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STATUS OF QUANTUM ELECTRODYNAMICS*

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I. INTRODUCTION

This is an excellent time for review and reflection on the status of quantum electrodynamics since a great deal of the high energy test program has now been completed, several important new atomic physics measurements have been reported, and the value for α, the fine structure constant, has been canonized. It certainly is time to consider what should be done further, both experimentally and theoretically.

At the simplest level, quantum electrodynamics is restricted to the description of electrons, muons and photons in isolation except for known external electromagnetic sources. Unfortunately many of the important tests of the theory are somewhat influenced by hadron dynamics and the complications are certain to increase in the future. To a certain extent we have a ready scapegoat if any discrepancies show up. Conversely, important limits on hadron physics can be inferred from some of the QED checks.

The basic equations of QED give the impression of being as simple as possible.[1] Because of the efforts of Feynman, Schwinger, Tomonoga and many others, the theory possesses a systematic calculational scheme which includes an incredible, mathematically heuristic, renormalization procedure. Yet over the years it has eventually met every experimental challenge with quantitative success. Its range of application has been extended from atomic (10^{-8} cm) to electron (10^{-11}cm) to nuclear dimensions ($\approx 10^{-14}$ cm). Adding in the classical aspects of Maxwell's equations, the total range covers more than 23 decades to 10^9 cm, where the r^{-3} fall off of the earth's magnetic field has been verified.[2] The basic symmetry properties of QED, conservation laws, scalar constancy of α, c, etc. have all been checked to various degrees of precision.[3]

The plan of this review is as follows:

The high energy tests are discussed in the next section with special emphasis on the colliding beam experiments. The high energy tests are essential for detecting possible new interactions or deviations at high momentum transfer (short distance), but they are only sensitive to the Born diagrams of QED. Tests of the higher order corrections, including those involving renormalization, are realizable because of the very high precision atomic hyperfine and fine structure measurements and the precise determination of the electron and muon anomolous moments. These are discussed in Sections III and IV. In Section V, we discuss some recent speculations about QED and its relevance as a model for hadron dynamics.

II. THE HIGH ENERGY TESTS OF QUANTUM ELECTRODYNAMICS

The verdict as far as the high energy tests are concerned is that Maxwell's equations with the Dirac form of the current for the electron and muon are correct. Specifically we can state that nearly all the predictions for the Born amplitudes are confirmed in detail by some 20 or so experiments in the energy and momentum transfers accessible by current accelerators.

The Born diagrams which are involved in these tests are shown in Fig. 1. In general only the Born terms can be checked. Radiative and higher Born corrections are at the 3% level and (with one exception[4] — an asymmetric electron pair production measurement on lead) play a minor role since normalization and statistical errors are typically 5% or larger.

The nuclear vertex in the Bethe-Heitler graphs can, at least in principle, be completely determined via electron scattering measurements of the elastic and inelastic form factors. The influence of unknown features of the Compton amplitude for virtual photons can be minimized in the pair production and bremsstrahlung tests by symmetric kinematics for the final state.

On the other hand, it is often possible to choose the kinematics to emphasize the Compton amplitude in the resonance regions[4,5]

$$m^2_{e^+e^-} \approx m^2_\rho,\ m^2_\omega,\ m^2_\phi$$

or[6]

$$s \approx m^2_{N*}$$

and even measure its phase by interference with the Bethe-Heitler amplitudes, producing cross sections asymmetric in the interchange of electron and positron. These measurements, off-shoots of the QED tests, have provided leptonic branching ratios of the vector mesons as well as the magnitude and phase of their production amplitudes.

1. Electron-Positron and Electron-Electron Elastic Scattering

A particularly interesting new result is Orsay measurement[7] of e^+e^- large angle elastic scattering at $E_{cm} = \sqrt{s} \approx 1020$ MeV. An absolute measurement of the Bhabha rate was made, relative to the calculable rate for γ-rays emitted along the beam line from the double bremsstrahlung reaction $e^+e^- \to e^+e^- + 2\gamma$. The energy spectrum for the normalization reaction agreed extremely well with the theoretical spectrum, and the agreement between theory and experiment for the elastic scattering was very good. The most convenient method of cataloguing the results of such experiments is to limit the possible modification of the photon propagator in each Born diagram

$$\frac{1}{q^2} \to \frac{1}{q^2} \pm \frac{1}{q^2 - K^2} \tag{2.1}$$

corresponding to adding a positive or negative metric heavy photon pole in the propagator. The negative metric case does not necessarily violate any physical principle like unitarity or macrocausality, as has been recently emphasized by Lee and Wick[8]—and it has the added virtue of providing convergence to calculations of electromagnetic mass shifts of the hadrons (e.g., the n-p mass difference) and radiative corrections to weak decays. We will discuss this further in the last section of this review.

The 95% confidence limit (allowing for systematic errors) for the mass K of a pole in the propagator of either metric from e^+e^- scattering is

$$K > 2.5 \text{ GeV}$$

It should be noted that the experiment was mostly sensitive to modification of the space-like photon propagator.

The e^+e^- experiment is complimentary to the 1968 measurement at the Princeton-Stanford storage rings of e^-e^- elastic scattering at $\sqrt{s} \cong 1100$ MeV.[9] In this experiment the angular dependence of the cross section was tested but not the normalization. The results, assuming a photon modification factor $(1-q^2/K^2)^{-1}$ is

$$K^{-2} = -0.06 \pm 0.06 \text{ (GeV)}^{-2} \qquad \text{(Statistical error only)}$$

which implies with 95% confidence

$$K > 4 \text{ GeV}.$$

The corresponding limit on a possible positive metric heavy photon is

$$K > 2.4 \text{ GeV}.$$

Preliminary results have been announced for two other important e^+e^- annihilation experiments. Annihilation into two photons has been measured at Novosibirsk[9a] and found to agree with QED. A fermion propagator cutoff limit of 1.5 GeV (95% confidence) was obtained. Annihilation of e^+e^- into a muon pair has been measured at Orsay[9b] giving a photon propagator limit of $K > 1.3$ GeV. This provides the best limit on the time-like photon propagator, far exceeding limits established from positronium.

2. Bethe-Heitler Pair Production and Bremsstrahlung

In the last two years several new measurements of pair production and bremsstrahlung of muons and electrons have been reported. All are in agreement with theory within experimental errors. This includes new measurements of symmetric[10] and asymmetric[11] wide angle electron pair production on carbon at CEA, and from Daresbury,[12] the first measurements of wide angle electron pair production on hydrogen. It should also be noted that the total Bethe-Heitler pair production cross section has now been checked to within 1% at energies up to $k = 3.6$ GeV.[13]

The results of all the recent high energy experiments are shown in Table 1. Although the method of parameterizing any breakdown of theory is somewhat arbitrary, we presumably want to keep gauge invariance. A modification of a lepton propagator then requires a corresponding change in the lepton-photon vertex and in general induces multiphoton vertices.[14,15] It turns out that a reasonable modification of the the pair production and bremsstrahlung amplitude should depend on the second or higher even power of the off shell fermion momentum squared. Accordingly all of these experiments have been parameterized according to[16]

$$\frac{\sigma_{exp}}{\sigma_{Th}} = \left(1 \pm \frac{m^4}{\Lambda^4}\right) \qquad (2.2)$$

where $m = m_{e^+e^-}$ or $m_{e\gamma}$, etc. is the invariant mass of the final state. (In the case of symmetric pair production $m^2_{e^+e^-}$ is $\approx$ twice the mass squared of the off shell fermion.) The cutoff limit quoted is the 95% confidence level without considering systematic errors; I have consistently taken the sign of the modification to give the minimum Λ.

3. Tridents — Pair Production by Leptons

A high energy test of a trident cross section, electrons producing muon pairs on carbon, has been reported to this conference by a Northeastern-CEA group.[17] The measurements agree with theory if and only if one allows for interference of the virtual Compton amplitude with the time-like photon Bethe-Heitler amplitudes. The results were consistent with the conventional phenomenological model for the

Compton amplitude based on a diffractively produced rho decaying into mu pairs. A heavy photon pole of mass less than 400 MeV in the time-like propagator is excluded by this measurement, although this result is dependent on the model for the Compton amplitude.

Finally, we note happily that the analysis of the Brookhaven muon trident experiment (the direct production of a muon pair by an incident muon in the field of a lead nucleus) has been completed.[18] In a paper submitted to this conference the Harvard-University of Massachusetts-McGill collaborators report that the observed number of events (89 ± 9.5) agrees well with the theoretical prediction (82 ± 2) for the experimental acceptance. The experiment was sensitive enough to check for the interference of the exchange and direct graphs — theory without interference would predict ~113 events.

Further, a depression of the cross section at low pair mass for the like-charged muons due to the exclusion principle has been confirmed. A distinct depression at low invariant pair mass for Fermi-Dirac particles occurs since the two identical muons have similar directions and energies and therefore similar wave functions. Further details will be discussed by Toner in his talk on muon physics.

Thus, the Born diagrams of QED have been directly confronted by experiment. For anyone looking for that elusive sign of breakdown of the theory at momentum transfers of the order of a nucleon mass, there can be no joy. All the combinations of virtual electron, muon, and photon propagators and vertices have been checked; the possible cutoffs have been pushed to 1 GeV or higher. Figure 2 summarizes the situation using Eq. (2.1) to modify the photon propagator; a quartic modification factor $(1 \pm p^4/\Lambda^4)$ was assumed for the lepton propagators.

4. High Energy Tests of Electron-Muon Universality

One possible origin for the μ-e mass difference might be the existence of new couplings of muon and electron or a coupling special to muons. Nothing of this sort has shown up in the μ-pair or trident experiments discussed above. The ratio of μ-p to e-p elastic scattering has been checked in a Brookhaven experiment to be independent of q^2 up to $(1.1 \text{ GeV/c})^2$,[19,20] although an 8% normalization discrepancy was present. If form factors for the electron and muon vertices are introduced, the results imply to 95% confidence

$$\left|\frac{1}{\Lambda_e^2} - \frac{1}{\Lambda_\mu^2}\right| < (2.04 \text{ GeV})^{-2} \qquad (2.3)$$

As we shall see, the limit on Λ_μ from the anomalous moment measurement is considerably better, ($\Lambda_\mu > 7$ GeV). Thus we can use Eq. (2.3) to establish to 95% confidence[21] $\Lambda_e > 1.8$ GeV.

Electron-muon universality for coupling to time-like photons can be checked from the decay branching ratios of vector mesons into e^+e^- versus $\mu^+\mu^-$. Considerable caution should be taken in combining colliding beam and photoproduction data since the ρ-ω and $\rho\omega\phi$ interference problems are very complicated and the definitions of the resonance spectra are different.[22]

For the ρ meson the branching ratios are[18,22]

$$\frac{\Gamma(\rho \to \mu^+\mu^-)}{\Gamma(\rho \to \text{all})} = (7.9 \pm 2.0) \times 10^{-5}$$

and[22a]

$$\frac{\Gamma(\rho \to e^+e^-)}{\Gamma(\rho \to \text{all})} = (6.5 \pm 1.4) \times 10^{-5}$$

from photoproduction experiments, and

$$\frac{\Gamma(\rho \to e^+e^-)}{(\rho \to \text{all})} = (5.9 \pm 0.7) \times 10^{-5}$$

from an average of colliding beam measurement.[22b,22c]

Preliminary results for the branching ratio of ϕ into muon pairs has been reported by groups from Northeastern[23] and Cornell[24]

$$\frac{\Gamma(\phi \to \mu^+\mu^-)}{\Gamma(\phi \to \text{all})} = \begin{cases} (2.34 \pm 1.01) \times 10^{-4} & \text{Northeastern} \\ (2.1 \pm 0.3) \times 10^{-4} & \text{Cornell (preliminary statistical error only)} \end{cases}$$

These results may be compared with the DESY-MIT[24b] measurement of the e^+e^- branching ratio (from photoproduction of ϕ's)

$$\frac{(\phi \to e^+e^-)}{\Gamma(\phi \to \text{all})} = (2.9 \pm 0.8) \times 10^{-4}$$

and a new result from colliding beam measurements at Orsay[9b,25]

$$\frac{\Gamma(\phi \to e^+e^-)}{\Gamma(\phi \to \text{all})} = (3.73 \pm 0.25) \times 10^{-4}$$

Further experiments are obviously required here, especially a simultaneous measurement of the e^+e^- and $\mu^+\mu^-$ decay modes; this is to be done next year at Daresbury.[26]

We should also record here that muon number is conserved to high accuracy because of the limit[27,28]

$$\frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to e\nu\bar{\nu})} < 0.6 \times 10^{-8}$$

If this conservation law were multiplicative, the reaction

$$e^-e^- \to \mu^-\mu^-$$

would be possible. An upper limit on the cross section has been established at the Stanford storage ring[29]

$$\sigma < 0.67 \times 10^{-32}\ \text{cm}^2 \ (95\%\ \text{conf})$$

If the reaction were to occur through the exchange of a heavy vector photon, the limit on the simplest Dirac coupling is

$$\frac{f^2}{4\pi} \frac{1}{M_V^2} \approx \frac{10^{-3}}{(\mathrm{GeV})^2}$$

Three searches[30-33] for a heavy lepton in the reaction

$$e + p \rightarrow e^* + p \rightarrow e + \gamma + p$$

gave negative results for m_{e^*} in the range 100 MeV to 1300 MeV. This is of course consistent with the lack of deviation from ordinary theory in the wide angle electron pair and bremsstrahlung experiments.[34]

5. Future High Energy Experiments

It is clear that the most interesting tests of QED at high energies will come from the colliding beam elastic scattering and annihilation experiments. Measurements which would be sensitive to high mass modifications of the photon propagator and lepton vertices may be possible at center-of-mass energies $\sqrt{s}$ as large as 6 GeV at DESY and SLAC and even higher at CEA. The small virtual hadron corrections should cause no difficulty and even could be interesting to observe if the interference effects could be measured.

The pair production and bremsstrahlung experiments may be pushed to higher invariant mass with the advent of higher energy accelerators. This task may be somewhat easier since the deep inelastic nucleon form factors will not severely cutoff the cross sections. Certainly the latter tests will be as much a contest to unravel the severe complications due to hadron processes as a test of QED.

III. THE LOW-ENERGY TESTS OF ELECTRODYNAMICS

We next come to the study of the precise atomic physics and static tests of QED. This field has a rather aesthetic sense of its own and its goals are broader than the pursuit of that elusive sign of breakdown of field theory. First, there are small but very interesting hadronic and weak interaction effects buried in these tests which are very much worth pursuing. Second, the hydrogen atom is the fundamental two-body system and perhaps the most important tool of physics; 56 years after the Bohr theory the challenge is still there to calculate its properties (as well as the static properties of the leptons) to the highest accuracy possible.

1. The Determination of the Fine Structure Constant

All of the precision tests of electrodynamics hinge on the value of the fine structure constant α. Remarkably, its most precise determination has not come from the atomic physics measurements, but rather from a combination of values from very diverse fields:

$$\alpha^{-2} = \frac{1}{4Ry_\infty} \frac{1}{\gamma_p'} \frac{\mu_p'}{\mu_B} \frac{2e}{h} \frac{c\Omega_{abs}}{\Omega_{NBS}} \tag{3.1}$$

where the Rydberg Ry_∞, the proton gyromagnetic ratio in water γ_p', the magnetic moment μ_p'/μ_B of the proton (in a water sample) in units of the electron Bohr magneton, and $c\Omega_{abs}/\Omega_{NBS}$ are known to one or two parts per million, and the ratio 2e/h is determined to about 2 ppm via the A. C. Josephson effect in superconductors by Parker, Taylor, and Langenberg.[35,36] Their value of e/h has recently been confirmed by Petley and Morms[37] to within 1 ppm. Also, Clarke[38] has compared the induced steps in Josephson junctions of different materials (Sn, Pb, In) exposed to the same rf field using a sensitive differential method. It was found that the steps occurred at the same voltage, independent of junction material, to within 1 part in 10^8. I understand Clarke's measurements have now been repeated with a resistance in the circuit; this ends speculation that a flux circuit completed through the superconducting volt-meter could have been preventing a potential gradient. The Josephson relation seems to be based on general quantum-mechanical arguments,[36] and since the results are independent of macroscopic parameters it does not seem imprudent to trust the new order of magnitude of precision it gives to e/h over previous determinations.

In a massive effort, to appear shortly in the Reviews of Modern Physics, Taylor, Parker and Langenberg[39] have critically reanalyzed all the experiments relevant to the determination of the fundamental constants and have derived a new least square fit to their values. Their result for the fine structure constant, derived from relations such as (3.1) is

$$\alpha^{-1} = 137.03608 \pm .00026 \quad (1.9 \text{ ppm}) \tag{3.2}$$

(This is the "WQED" value, derived without using any measurements dependent on quantum electrodynamics.) As we shall discuss later, this value of α is consistent with all but one other experimental determination.

Another accomplishment of this group, nearly as valuable, is a complete and careful reassessment of the accuracy of the atomic physics tests of QED including critical criteria (based on statistical and systematic errors) for the evaluation of the one standard deviation limits of the various energy level measurements. This allows a critical confrontation of theory and experiment which does not rely on a vague and inconsistent comparison based on "limits of error." In the following, we shall adopt the Taylor et al., assignment of the one standard deviation error; typically their value was about two-thirds of the limit of error assigned by the experimentalists.

For convenience, we show in Fig. 3 the low-lying level structure of the hydrogen spectrum and characteristic orders of magnitude of the energy intervals.

2. Fine Structure in Hydrogen

The $2P_{3/2} - 2P_{1/2}$ fine structure separation in hydrogen is given by a very simple formula[39]

$$\Delta E_{n=2} = \frac{2Z^2 Ry_\infty (Z\alpha)^2 c}{4n^3} \left\{ \left(1 + \frac{5}{8}(Z\alpha)^2\right)\left(1 - \frac{m_e}{M}\right) + 2a_e\left(1 - \frac{2m_e}{M}\right) - \frac{\alpha}{\pi}(Z\alpha)^2 \log(Z\alpha)^{-2} \right\} \quad (3.3)$$

(As is common, we retain the dependence on Z to indicate the dependence on nucleon charge.) This is just the Dirac-Sommerfield formula corrected for reduced-mass effects[40] and the electron anomalous magnetic moment a_e. The last term, a radiative correction, only amounts to 1.2 ppm and comes from the same terms which give the Lamb shift for the S-states.[41,42] The neglected terms are typically of order 0.3 ppm from neglected radiative and recoil corrections.[39,43]

This is then the most reliable of the theoretical formulae. Using the Taylor et al.,[39] determination of $\alpha^{-1} = 137.03608(26)$, the theoretical prediction accurate to one standard deviation is

$$\Delta E_{n=2}^{th}(H) = 10969.026 \pm .042 \text{ MHz} \quad (3.4)$$

A very novel technique was used by Metcalf, Baird, and Brandenberger[44,45] to determine the fine structure separation directly. They made use of the fact that the angular distribution of resonant scattering of Lyman α radiation on a hydrogen atom changes anomalously when the $2P_{3/2}$ and $2P_{1/2}$ cross (by application of a magnetic field). A precise Zeeman theory[46,47] valid to at least 1 ppm was then used to infer the fine structure interval at zero field. Their result was

$$\Delta E_{n=2}^{exp} = 10969.127 \pm .095 \text{ MHz} \quad (3.5)$$

or using this result to determine α,

$$\alpha_{MBB}^{-1} = 137.0354 \pm .0006 \ .$$

This is a reassuring confirmation. The determination of α from the Dayhoff et al.,[123] measurement of the $2S_{1/2} - 2P_{3/2}$ splitting in deuterium has now been superseded by the new measurements discussed in the next section.

3. The Lamb Shift ($2S_{1/2} - 2P_{1/2}$)

It would be impossible to overestimate the importance of the measurements of Lamb and his co-workers[123] on the development of quantum electrodynamics. It is interesting to note that subsequent measurements of the $2S_{1/2} - 2P_{1/2}$ separation have shown no significant improvement over the Lamb-Dayhoff-Triebwasser-Retherford experiments, although rather new techniques have been developed. It is also rather paradoxical that the Lamb shifts in H and D remain the only tests of QED which are seriously in disagreement with theory.

In Table 2 we list all the experimental measurements relevant to the $2S_{1/2} - 2P_{1/2}$ separation in hydrogen. The first two results are those usually quoted for the Lamb[123] and Robiscoe[133] experiments. The error limits are the one standard deviation errors assigned by Taylor et al., the usually quoted result is $\pm$ 0.10 for a limit of error. These results are now outdated because Robiscoe[134] has discovered a correction to the atomic beams experiments which raises the experimental values for the Lamb shift. His recent measurements have shown that the velocity distribution of atoms which had been assumed previously in calculating the Stark corrections to the line shape was incorrect. This changes the inferred position of the line center and raises his results for the Lamb interval slightly (.04 MHz).

Robiscoe has also applied a corresponding correction to the 1953 result of Triebwasser, Dayhoff and Lamb and we have included his revision on the table, although there may be some question whether this is appropriate. It should be noted that the results of the individual TDL (αe) and (αf) runs become separated by 0.23 MHz if the velocity distribution correction is applied, which is inconsistent with the assigned errors.

There have recently been three new measurements of the large interval ($2P_{3/2} - 2S_{1/2}$) in hydrogen. The measurement by Kaufman et al.,[135] uses the so-called bottle method, where metastable atoms in an rf cavity are excited to the $2P_{3/2}$ state; the signal is the decay radiation of the upper level. The other two new measurements use the level-crossing atomic beam method developed by Lichten and Robiscoe.

These results can be directly compared to the theoretical prediction for the $2P_{3/2} - 2S_{1/2}$ separation (9911.471 ± 0.051 MHz), but it is more enlightening to employ the theoretical value of the fine structure separation to obtain new values for the Lamb interval. The resulting discrepancy with the theoretical value[39,48,49,50]

$$\tilde{S}_{th} = 1057.555 \pm .086 \quad (\pm 3\sigma)$$

seems serious for four out of the five measurements.

The error limit on the theory is meant to represent generous bounds on uncalculated higher order radiative corrections.[57] Certainly further effort should be made to reduce the uncertainty in the $\alpha(Z\alpha)^6 m$ contribution to the Lamb shift.

The other Lamb shifts which have been measured are included in Table 3. The deuteron measurements indicate the same discrepancy with theory as hydrogen. The remaining measurements have considerably larger relative errors than the H and D work. At the least we can hope that further experimental work will be done, not only to obtain further improvements in accuracy, but also to resolve the disagreements among experiments. Certainly, the new discovery of a systematic error by Robiscoe makes one take the disagreement with theory less seriously.

4. The Theory of the Hydrogenic Atom

Although it seems unlikely that the disagreement between the Lamb shift theory and experiment is reflecting any breakdown[51,52] of electrodynamics, it is certainly possible there is a misapplication of the theory. In order to place the possible discrepancy in proper context, let us briefly review the theoretical calculations. If one analyses photon resonant scattering on a hydrogenic atom, one is inevitably led to the problem of determining the eigenvalues of the Bethe-Salpeter equation with the typical kernels shown in Fig. 4.[53] In the limit where the nucleon mass $M \to \infty$, the one-photon exchange kernel gives the Schrödinger equation for non-relativisitc electrons in a Coulomb field. All of the crossed photon kernels are needed to establish the Dirac equation in this limit, with its famous degeneracy of the $2P_{1/2}$ and $2S_{1/2}$ levels. This degeneracy, however, is rather delicate and is removed by any modification of the Coulomb interaction.

In particular, modifications occur from (non-reduced mass) recoil corrections, finite nucleon size R_p, vacuum polarization corrections, as well as the most important contribution to the level shift, the self-energy correction to the bound electron. The latter effectively gives the electron "size." The various contribution terms can be classified in powers of the available small parameters, α, m_e/M, $Z\alpha$, and $Z\alpha m_e R_p = R_p/a_0$, where $Z\alpha$ indicates the dependence on the Coulomb charge. As is so often the case when the Coulomb potential is involved, any expansion in powers of the binding potential must be handled with great care. In fact, the contributions to the level shift from the self-energy corrections such as the Bethe result and the recoil corrections (from one transversely polarized photon exchange) are functions of log $Z\alpha$ and all orders in the Coulomb potential must be calculated. Over the years there has been considerable progress in the solution of the technical problems, especially the work of Erickson and Yennie[48,49] on the self-energy corrections, and Salpeter[54] on the recoil corrections. Also this past year Grotch and Yennie[40] have developed a very convenient effective potential method to handle the m_e/M corrections, and have verified the previous calculations.

The various terms that comprise the theoretical prediction for the Lamb shift in hydrogen are listed in Table 4.[48,49] The only experimentally important term which has not been checked independently is the fourth order self-energy calculation of Soto.[50] All that is required is the α^2/π^2 coefficient of the rms charge radius for the Dirac form factor of the electron vertex; several groups are now checking this contribution.

There has also been progress in the past year using dispersion theory techniques to evaluate the self-energy corrections to the non-relativistic Lamb shift.[55,56] It is possible these methods will eventually give an analytic result for the entire second order self-energy correction (all orders in $Z\alpha$).

From the present data on L_i^{++} and He^+ the most likely anomaly in the theoretical result would depend on Z^4.[57] The lowest order self-energy and vacuum polarization corrections are certainly correct. Accordingly the recheck of the fourth order QED results seems essential. The final Z^4 candidate is the nuclear size correction[58,59] which is simply proportional to the rms proton charge radius

$$r_{ch}^2 \equiv 6 \left. \frac{dG_{E(q^2)}}{dq^2} \right|_{q^2 \approx (Z\alpha m_e)^2} \tag{3.6}$$

Two years ago Barrett et al.,[60] speculated whether this value can be extrapolated from the results of high energy electron-proton scattering. Of course, as long as the form factor is analytic about $q^2 = 0$, all our ideas on hadron dynamics assure us that a linear extrapolation $|q^2| \approx 4m_\pi^2$ to $|q^2| = 0$ is correct. The odd thing is that some measurements at the lowest points in $|q^2|$ $\left(\text{down to } |q^2| = 0.30\ F^{-2} \approx (100\ \text{MeV})^2\right)$ show an increase in the slope of the charge form factor as $|q^2|$ decreased. For example, an Orsay group, Dudelzak et al.,[61] reported (with some embarrassment) that r_{ch} increased from 0.82 ± 0.02f to 0.86 ± 0.03f to 0.92 ± 0.06f as data were progressively restricted to the smallest

$|q^2|$ points. Similar data were reported by Yount and Pine,[62] but this effect was not confirmed by Drickey and Hand[63] and this indicates the anomalous effect was probably statistical. Still, an extrapolated value of $r_{ch} \approx 1.3$f would fix up the Lamb shift discrepancies and would not significantly affect the comparison of theory and experiment in the ground state hyperfine splitting of hydrogen, assuming a "halo-type" charge distribution with 1 to 2% of the proton charge distributed over an rms radius of ~8f. However, a long-tailed distribution could cause a disagreement between the charge distributions inferred from electron scattering and muonic X-rays for light and heavy nuclei. The most precise X-ray spectra have been measured for μ-Bi and μ-Pb but analyses are complicated by muonic Lamb shifts and nuclear polarization. In a preliminary analysis, however, a Stanford-Detroit, Illinois Collaboration[63a] find that comparison of muonic calcium 40 X-rays with electron-calcium scattering probably is sensitive enough to rule out an anomalous distribution of the required size to fit perhaps half or less of the Lamb shift discrepancy. The definitive test, of course, would be precise low momentum-transfer electron-proton scattering measurements.

5. Muonic Hydrogen

The Lamb shift discrepancy could be clarified if the Lamb interval could be measured in the μ-p system, the basic μ-mesic atom. In this fascinating system, the characteristic momentum transfers are 200 times larger than that of ordinary hydrogen. Because of the huge effect of electron pair vacuum polarization the $2S_{1/2}$ level lies well below the $2P_{1/2}$ energy; the interval is 25 times larger than the $2P_{1/2} - 2P_{3/2}$ separation. The vacuum polarization shift is 60 times larger than the proton size correction which in turn is five times larger than the effect of the self-energy radiative correction to the muon.[64] Although the experimental difficulties seem very formidable, new high energy sources of muons may make these studies possible. E. Zavattini[65] of CERN has discussed an experiment where the 1S-2P transition is induced producing an enhancement of the counting rate of the 2P-2S decay. The Lamb shift in high Z hydrogenic ions could also possibly be measured by Bashkin's beam foil method.[66] Such measurements could further isolate the Z dependence of the discrepancy.

6. The Hyperfine Splitting

The theoretical formula for the ground state hyperfine splitting in hydrogenic atoms is[39,67]

$$\nu_{hfs} = \frac{16}{3}\,\alpha^2 R_\infty c \frac{\mu_N}{\mu_B}\left(1+\frac{m_e}{m_N}\right)^{-3}\Big\{1 + a_e + \frac{3}{2}(Z\alpha)^2 + \alpha(Z\alpha)\left(-\frac{5}{2}+\log 2\right)$$

$$+ \frac{\alpha}{\pi}(Z\alpha)^2\left[-\frac{2}{3}\log^2(Z\alpha)^{-2} + \left(\frac{37}{72}+\frac{4}{15}-\frac{8}{3}\log 2\right)\log(Z\alpha)^{-2} + 18.36 \pm 5\right] + \delta_N\Big\} \qquad (3.7)$$

which takes the form of the Fermi frequency modified by corrections due to the electron anomalous moment, v^2/c^2 corrections from the Dirac equation, second order radiative corrections, and nuclear size and polarization contributions. For a proton with a static charge distribution[40,68,69]

$$\delta_p^{static} = -34.4 \pm 0.9 \text{ ppm} \qquad (3.8)$$

Polarization corrections due to an s-channel N* resonance contributing in the two photon exchange diagrams seems to give only a 1 ppm effect.[40] The two photon coupling of the electron to the proton through an axial vector meson is probably negligible because of the experimental equality of e^+ and e^- scattering at high momentum transfer. Current algebra calculations of the polarization also lead to small corrections.[70,71] On the other hand bound state models can be given[72] which would not rule out corrections of the order of 5 ppm or larger.

Experimentally Vessot et al.,[73] obtained

$$\nu_H^{expt} = 1420.405\,751\,7864\ (17)\ \text{MHz}\ (\pm 1.2/10^{13})$$

based on the hydrogen masser technique of Crampton, Kleppner and Ramsey.[74] Using $\alpha^{-1} = 136.03608\ (26)$ one finds

$$\frac{\nu_H^{expt} - \nu_H^{th}}{\nu_H^{expt}} = 2.5 \pm 4.0\ \text{ppm} - \delta_P^{pol}$$

which is consistent with a small polarization correction. Unfortunately, the tremendous precision of ν_H^{expt}/ν_{fermi} cannot be utilized as a check on QED until a better understanding of δ_P^{pol} can be made.

This hadron dynamics problem is avoided by studying the hyperfine splitting of the ground state of muonium (μ^+e^-). The dynamics of this atom is completely specified by quantum electrodynamics and the Bethe-Salpeter equation; unlike positronium the calculation of its energy levels is a tractable problem. The hyperfine splitting of the ground state is still given by Eq. (3.7) but the nuclear correction is just[75,76]

$$\delta_\mu = \frac{-3\alpha}{\pi} \frac{m_e}{m_\mu} \log \frac{m_\mu}{m_e} = -179.7\ \text{ppm} \qquad (3.9)$$

with, of course, zero polarization correction. In a report to this conference the Yale group of Crane, Amato, Hughes, Lazarus, du Pulitz, and Thompson,[77] have extended their measurements of last year, verifying a linear dependence of $\nu_{(\mu e)hfs}$ on buffer gas pressure. Their new result is

$$\nu_{(\mu e)}^{expt} = 4463.248 \pm .031\ \text{MHz}$$

which is consistent with their previous results averaged over high and low field measurements.

A new result has also been reported from the University of Chicago by Ehrlich, Hofer, Magon, Stowell, Swanson and Telegdi.[79] Their method is a variant of the Yale experiment although lower gas pressures were used to reduce uncertainty from extrapolation, and the external Zeeman field B is chosen such that the microwave resonance frequency for the transition $(F, M_f) = (1,1) \rightarrow (1.0)$ is to first order field-independent: $\partial\nu/\partial B = 0$. The result is

$$\nu_{(\mu e)}^{expt} = 4463.317 \pm 0.021 \text{ MHz}$$

which is within 2σ of the Yale result.

The main trouble in comparing with theory is that the muon moment in Bohr magnetons or alternatively the electron/muon mass ratio is not known to sufficient precision. We can use the measured μ_μ'/μ_p' ratio in water if we assume the Ruderman[80] estimate of the correction for diamagnetic shielding of the muon moment:

$$\mu_\mu = (1 + \sigma_\mu)\mu_\mu', \quad \sigma_\mu = 10 \text{ ppm}$$

which is a small correction compared to

$$\mu_p = (1 + \sigma_p)\mu_p', \quad \sigma_p = 26 \text{ ppm}.$$

Then with $\alpha^{-1} = 137.03608\ (26)$,

$$\nu_{hfs}^{th} = 4463.272 \pm .061 \text{ MHz } (\pm 14 \text{ ppm}) \qquad (3.10)$$

which beautifully overlaps both measurements. The great excitement in the muonic X-rays will begin when the muon moment is directly measured. Telegdi expects to do this with a double resonance measurement of the muonium Zeeman structure.[81]

A graphical representation of the agreement between theory expressed in terms of values for α is shown in Fig. 5.[82]

IV. THE ANOMALOUS MOMENTS

1. The Electron Moment

The basic test of quantum electrodynamics is surely the anomalous moment $a_e = (g-2)/2$ of the electron since the experiment can be idealized as a measurement of the static electron in isolation from other dynamics. [Contributions from hadrons come in at the $\alpha^2/\pi^2 \left(m_e^2/m_\pi^2\right) \sim 10^{-9}$ level.] The measurement of Wilkinson and Crane,[83] (corrected ~60 ppm by Rich[84] for relativistic effects and a re-evaluation of corrections to the mean magnetic field acting on the precessing electrons) yields the results

$$a_e^{exp} = 0.001\,159\,557\,(30)$$
$$= 1/2\,(\alpha/\pi) - (0.3285)\,\alpha^2/\pi^2 - (6.4 \pm 2.4)(\alpha^3/\pi^3) \qquad (4.1)$$

if $\alpha^1 = 137.03608\,(26)$.

The sixth order coefficient is uncomfortably large compared to theoretical estimates. This theoretical result consists of $\sim(0.13)\,\alpha^3/\pi^3$ from the Drell-Pagels-Parsons[85,86] dispersion theory estimate of the three-photon corrections and the new results of $(0.055)\,\alpha^3/\pi^3$ by Mignaco and Remiddi[87] for the contribution of the fourth order vacuum polarization[88] to the sixth order moment. (The α^3/π^3 contribution from second order vacuum polarization has not been calculated.)

An interesting feature which does not occur until sixth order is the contribution to the anomalous moment from photon-photon scattering (three-photon exchange). (See Fig. 6.) A numerical result has now been obtained by Aldins, Brodsky, Dufner and Kinoshita.[89] Their result is

$$(\Delta a_e)_{\gamma\gamma} = (0.36 \pm 0.04)\,\alpha^3/\pi^3 \qquad (4.2)$$

It might be noted that these contributions contribute to an electron-photon cut when the magnetic moment is evaluated using a dispersion relation for the vertex in the off-shell fermion mass. Thus it would not be estimated as a small contribution using the procedures of Ref. 85. The error limit represents a 2σ confidence limit on a numerical integration over seven dimensions. Combining this result with the previous calculations, the best current estimate of the α^3/π^3 coefficient is about $+0.5$. Clearly we should wait until the expected new measurements are in and further theoretical work is done before drawing any conclusions about QED. For now we can note that theory and experiment agree at the 70 ppm level and differ in the total moment g/2 only in the eighth significant figure.

The theoretical calculation of the entire sixth order moment does not seem so formidable now since the development of powerful algebraic computer methods by people such as Hearn,[90] Veltman,[91] Levine,[92] Calmet and Perrottet,[93] and Campbell.[94] In the photon-photon scattering contribution of Aldins et al., all the traces, and reduction to Feynman parametric form were done automatically by REDUCE, a LISP-based algebraic computation program developed by A. C. Hearn.[90] The resulting seven dimensional integrand was punched out in Fortran form and then numerically integrated using a novel program (originally written by G. Sheppey at CERN) which on successive iterations, improves the Riemann integration grid through a random variable sampling technique.

2. The Muon Moment

In the last few years increasingly accurate measurements of the magnetic moment of the muon have been performed at CERN. The most recent value of the anomalous part of the muon g factor is[95]

$$\begin{aligned} a_\mu^{exp} &= (116616 \pm 31) \times 10^{-8} \\ &= \alpha/2\pi + 0.76578\ (\alpha^2/\pi^2) \\ &\quad + (49 \pm 25)\ \alpha^3/\pi^3 \end{aligned} \tag{4.3}$$

assuming again $\alpha^{-1} = 137.03608\ (26)$. Theoretically, the difference between the electron and muon moment arises from the typical diagrams shown in Fig. 7. In sixth order all the contributions containing electron-positron vacuum polarization loops of second and fourth order sum to[96-100]

$$(\Delta a)^{6}_{\mathrm{v.p.}} = \left[\frac{2}{9}\left(\log \frac{m_\mu}{m_e}\right)^2 - 1.114 \log \frac{m_\mu}{m_e} + 2.44 \pm 0.5\right]\left(\frac{\alpha}{\pi}\right)^3 \tag{4.4}$$

$$= (2.82 \pm 0.5)\ \frac{\alpha^3}{\pi^3}\ .$$

(There is a rather large numerical cancellation of $\log^2(m_\mu/m_e)$ and $\log(m_\mu/m_e)$ contributions.) The assumption that neglected constants in this result are small has been partially checked by Lautrup and de Rafael.[101]

The photon-photon scattering contribution to $a_\mu - a_e$ has been evaluated numerically by Aldins et al.[102] Using the same method discussed above, the result is

$$(\Delta a_\mu)_{\gamma-\gamma} = (18.4 \pm 1.1)\ \frac{\alpha^3}{\pi^3} \tag{4.5}$$

This large coefficient is a result of a $\log m_\mu/m_e$ dependence of the result for large m_μ/m_e.

$$(\Delta a_\mu)_{\gamma-\gamma} \Rightarrow \left[(6.4 \pm 0.1) \log \frac{m_\mu}{m_e} + \mathrm{const}\right]\left(\frac{\alpha}{\pi}\right)^3 \tag{4.6}$$

The error limits in (4.5) and (4.6) are 2σ confidence limits for seven and five dimensional numerical integrations respectively.

The hadronic vacuum polarization contribution can be exactly related to an integral over the cross section for e^+e^- annihilation into hadrons. The Orsay colliding beam data has been integrated over the ρ, ω, ϕ regions to give the contribution[103]

$$\begin{aligned}(\Delta a_\mu)_{\rho,\omega,\phi} &= (6.5 \pm 0.5) \times 10^{-8} \\ &= (5.0 \pm 0.4)\ (\alpha/\pi)^3\end{aligned} \tag{4.7}$$

Estimates for the weak interaction contribution, although cutoff dependent, give $(\Delta a_\mu)_{weak} \sim 1 \times 10^{-8}$.

The theoretical prediction for $a_\mu - a_e$ is then

$$(a_\mu - a_e)^{(6)} = (27 \pm 3) \frac{\alpha^3}{\pi^3} \tag{4.8}$$

Since the sixth order coefficient of a_e^{th} is less than 1 we can adopt $(27 \pm 3)\, \alpha^3/\pi^3$ as the sixth order result for a_μ^{th}. Remarkably theory and experiment $(49 \pm 25)\, \alpha^3/\pi^3$ agree within one standard deviation.

It should be emphasized that <u>all the QED contributions</u> to $a_\mu - a_e$ have been calculated or estimated through sixth order.

The greatest uncertainty in the calculation of a a_μ arises from further positive contributions from hadrons beyond the ϕ. Theoretical bounds have not proved expecially useful (see Bell and de Rafael[104] for a critique). The best statement we can make is that the present agreement of theory and experiment bound the integral[102]

$$\int_{s>m_\phi^2}^{\infty} \frac{\sigma_{e^+e^- \to hadrons}(s)}{s}\, ds < 8.2\, \mu b$$

i.e., the contribution of all hadrons to the interval (1) is not more than three or four times the contribution of the ρ. Thus the muon moment measurement provides a bound for the electromagnetic couplings to <u>all</u>, hadrons, and indicates that a large portion of the annihilation cross section has already been measured.

Since the uncertainty in the hadronic contribution is in the positive direction we can still use the comparison of theory and experiment to place a 95% confidence limit on a heavy photon of negative metric

$$-\frac{\delta a}{a} = \frac{2}{3} \frac{m_\mu^2}{\Lambda_\gamma^2} < 28\, \frac{\alpha^3}{\pi^3}$$

or

$$\Lambda_\gamma > 5 \text{ GeV}$$

The implications of the agreement between the theoretical and experimental values for the muon moment for various speculative theories has been summarized by Bailey and Picasso.[105]

V. IS QUANTUM ELECTRODYNAMICS A MODEL FOR HADRON PHYSICS

The hypothesis that even hadron physics can eventually be understood in the framework of local field theory has been a lingering hope for many years, a hope buoyed by the successes of quantum electrodynamics and the application to weak interactions. Some recent work has cast a critical light on this question.

(1) There is evidence that mesons and nucleons are sufficiently composite that the direct use of local field theory is inappropriate. Just as one example, the successes of duality amplitudes which describe both s and t channel structure and which, presumably, are reflecting composite structure, support this view. Local fields could still be correct for the constituents; the various successes of current algebras give some hope for this.

(2) The more specific question of whether a local field theory such as quantum electrodynamics can exhibit Regge behavior has been investigated this past year by Cheng and Wu,[106] Chang and Ma[107,108] and Abarbanel and Itzykson.[109] At high energies it is now possible to rather compactly sum large (but restricted) sets of Feynman amplitudes to all orders in perturbation theory. For example, for electron-electron scattering all graphs containing the exchange of arbitrary numbers of photon lines (ladder plus non-ladder) yield a high energy scattering amplitude of the form isf (t) which is definitely not of the Regge form $s^{\alpha(t)}$. However, as Blankenbecler[110] has emphasized, the results of such calculations can be taken

as an effective potential which may be iterated in the t channel (see Fig. 8). There is no reason why some type of Regge behavior will not ensue.

(3) It is reasonable to assume that the observed mass differences between particles in the same isospin multiplet such as the n-p mass difference are due to the electromagnetic interactions.[111] If one neglects the strong interactions one finds that in order α the electromagnetic mass differences are logarithmically divergent. There is now evidence that such infinities will not be removed by the smearing effect of strong interactions. Bjorken[70,71] has shown that the coefficient of the logarithmic divergence is proportional to the expectation value of the equal time commutator of the hadronic electromagnetic current with its derivative:

$$<A \left| \left[\frac{\partial}{\partial t} j_\mu (\vec{x}, t), j^\mu (\vec{x}, 0) \right]_{t=0} \right| A>.$$

In general one cannot infer that this value is zero or constant across the states $|A>$ in an isospin multiplet. The exception is in the gauge field algebra in which the commutator is a c number; however, in the simplest models j_μ is an intermediate boson field which through its coupling again leads to infinities in the mass differences. (As Lee has said, "Such methods remind one of a highly organized bureaucracy where difficulties are transferred but never solved".)[112]

There are, however, possibilities for making the electromagnetic shifts finite in field theory. Perhaps results in second order perturbation theory are infinite but higher order corrections from weak or electromagnetic interactions provide an effective cutoff. Another way out has the attractive feature that all radiative corrections would be finite (even electron mass renormalization) if all this time we have been misjudging the form of the photon propagator at high q^2. As Lee and Wick[8] have proposed, the addition of a negative metric heavy mass pole in the photon propagator

$$\frac{1}{q^2} \to \frac{1}{q^2} - \frac{1}{q^2 - M^2} = \frac{1}{q^2} \left(\frac{M^2}{M^2 - q^2} \right)$$

yields the desired extra convergence factor. (A similar procedure can be carried out to make higher order weak interactions convergent.) If the negative metric particles are unstable then conflicts with unitarity and macrocausality can apparently be avoided. The experimental tests we discussed above [e-e scattering[7,9] and $(g-2)_\mu$[95]] already place a limit on a modification of the above type:

$$M > 5 \text{ GeV} \qquad (95\% \text{ conf}).$$

It is conceivable that some of the known unstable hadrons could be negative metric hadrons. For example, the ρ could be the first of a series of negative metric 1^- mesons the sum of whose couplings could cancel against the divergent couplings of the photon interactions. However, the phase of the Compton amplitude has been measured in the asymmetric pair production[2] and trident[72] experiments and is apparently consistent with the diffraction production of the ρ plus Hermitian couplings.

VI. ON FUTURE PROGRESS

Several people at this conference have suggested that this might be the last summary talk on quantum electrodynamics. I believe that QED will be very much a live subject as long as there are critical discrepancies to resolve, such as those now present in the Lamb shift and the electron anomalous moment, and opportunities to extend its range of validity such as in muonium, muonic hydrogen, and the clashing beam experiments ($e^-e^- \to e^-e^-$, $e^-e^+ \to e^-e^+$, $e^-e^+ \to \mu^-\mu^+$). Finally the complications of hadron dynamics can often be turned to advantage, as witnessed by our increased knowledge of hadronic Compton and polarization amplitudes.

The limit on the positron-electron annihilation cross sections integrated over the entire hadronic spectrum, obtained from the anomalous moment of the muon is surely a remarkable link between QED and the unexplored regions of hadron physics.

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TABLE 1

Recent High Energy Tests of QED

Experiment	References	Parametrization	95% Confidence Limit
$e^- + e^- \to e^- + e^-$ $\sqrt{s} = m_{e^+e^-} \sim 1110$ MeV/c^2	Barber et al. (Stanford-Princeton, 1968)	$F(q^2) = (1 - q^2/K^2)^{-1}$ $F(q^2) = (1 + q^2/K^2)^{-1}$	$K > 2.4$ GeV/c^2 $K > 4.0$ GeV/c^2
$e^+ + e^- \to e^+ + e^-$ $\sqrt{s} = m_{e^+e^-} \sim 1020$ MeV/c^2	Augustin et al. (Orsay, 1969)	$F(q^2) = (1 + q^2/K^2)^{-1}$	$K > 2.6$ GeV
$\gamma + C \to C + e^+ + e^-$ $m_{e^+e^-} \leq 900$ MeV/c^2	Alvensleben et al. (DESY-MIT, 1968)	$\frac{\sigma_{exp}}{\sigma_{Th}} = 1 + m^4/\Lambda^4$	$\Lambda > 1.4$ GeV/c^2
$\gamma + C \to C + e^+ + e^-$ $m_{e^+e^-} \leq 444$ MeV/c^2	Tenenbaum et al. (Harvard, 1969)	$\frac{\sigma_{exp}}{\sigma_{Th}} = 1 + m^4/\Lambda^4$	$\Lambda > 0.8$ GeV/c^2
$\gamma + p \to p + e^+ + e^-$ $m_{e^+e^-} \leq 490$ MeV/c^2	Biggs et al. (Daresbury, 1969)	$\frac{\sigma_{exp}}{\sigma_{Th}} = 1 + m^4/\Lambda^4$	$\Lambda > 0.7$ GeV/c^2

[See also Cohen et al., MIT, 1969; Eisenhandler et al., Cornell, 1967]

(Table 1 con't.)

Experiment	References	Parametrization	95% Confidence Limit
$\gamma + C \rightarrow C + \mu^+ + \mu^-$ $m_{\mu^+\mu^-} \leq 1225$ MeV/c^2	Hayes et al. (Cornell, 1969)	$\frac{\sigma_{exp}}{\sigma_{Th}} = 1 - m^4/\Lambda^4$	$\Lambda > 1.5$ GeV/c^2
[See also dePagter et al., Northeastern, 1966; Quinn et al., Stanford, 1968]			
$e^- + C \rightarrow e^- + C + \gamma$ $m_{e\gamma} \leq 1030$ MeV/c^2	Siemann et al. (Cornell, 1969)	$\frac{\sigma_{exp}}{\sigma_{Th}} = 1 + m^4/\Lambda^4$	$\Lambda > 1.5$ GeV/c^2
$\mu + C \rightarrow \mu + C + \gamma$ $m_{\mu\gamma} \leq 650$ MeV/c^2	Liberman et al. (Harvard-Case-McGill)	$\frac{\sigma_{exp}}{\sigma_{Th}} = 1 + m^4/\Lambda^4$	$\Lambda > 0.7$ GeV/c^2
[See also Bernardini et al., Frascati, 1967]			

TABLE 2

The Lamb Shift in Hydrogen (in MHz)

$(2S_{\frac{1}{2}} - 2P_{\frac{1}{2}})$: $\mathscr{S}_{Th} = 1057.555 \pm 0.086$ ($\pm 3\sigma$)

Reference	$\mathscr{S}_{exp}$ ($\pm 1\sigma$)	Disc. ($\pm 1\sigma$)
Triebwasser, Dayhoff, Lamb (TDL, 1953)	1057.77 ± 0.06	0.21 ± 0.07
Robiscoe and Cosens (RC, 1968)	1057.86 ± 0.06	0.30 ± 0.07
TDL (Revised by RC, 1969)	1057.86 ± 0.06	0.30 ± 0.07
RC (Revised, 1969)	1057.90 ± 0.06	0.34 ± 0.07
Kaufman, Lea, Leventhal, Lamb (1968)	1057.65 ± 0.05	0.09 ± 0.06
$\Delta E - \mathscr{S} = 9911.377 \pm 0.026$		
Shyn, Williams, Robiscoe, Rebane (1969)	1057.82 ± 0.07	0.26 ± 0.08
$\Delta E - \mathscr{S} = 9911.213 \pm 0.058$		
Vorburger and Cosens (1969)	1057.85 ± 0.06	0.29 ± 0.06
$\Delta E - \mathscr{S} = 9911.176 \pm 0.050$		

Note: $\alpha^{-1} = 137.03608 \pm 0.00026$

$(2P_{\frac{3}{2}} - 2P_{\frac{1}{2}})$: $\Delta E_{Th} = 10969.026 \pm 0.042$

$(2P_{\frac{3}{2}} - 2S_{\frac{1}{2}})$: $(\Delta E - \mathscr{S})_{Th} = 9911.471 \pm 0.051$

TABLE 3

Other Lamb Shifts (in MHz)

	Theory ($\pm 3\sigma$)	Experiment ($\pm 1\sigma$)	Discrepancy ($\pm 1\sigma$)	Reference
D	1058.82 ± 0.14	1059.00 ± 0.06	0.18 ± 0.08	Treibwasser, Dayhoff, Lamb (1953)
(n = 2)		1059.24 ± 0.06	0.42 ± 0.08	Cosens (1968)
H_e^+	14038.9 ± 4.1	14040.2 ± 1.8	1.3 ± 2.2	Lipworth, Novick (1957)
(n = 2)		14045.4 ± 1.2	6.5 ± 1.8	Narasimham (1968)
H_e^+ (n = 3)	4182.7 ± 1.2	4181.7 ± 1.0	1.0 ± 1.5	Mader, Leventhal (1968)
H_e^+	1768.34 ± 0.51	1766.0 ± 7.5	-2.3 ± 7.5	Hatfield, Hughes (1957)
(n = 4)		1768.0 ± 5.0	0.0 ± 5.0	R. Jacobs (1969)($\Delta E-\mathcal{S}$)
L_i^{++}	62743.0 ± 45.0	63031.0 ± 327.0	288.0 ± 3330.0	C. Fan, M. Garcia-Munoz, I. A. Sellin (1967)
Hg	41 Ry			Brown and Mayer (1959)
(K-shell shift)		25.0 ± 9.0	16.0 ± 9.0	Coulthand (1967)

TABLE 4

VARIOUS CONTRIBUTIONS TO THE LAMB SHIFT IN H

DESCRIPTION	ORDER	MAGNITUDE (MHz)
2^{nd} ORDER — SELF-ENERGY	$\alpha(Z\alpha)^4 m\{\log Z\alpha, 1\}$	1079.32 ± .01
($\alpha^{-1} = 137.0361$)	$\alpha(Z\alpha)^5 m$	7.14
	$\alpha(Z\alpha)^6 m\{\log^2 Z\alpha, \log Z\alpha, 1\}$	- 0.38 ± .04
2^{nd} ORDER — VAC. POL.	$\alpha(Z\alpha)^4 m$	- 27.13
4^{th} ORDER — SELF-ENERGY	$\alpha^2 (Z\alpha)^4 m$ $F_1'(0)$	0.10
	$\alpha^2 (Z\alpha)^4 m$ $F_2(0)$	- 0.10
	$\alpha^2 (Z\alpha)^5 m$	± .02
4^{th} ORDER — VAC. POL.	$\alpha^2 (Z\alpha)^4 m$	- 0.24
REDUCED MASS CORRECTIONS	$\alpha(Z\alpha)^4 \frac{m}{M} m\{\log Z\alpha, 1\}$	- 1.64
RECOIL	$(Z\alpha)^5 \frac{m}{M} m\{\log Z\alpha, 1\}$	0.36 ± .01
PROTON SIZE	$(Z\alpha)^4 (mR_N)^2 m$	0.13
		1057.56 ± .08

FIGURE CAPTIONS

1. Feynman amplitudes contributing to the high energy tests of quantum electrodynamics. In the pair production, bremsstrahlung, and trident Bethe-Heitler diagrams, the photon from the nucleon vertex attaches at each of the open circles. In the case of $\mu + A \rightarrow \mu + \mu^{+} + \mu + A$, there are eight Bethe-Heitler diagrams including those obtained by interchanging the final leptons of like charge. The virtual Compton amplitude on the nucleus is represented by the cross-hatched diagram.

2. Composite picture of the high energy measurements, with 95% confidence limits (in GeV), on possible modifications of the photon and lepton space-like and time-like propagators. See Table 1 and text.

3. The n = 1 and n = 2 level structure of atomic hydrogen.

4. Typical Bethe-Salpeter kernels for the hydrogen atom.

5. Graphical comparison[39] of several least-squares adjusted values of α^{-1} with values obtained from QED experiments.[39] Note that two different values are given for the Kaufman, Lamb, Lea and Leventhal[135] measurement of $\Delta E_H - \mathcal{S}_H$. The point labelled "($\Delta E_H - \mathcal{S}_H$)" was obtained by combining the KLLL results with the average of the Triebwasser, Dayhoff, and Lamb,[123] and Robiscoe[133] measurements of $\mathcal{S}_H$ and then calculating α^{-1} from the theoretical equation for ΔE_H. The point labelled "$\Delta E_H - \mathcal{S}_H$" was calculated directly from the KLLL result using the theoretical equation for $\Delta E_H - \mathcal{S}_H$. The errors for the KLLL and Metcalf, Brandenberger, and Baird[44] (MBB) values are based on the uncertainties assigned by the experimenters. The value for α used prior to 1967 is from Cohen and DuMond.[82]

6. Photon-photon scattering contribution to the 6th order anomalous moment of the electron.

7. Representative contributions to the difference of muon and electron anomalous moments.

8. Electron-electron scattering. The ladder and non-ladder photon exchange amplitudes in the s-channel are summed to an effective potential V. Further Feynman amplitudes are obtained by iterating V in the t-channel.