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HADRON STATES OF EVEN CHARGE CONJUGATION FROM ELECTRON-POSITRON ANNIHILATION*

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ABSTRACT

We show how electron-positron storage rings may be used to obtain information on hadronic states with even charge conjugation. This is illustrated by studying the reaction $e^- e^+ \rightarrow \pi^- \pi^+ \gamma$, the analysis of which should reveal information on even-C dipion resonances, such as the existence of the ϵ-meson.

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Colliding electron-positron beams have the attractive property that, to order α, the emerging hadronic state must have odd C and J = 1. This allows the clean study of hadronic states with these quantum numbers and provides, for example, beautiful new data on electromagnetic form factors. In this letter we show how electron-positron storage rings can be used to study hadronic states of even charge conjugation in the reaction

$$e^- + e^+ \rightarrow A + \gamma$$

Here, A is any neutral hadronic state and γ is a hard photon. To order α^3 in the cross section, the contribution of states with odd charge conjugation can be removed using only quantum electrodynamics and knowledge of the cross section for the reaction

$$e^- + e^+ \rightarrow A .$$

To illustrate the method, let us consider[1]

$$e^- + e^+ \rightarrow \pi^- + \pi^+ + \gamma \quad . \tag{1}$$

The application of the method to other hadronic systems will be immediately apparent. We choose to study the $\pi\pi$ system because observation of reaction (1) should resolve the question of the existence of an I = 0, J = 0 dipion resonance. This resonance (called an ϵ- or σ-meson) has been predicted[2] to have a mass approximately the same as the ρ-meson; estimates of its width vary from 150 MeV to 500 MeV.[3] In this letter we describe the essential features of the method, leaving the details for a lengthier publication[4].

To order e^3, the amplitude for the process $e^- e^+ \rightarrow \pi^- \pi^+ \gamma$ may be represented in terms of the Feynman diagrams of quantum electrodynamics as the sum of two contributions, which are distinguished by whether the source of the emitted photon is the leptons or otherwise. (See Figure 1). Class B diagrams have the final photon emitted by the leptons, while Class A diagrams include all other possibilities. Let us assume the validity of charge conjugation (C) invariance[5]. Recalling that the electromagnetic current has C-minus, we notice that the dipion system emerging from diagrams A have C-even; from diagrams B, C-odd. Since a $\pi^+\pi^-$ system with definite angular momentum J has $C = (-)^J$, A (B) produces pions with J-even (-odd). By the generalized Pauli principle, then, the pions in A (B) have isospin $I = 0$ or 2 $(I=1)$. From the observation of charged pion production only, there is no way to separate the $I = 0$ and $I = 2$ contributions. Finally, in Class B, since the pions are coupled to a four-vector current, they can have only $J = 1$.

We recognize the "blob" in diagrams B as the pion form factor $F_\pi(t)$, where t is the invariant mass-squared of the dipion system. The magnitude of the pion form factor is measured[6] in $e^- e^+ \rightarrow \pi^- \pi^+$, which is described by diagram B without the external photon emission[7]. Consequently, once the cross section for $e^- e^+ \rightarrow \pi^- \pi^+$ is known, the <u>magnitude</u> of B can be calculated exactly, and we will assume henceforth it is known.

The cross section $d\sigma_{-+}$ for $e^- e^+ \rightarrow \pi^- \pi^+ \gamma$ is proportional to $|A+B|^2$. Under the exchange of the charges of the pions, since B has C-odd, it changes sign, while A, with C-even, does not. Consequently, the cross-section for $e^- e^+ \rightarrow \pi^+ \pi^- \gamma$ has $d\sigma_{+-} \propto |A-B|^2$. If one does an experiment in which he does not distinguish the charges of the pions, he measures $d\sigma_{\text{charged}} = d\sigma_{-+} + d\sigma_{+-} \propto |A|^2 + |B|^2$. According to our remarks above, the magnitude $|B|$ is known and so can be subtracted from the total cross-section for charged pion production, leaving only $|A|^2$. Thus, in a model-independent way, the contribution of even angular momentum states can be isolated. If there is an even-spin resonance, one excepts to see a bump in $|A|^2$ as the dipion invariant mass passes through the resonance (Figure 2).

If, however, a more detailed experiment is performed, and the charges of the pions are identified, then one knows $d\sigma_{-+}$ and $d\sigma_{+-}$ separately. Hence, the interference term between A and B may be easily isolated from the difference:

$$d\sigma_{-+} - d\sigma_{+-} \propto \mathrm{Re}(A^*B) .$$

This yields the relative phase between A and B.

Although there is no final-state interaction theorem for the process under consideration, there are nevertheless many similarities between the analysis of $K_{\ell 4}$ decay by Pais and Treiman[8] and this analysis. In particular, $e^- e^+ \to \pi^- \pi^+ \gamma$ depends on five kinematical invariants, aside from spin variables. However, our understanding of quantum electrodynamics allows us to decompose the cross section in such a way that the dependence on two of the five invariants is explicit. As with $K_{\ell 4}$, one can show that, given the differential unpolarized cross section, one can extract the magnitudes[9] and relative phases of all unknown scalar amplitudes.

If we wish to use this reaction to determine the existence of a scalar dipion resonance (the ϵ-meson), what would be the "best" experimental arrangement? By G-parity it is easy to see that both the emitted photon and the virtual photon must have the same G-parity, that is, they are either both isovector or both isoscalar photons. However, in B, the virtual photon couples to two pions, and so must be an isovector. Thus, B receives contributions from only isovector photons, whereas A receives contributions from both isoscalar and isovector photons. We can enhance the contribution of A with respect to B by setting the colliding beam energy to an isoscalar resonance. Since the photon emitted in the sequence $e^- e^+ \to \epsilon\gamma$, $\epsilon \to \pi^- \pi^+$ will certainly be internal bremsstrahlung, it is necessary to work with fairly hard photons in order that this contribution be seen above the $1/E_\gamma$ spectrum, typical of bremsstrahlung from external legs. Since we anticipate that the ϵ mass will be about equal to the ρ mass, we must work at incident energies on the order of 800-1000 MeV. The ϕ-meson resonance is very conveniently located at 1019 MeV; not only is it a very narrow isoscalar resonance, but also, in the decay $\phi \to \epsilon\gamma$, the photon will carry off around 250 MeV. So we suggest setting $s = m_\phi^2$.

External bremsstrahlung contributions become very large in the direction of travel of charged particles. Consequently, observations of the photon considerably away from the direction of the pions and the leptons would be best. So one should look for events in which, in the center of mass of the electron-positron pair, the photon is emitted to the left, say, of the collision axis; the pions, to the right. A crude model, based on vector-meson dominance, (Figure 3) indicates that the contribution of A to the differential cross-section at a beam energy on the ϕ peak, is of the same order of magnitude as that coming from B and, in fact, for some configurations may considerably exceed B. (We assumed the coupling $g_{\epsilon\phi\phi}$ to be the same order of magnitude as $g_{\epsilon\pi\pi}$.) The cross-section will thus be on the order of 10^{-2}- 10^{-3} μb.

The details of this analysis will be given elsewhere[2]. Clearly, the method of analysis indicated here applies equally well to such processes as $e^- e^+ \to K^- K^+ \gamma$ or $e^- e^+ \to p\bar{p}\gamma$, as well as to more than two particle hadronic states. Thus it is possible to extract information from colliding beam experiments on hadronic states with even charge conjugation.

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References

1. This reaction has been discussed previously by F. M. Renard, Dept. de Physique Mathematique, Montpellier, France, Preprint No. PM/69/2, April 1969. Contributions from photon emission by leptons are ignored and the vector dominance model is assumed at the outset. For a discussion of a related reaction, viz., $e^- e^- \to \pi^- \pi^+ e^- e^-$, see F. Calogoro and C. Zemach, Phys. Rev. 120, 1860 (1960) and P. C. DeCelles and J. F. Goehl, Jr., Phys. Rev., to be published.
2. The following references provide a sampling of theoretical discussions bearing on the existence of the ϵ. C. Lovelace et al., Phys. Letters 22, 332 (1966); F. J. Gilman and H. Harari, Phys. Rev. 165, 1803 (1968); S. Weinberg, Phys. Rev. Letters 22, 1023 (1969). For a summary of applications of Veneziano's model to $\pi\pi$ scattering, see C. Lovelace, talk presented at the Argonne Conference on the $\pi\pi$ and $K\pi$ Interactions, May, 1969.
3. Since writing this note, we received a report on a measurement of $\pi^- p \to \pi^0 \pi^0 n$ which shows quite distinctly a peak in the dipion invariant mass spectrum which is probably due to the ϵ. P. Sonderegger and P. Bonamy, Lund International Conference on Elementary Particles, June, 1969. We review the experimental and theoretical background elsewhere.[4]
4. M. J. Creutz and M. B. Einhorn, to be published.
5. Of course, every annihilation experiment should first be used to check C-invariance. See A. Pais and S. B. Treiman, Phys. Letters 29B, 308 (1969).
6. J. E. Augustin et al., Phys. Letters 28B, 508 (1969); V. L. Auslander et al., Phys. Letters 25B, 433 (1967). We anticipate that these measurements will be improved and extended over a wider energy range by subsequent experiments.

7. The distinction between the processes depends on identifying the photon as "hard". For "soft" photons, one thinks of diagram B as the radiative corrections to $e^- e^+ \rightarrow \pi^- \pi^+$. See Augustin, reference 6.

8. A. Pais and S. B. Treiman, Phys. Rev. 168, 1858 (1968).

9. It is interesting that in this experiment alone, $|F_\pi|$ can be determined. This will provide a consistency check on the data as well as an independent determination of the overall normalization. This should help eliminate uncertainties due to the incident flux.

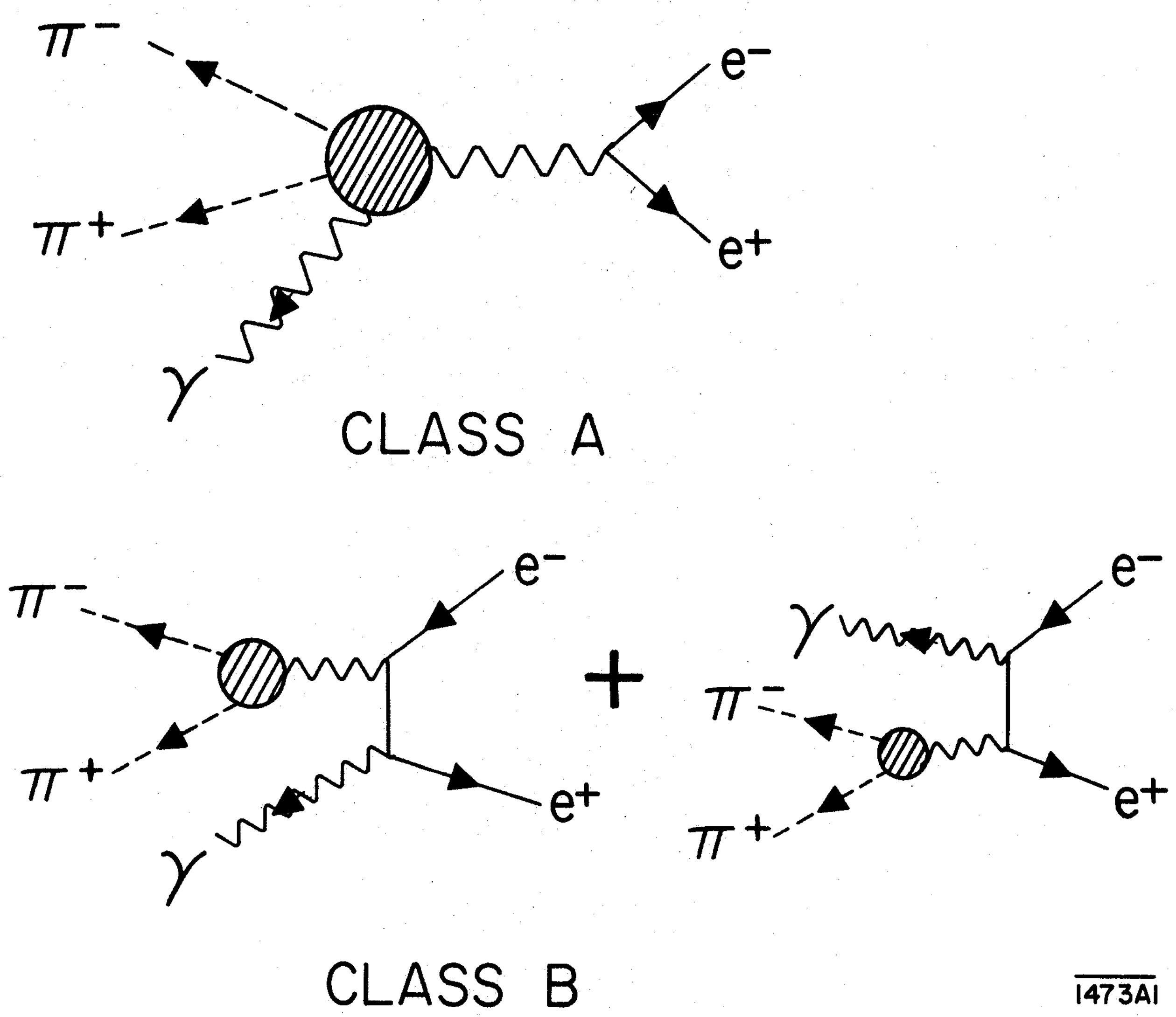

Figure 1 — Classes of diagrams for $e^- + e^+ \rightarrow \pi^+ + \pi^- + \gamma$.

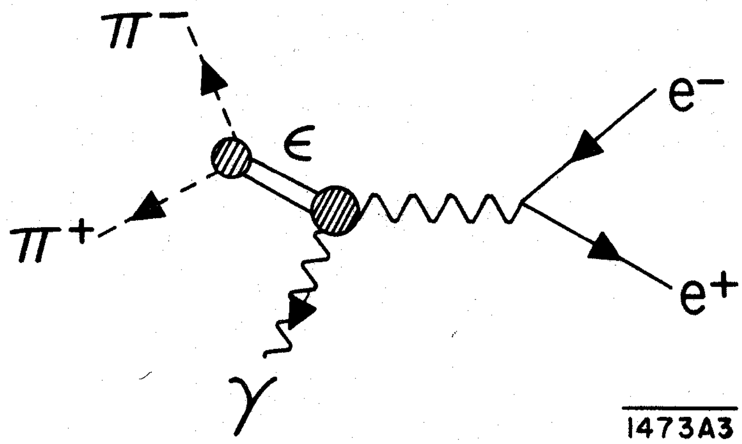

Figure 2 — Contribution of the ϵ-meson.

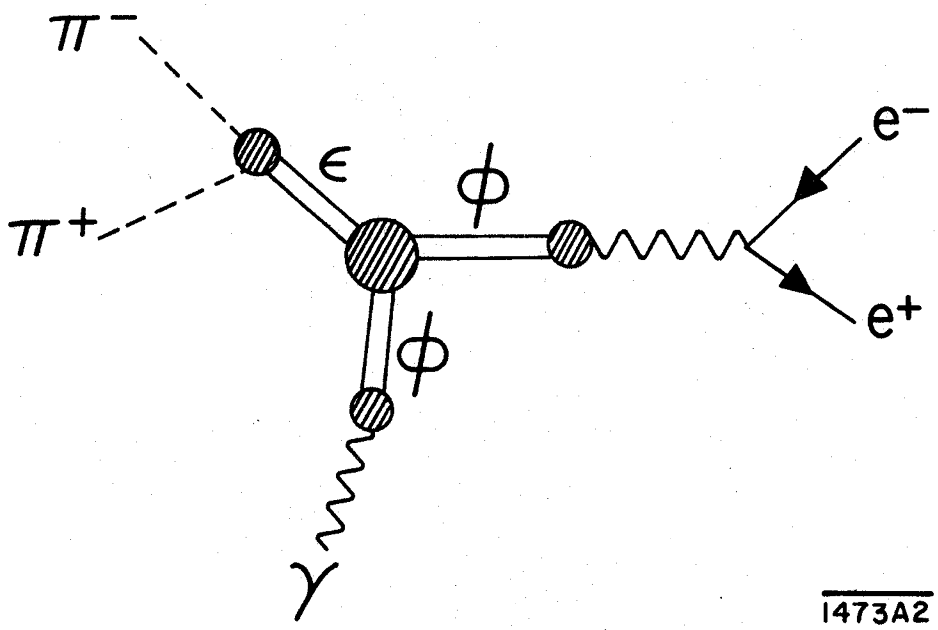

Figure 3 — Vector dominance model.