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Critical Power Dissipation in a Superconductor*

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Abstract

The magnetic breakdown field, H_p', is calculated which, if applied at the surface of a superconductor, produces a critical power dissipation leading to a steep rise in the power loss. Generally $H_p' < H_c$, the critical field of the material. The functional dependence of the Q of a microwave cavity for values of H_p near H_p' is also found.

As the magnetic field, $H_p \cos \omega t$, applied to the surface of a superconductor is increased, a critical value of $H_p = H_p'$ is reached at which the power dissipation rises sharply. Generally $H_p' < H_c$, the critical field of the material. The object of this paper is to calculate H_p'.

The fact that H_p' can be well below H_c (H_{c1} for type II) in both low and high frequency measurements has been blamed on surface defects such as impurities and dislocations; protrusions which cause local magnetic field enhancement; and trapped flux. Halbritter[1] realized that a suggestion by Easson *et al.*,[2] for the low frequency case might be applicable to superconducting microwave cavities. Easson *et al.*,[2] suggested that in type II the transition to the normal state is caused by a temperature rise above T_c due to ac losses rather than by the peak

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current in the sample exceeding the thermodynamic critical value. However it appears that neither Halbritter, Easson *et al.*, nor anyone else has pursued this suggestion in terms of a theoretical analysis which relates the magnetic breakdown field to the thermal and electrical properties of a superconductor.

For simplicity, let us consider a cylindrical normal region of radius a within the penetration depth, λ, which may be a fluxoid, etc. The power dissipation per unit area of the cylinder is

$$P_a = \frac{1}{2} R(FH_p)^2 \tag{1}$$

where FH_p represents the proper spatial average of the magnetic field across the cylinder whose center lies a distance d from the conducting surface. R is the effective normal state surface resistance of the cylinder. $R = R_0 + R(T) \doteq R_0$, as the temperature excursion is not great.

Assuming cylindrical symmetry as a fair approximation, and using cylindrical coordinates centered at the fluxoid, the heat flow equation is $P_a\left(\frac{a}{r}\right) = -K\frac{dT}{dr}$, where the superconducting thermal conductivity for many metals is roughly given by:

$$K = \begin{cases} k_1(T - T_1) \\ k_2T^{-3/2} + C_2 \\ k_3T^3 \end{cases} \doteq \begin{cases} k_1T & T_a \geq T \geq T_m \\ k_2T^{-3/2} & T_m \geq T \geq T_M \\ k_3T^3 & T_M \geq T \geq T_b \end{cases} \tag{2}$$

where T_a is the temperature at the periphery of the cylinder, T_b is the temperature of the outer surface of the conductor which is approximately the bath temperature neglecting Kapitza resistance, T_M is where K is maximum, and T_m is where K is minimum. (It makes only a small difference whether $T^{-3/2}$ or $\exp\left(\frac{g}{T}\right)$ is used.)

If the cylinder center is a distance b from the outer surface, the approximate solution is:

$$H_p = \frac{1}{F}\left[\frac{k_1\left(T_a^2 - T_m^2\right) + 2C_1}{NRa\ell n\left(\frac{b}{a}\right)}\right]^{1/2} \tag{3}$$

where N is a factor to correct for the departure from cylindrical symmetry, and

$$C_1 = 2k_2\left(T_M^{-\frac{1}{2}} - T_m^{-\frac{1}{2}}\right) + \frac{1}{4}k_3\left(T_M^4 - T_b^4\right) . \tag{4}$$

For

$$k_1T_a^2 \gg k_1T_m^2 - 2C_1,\ H_p \doteq \frac{1}{F}\left(\frac{k_1}{NRa\,\ell n\left(\frac{b}{a}\right)}\right)^{\frac{1}{2}} T_a\left[1 - \frac{1}{2}\left(\frac{k_1T_m^2 - 2C_1}{k_1T_a^2}\right) - \frac{1}{8}\left(\frac{k_1T_m^2 - 2C_1}{k_1T_a^2}\right)^2 - \ldots\right] \tag{5}$$

So at large T_a, H_p is approximately linear with T_a. If the superconductor is a microwave cavity, when $k_1T_a^2 \gg k_1T_m^2 - 2C_1$ and $T_a > \frac{1}{2}T_c$, then the cavity $Q \propto e^{\epsilon/2k_BT}$ where ϵ is the energy gap and k_B is the Boltzmann constant. If the cavity Q is dominated by the power loss around the cylinder (there may be more than one), $T = fT_a \propto H_p$, $(f < 1)$,

$$\Rightarrow Q \propto e^{D/H_p} \text{ for } H_p \text{ near } H_p' . \tag{6}$$

where

$$D \doteq \frac{\epsilon f}{2k_BF}\left[\frac{k_1}{NRa\,\ell n\left(\frac{b}{a}\right)}\right]^{1/2} . \tag{7}$$

When $T_a < \frac{1}{2}T_c$ and $k_1T_a^2 \gg k_1T_m^2 - 2C_1$, then for H_p near H_p',

$$Q \propto Te^{\epsilon/2k_BT} \propto H_pe^{D/H_p} . \tag{8}$$

Equation (8) gives a good representation of the data of Turneaure and Viet.[3]

When H_p reaches a point where a critical magnetic field is reached in the neighborhood, then the material surrounding the cylinder will go normal, leading to a sharp rise in the power dissipation and ultimately a run-away situation. In the case of a cavity, the Q will drop precipitously. The magnetic field in the neighborhood is $F\vec{H}_p \cos \omega t + \vec{H}_a$, where $\vec{H}_a$ is the magnetic field which exists at radius a due to the contribution from all sources besides the current in the cylinder. $\vec{H}_a$ may be an applied dc field, and/or the field penetration from a fluxoid. The worst case is when H_a and $F H_p \cos\omega t$ add together algebraically at peak value.

$$H_a + F H_p = H_c = H_0 \left[1 - \left(\frac{T_a}{T_c}\right)^2\right] \tag{9}$$

Combining Eq. (3) and (9) yields

$$H_p' = \frac{-\frac{1}{2} k_1 \frac{T_c^2}{H_0} + \left[\left(\frac{1}{2}k_1 T_c^2/H_0\right)^2 - 2NR\, a\ln\left(\frac{b}{a}\right)\left\{\frac{1}{2}k_1\left[T_m^2 - T_c^2\left(1 - \frac{H_a}{H_0}\right)\right] - C_1\right\}\right]^{1/2}}{NRF\, a\ln\left(\frac{b}{a}\right)} \tag{10}$$

This is the peak magnetic field at the surface of a superconductor which produces a critical power dissipation leading to a steep rise in the power loss. With the possible exception of R and F, all the parameters can be determined experimentally. The frequency dependence of H_p' may help to determine R.

In the case of Pb or Nb, if the normal region is a fluxoid, 2a can easily be $\gtrsim \lambda$, and an rf field might penetrate it fully if it is located at the conducting surface. The current in a conductor flows in such a manner as to minimize resistive losses for dc or low frequencies. However, at high frequencies, as in a GHz cavity, the stored electromagnetic energy is minimized and the current tends to flow more uniformly through the conducting surfaces. For an oscillating fluxoid which results when a high frequency current is perpendicular to the fluxoid, Gittleman and Rosenblum[3] have shown that the effective resistivity of the fluxoid is $\sim\rho_n$, the normal bulk resistivity. From the Wiedemann-Franz law $L/\rho_n = k_{1n} \doteq k_1$, where $L = 2.45 \times 10^{-8}$ W-$\Omega/^{o}K^2$, and $k_{1n}T$ is the normal state thermal conductivity.

We are now in a position to make a predictive comparative estimate (see Table I) of the relative importance of the various parameters as they affect the magnetic breakdown field H_p' at which, for example, the Q of a niobium cavity would be seriously degraded. Take $T_c = 9.5^{o}K$, $N=2$ and $\ell n\left(\frac{b}{a}\right) \sim 12$. The thermal conductivity values for group A were obtained from the highest values of Calverly et al.,[4] group B from Styles and Weaver,[5] and group C from H. Brechna.[6]

Turneaure and Weissman[7] have made measurements on niobium cavities. They report magnetic breakdown fields which range from 290 to 436 Oe. Similarly Nb measurements by Turneaure and Viet[3] have extended the breakdown field in the range from 710 to 1080 Oe, primarily by improved vacuum heat treating of the Nb cavities. The theoretical predictions of Table I are consistent with this entire range, as well as the fact that the thermal conductivity improves with vacuum heat treating.

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TABLE I

Comparative Predicted Values of H_p' for Nb

	H_p'	H_0	H_a	T_b °K	k_1 $\frac{W}{cm\,°K^2}$	C_1 $\frac{W}{cm}$	T_a °K
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A	1000	4000	2400	1.0	0.10	28.8	3.6
	960	4000	2400	1.8	0.10	20.9	3.8
B	690	4000	2400	1.0	0.045	~ 0	4.5
	660	4000	2400	1.8	0.045	~ 0	4.6
C	300	4000	2400	1.0	0.016	~ 0	5.4
	290	4000	2400	1.8	0.016	~ 0	5.4

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