 GeV). The remaining low-ν contribution may be separately computed with the help of the data[9] from CERN.

From Eq. (4) it is straightforward to show that

$$f_n(\nu) = \frac{1}{n!} \frac{d^{n+1}}{d\nu^{n+1}} \nu^n \sigma_n(\nu) \tag{5}$$

where

$$\sigma_n(E_\nu) = \int_0^{E_\nu} dE_\mu \left(\frac{d\sigma_n}{dE_\mu}\right) \tag{6}$$

as defined by (4).

To compute the rate R we insert (4) into (1) and change orders of integration.

$$R = C \sum_{n=0}^{2} \int_0^\infty d\nu \int_\nu^\infty dE_\nu \; E_\nu^{-\gamma-n} (E_\nu - \nu)^{n+1} f_n(\nu)$$

$$= C \sum_{n=0}^{2} \frac{\Gamma(\gamma-2)\,(n+1)!}{\Gamma(\gamma+n)} \int_0^\infty d\nu \; \nu^{2-\gamma} f_n(\nu) \tag{7}$$

Using (5), we obtain after integrating by parts[10]

$$R = C \sum_{n=0}^{2} \left(\frac{n+1}{n+\gamma-1}\right) \int_0^\infty d\nu \; \nu^{1-\gamma} \; \sigma_n(\nu) \tag{8}$$

For $\gamma \approx 3$, as computed by Cowsik et al.,[4] and Osborne et al.,[3] we find from (8) and (3)

$$R \approx C \int_0^\infty d\nu \; \nu^{-2} \left\{ \frac{1}{2}\, \sigma_0(\nu) + \frac{2}{3}\, \sigma_1(\nu) + \frac{3}{4}\, \sigma_2(\nu) \right\}$$

$$\equiv C \int_0^\infty d\nu \; \nu^{-2} \left(\sigma_0(\nu) + \sigma_1(\nu) + \sigma_2(\nu)\right) \epsilon \tag{9}$$

We see that ϵ must lie between $\sim \frac{1}{2}$ and $\sim \frac{3}{4}$, independently of energy with a slight modification possible when the contributions from low ν, which we have omitted, are included.

The estimate of ϵ can be sharpened by observing that upon including only the dominant $\Delta S = 0$ neutrino processes,[11] then charge-symmetry[8] equates $\bar{\sigma}_0$ for antineutrinos incident to σ_2 for neutrinos incident,[12] and $\bar{\sigma}_1$ to σ_1. Therefore there are good theoretical grounds for asserting

$$\begin{aligned} \bar{\sigma}_0(E) &\approx \sigma_2(E) \\ \bar{\sigma}_1(E) &\approx \sigma_1(E) \\ \bar{\sigma}_2(E) &\approx \sigma_0(E) \end{aligned} \tag{10}$$

Therefore were the ν and $\bar{\nu}$ fluxes equal, or were $\sigma_0 \approx \sigma_2$ as in Harari's model[13] or other "diffraction" models,[14] then the mean inelasticities would be

$$\epsilon \simeq \frac{2\gamma-1}{\gamma^2-1} \approx \frac{5}{8} \qquad n=0,2$$

and

$$\epsilon \approx \frac{2}{\gamma} \approx \frac{2}{3} \qquad n=1 \qquad (11)$$

Thus in the estimate $\epsilon = .62 \pm .12$ the quoted uncertainty is conservatively large.

We have neglected the effect of electromagnetic corrections, which will tend to reduce ϵ. While we know of no detailed calculation of this effect, a rough estimate gives a correction of at most a few percent.

In conclusion, we emphasize that a radical revision of present ideas of weak interactions, specifically a nonlocality of the lepton current, is implied by a value of ϵ much different from $.62 \pm .12$. The existence of nonlocality of the weak interaction arising from exchange of an intermediate boson between lepton and hadron does not modify the result (to the extent that W production by neutrinos can be neglected). Either the pure Fermi coupling or the W-exchange model satisfies the hypothesis of local lepton current.

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REFERENCES

1. F. Reines et al., Can. J. Phys. 46S, 350 (1968).

2. M. Menon et al. Can. J. Phys. 46S, 344 (1968).

3. J. Osborne, S. Said, and A. Wolfendale, Proc. Phys. Soc. (London) 86, 93 (1965).

4. R. Cowsik, Y. Pal, and S. Tandon, Proc. Indian Acad. Sci. 63A, 217 (1965).

5. G. Zatsepin and V. Kuzmin, Soviet Phys.—JETP 14, 1294 (1962).

6. An exponent slowly changing with E_ν will also be acceptable, as will be clear from the nature of the result.

7. For low values of E_ν, this is not a good assumption. However, even if the spectrum is cut off sharply for $E_\nu < E_0$, the main result, Eq. (7) is only modified by replacing $f_n(\nu)$ by $f_n(\nu)\,\phi(\nu)$, where $0 < \phi < 1$ for $\nu < E_0$ and $\phi \equiv 1$ for $\nu > E_0$. Thus only the contributions from low-mass hadron final states are affected; our major concern here is on the asymptotic region.

8. T. D. Lee and C. N. Yang, Phys. Rev. 126, 2239 (1962). Equation (4) follows from Lee and Yang's Eq. (49) and the approximation $P = \sqrt{\nu^2 + q^2} \approx \nu$. From kinematics, $q^2 \lesssim 2M_N\nu$ and the error ΔP in the approximation is $\approx q^2/2\nu$. ΔP is rigorously bounded above by $M \sim 1$ GeV; a more reasonable estimate gives $\langle \Delta P \rangle \lesssim 0.3$ GeV.

9. I. Buganov et al., Phys. Letters 30B, 364 (1969).

10. Because f_n is taken to vanish for $\nu < \nu_0$, there is no trouble with surface-terms coming from the limit $\nu = 0$. Because the observed rate is finite there can be no problem from $\nu = \infty$.

11. The $\Delta S = 1$ processes are suppressed by a factor $\tan^2 \theta_{\text{Cabibbo}} \approx .05$.

12. Specifically $\sigma_0(\bar{\nu} p) = \sigma_0(\nu n)$, etc.; for rock ($Z \approx N$) we then get the result in the text.

13. H. Harari, Phys. Rev. Letters 22, 1078 (1969).

14. H. Abarbanel, M. Goldberger, and S. Treiman, Phys. Rev. Letters 22, 500 (1969).